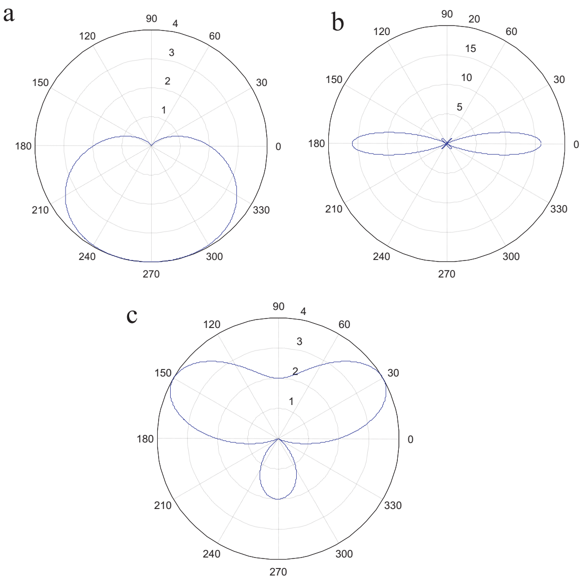
ECE 4370: Antenna Engineering
Solutions to TEST 2 (Fall 2012)

1. **Linear Antenna Arrays:** Below are the true patterns for each array factor. Full credit was given for anything remotely close.



Note: correct formula for array factors is

$$|AF| = \left| \frac{\sin\left(\frac{\pi}{\lambda} N d \sin\phi - N\beta/2\right)}{\sin\left(\frac{\pi}{\lambda} d \sin\phi - \beta/2\right)} \right| \quad \text{for } \theta = 90^\circ \qquad G_{\text{array}}(\theta,\phi) \propto G_{\text{element}}(\theta,\phi) \times |AF(\theta,\phi)|^2$$

2. **Broadband Antenna:**

   (a) According to our lecture on self-complimentary antennas, $Z_0 = \eta/2 = 189\,\Omega$.

   (b) The absolute size of the longest antenna dimension; at wavelengths of this scale, the structure is no longer self-complimentary.

3. **Directional Antennas:**

   (a) The phase distortion criteria for pyramidal horns is given as

$$A = \sqrt{3\lambda R_p + \frac{9\lambda^2}{16}} \qquad B = \sqrt{2\lambda R_p + \frac{\lambda^2}{4}}$$

   Note that the limit of large horn with respect to wavelength, λ, becomes

$$\lim_{R_p \gg \lambda} \frac{A}{B} = \lim_{R_p \gg \lambda} \frac{\sqrt{3\lambda R_p + \frac{9\lambda^2}{16}}}{\sqrt{2\lambda R_p + \frac{\lambda^2}{4}}} = \sqrt{\frac{3}{2}} = 1.22$$

(b) Keeping in mind that $\lambda = 0.06$ m at 5 GHz and that $A = \sqrt{3/2}B$:

$$G = 27\,\text{dBi} = 500 = \frac{4\pi \overbrace{A_{em}}^{AB}}{\lambda^2} \times 50\%$$

which implies $A = 59.0$ cm, $B = 48.4$ cm, and $R_p = 1.93$ m.

(c) For a long axial mode helix:

$$G = 500 = 15N \tan\alpha \longrightarrow N = 145$$

The circumference would be λ, implying a diameter of 1.9 cm and a total length of 2.0m. It is interesting to note that, for the same directivity, the helix winds up being almost the same length as the horn antenna.

(d) The helical antenna is circularly polarized while the pyramidal horn is linearly polarized.

4. **Electrically Small Antennas:** The two separate antennas would have $Q \approx 4.875$ and $a \approx 0.036$ m (700-860 MHz) and $Q \approx 4.12$ and $a \approx 0.0136$ m (1.92-2.45 GHz). The broadband antenna would require $Q \approx 0.9$, which would be achievable with $a \approx 0.03$ m. So the single antenna design would be the most compact design; this would not necessarily be true if the lower band had a higher Q-factor. The astute student also recognizes the the single antenna is slightly larger than the $\lambda/12$ threshold for the Chu-Harrington limit – which does not change the final answer (the broadband antenna can be made smaller than the sum of the two other antennas).