ECE 4370: Antenna Engineering
Solutions to TEST 1 (Fall 2012)

1. **Basic Radiating System:**

   (a) 3.75 cm

   (b) Magnetic Field Solution:

$$\tilde{\vec{\mathrm{H}}}(r,\theta,\phi) = \begin{cases} \frac{\pi I}{r}\exp(-jkr)\hat{\phi} & \text{for } -\frac{\pi}{8} \le \phi \le \frac{\pi}{8} \text{ and } \frac{\pi}{4} \le \theta \le \frac{3\pi}{4} \\ 0 & \text{elsewhere} \end{cases}$$

   (c) Poynting Vector Solution:

$$\vec{\mathrm{S}}_{av}(r,\theta,\phi) = \begin{cases} \frac{\pi^2 I^2 \eta}{2r^2}\hat{\mathrm{r}} & \text{for } -\frac{\pi}{8} \le \phi \le \frac{\pi}{8} \text{ and } \frac{\pi}{4} \le \theta \le \frac{3\pi}{4} \\ 0 & \text{elsewhere} \end{cases}$$

   (d) First compute total radiated power:

$$\begin{aligned} P_T &= \int_{\pi/4}^{3\pi/4}\int_{-\pi/8}^{+\pi/8} r^2 \|\vec{\mathrm{S}}_{av}\| \sin\theta \, d\theta \, d\phi \\ &= \int_{\pi/4}^{3\pi/4}\int_{-\pi/8}^{+\pi/8} r^2 \left[\frac{\pi^2 I^2 \eta}{2r^2}\right] \sin\theta \, d\theta \, d\phi \\ &= \frac{\pi^3 I^2 \eta \sqrt{2}}{8} \end{aligned}$$

   Directivity is then the radiated power relative to the isotropic power:

$$D(\theta,\phi) = \frac{\vec{\mathrm{S}}(\theta,\phi)}{P_T/(4\pi r^2)} = \frac{16}{\sqrt{2}} \text{ for } -\frac{\pi}{8} \le \phi \le \frac{\pi}{8} \text{ and } \frac{\pi}{4} \le \theta \le \frac{3\pi}{4}$$

   (e) 10.5 dBi

   (f) $\theta_{\text{HPBW}} = 90^\circ$ and $\phi_{\text{HPBW}} = 45^\circ$ (the sector widths)

   (g) Radiation Resistance:

$$R_{rad} = \frac{2P_T}{I^2} = \frac{\pi^3 \eta \sqrt{2}}{4} = 418\,\Omega$$

   (h) Received power:

$$\begin{aligned} P_R &= \underbrace{P_T + G_T}_{30\text{ dBm}} + \underbrace{G_R}_{0\text{ dBi}} \underbrace{- 20\log_{10}\left(\frac{4\pi}{\lambda}\right)}_{50.5} \underbrace{- 20\log_{10}(r)}_{60} \\ &= -80.5\,\text{dBm} \end{aligned}$$

(i) $D = \sqrt{r_{\text{ff}}\lambda} \approx 13.7$ cm

(j) z-directed current radiators are omnidirectional with respect to azimuth (no dependence on ϕ)

(k) The radiation pattern is exactly zero over a large portion of continuous space outside the sector; mathematically impossible for a finite antenna.