AAT1: Rayleigh Fading

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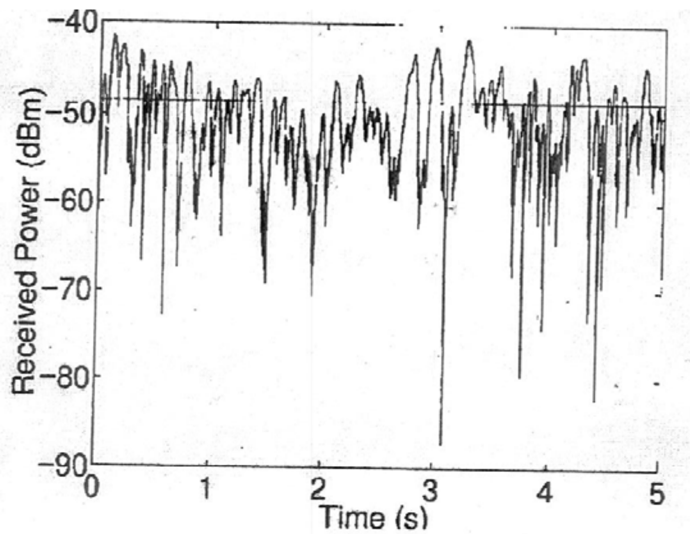
Example of Small-Scale Fading at 5.8 GHz



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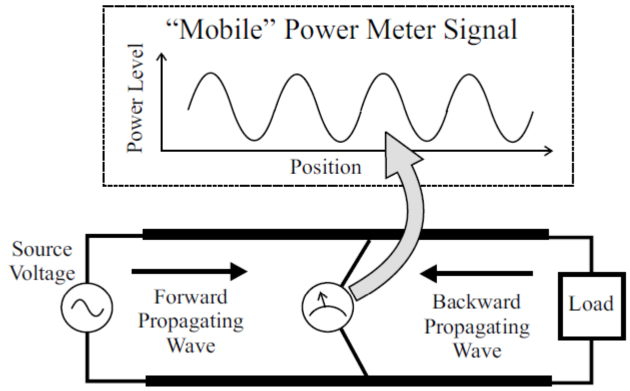
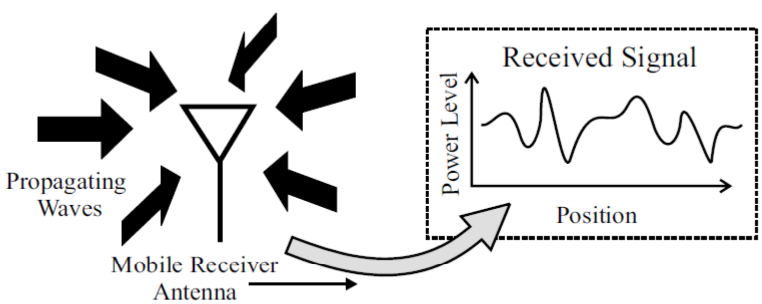
Wave Interference on a Transmission Line



Small-Scale Fading in a Moving Antenna



Physical Model for Fading

Voltage at Terminals of Antenna

$$\tilde{V}(\vec{r}) = A\hat{x}\cdot\left[\sum_{i=1}^{N} E_i\,\hat{x}\exp\left(j\left[\Phi_i - k\hat{k}_i\cdot\vec{r}\right]\right)\right]$$

$$= \sum_{i=1}^{N} V_i \exp\Big(j\underbrace{\left[\Phi_i - \tfrac{2\pi}{\lambda}\hat{k}_i\cdot\vec{r}\right]}_{\Phi_i \text{ - random variable uniformly distributed over } [0,\,2\pi)}\Big)$$

$$= \sum_{i=1}^{N} V_i \exp(j\Phi_i)$$

$\boxed{\text{the local area approximation}}$

Power {Volts-squared}: $|\tilde{V}(\vec{r})|^2 = P(\vec{r})$

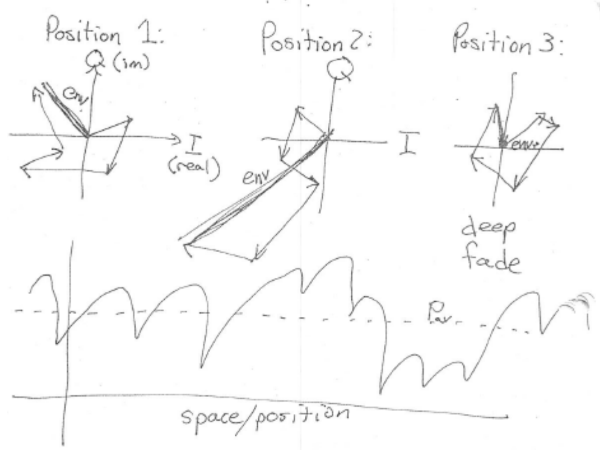
depends on equipment hooked to the antenna.

Envelope {Volts}: $|\tilde{V}(\vec{r})| = R(\vec{r})$

Power Mean: $E\{|\tilde{V}(\vec{r})|^2\} = \sum_{i=1}^{N} V_i^2$

Constructive and Destructive Interference

What does this tell us?



Review of PDFs

F - mathematical distribution describing the outcome of a random experiment.

R → envelope, outcome of an experiment (r.v.)

$f_R(\rho) =$ PDF.

$f_R(\rho) \geq 0$ always positive

$\int_A^B f_R(\rho)d\rho =$ Probability $A \leq R < B$

$\int_{-\infty}^{\infty} f_R(\rho)d\rho = 1$ max probability

Histogram – "Binned" plot ~~pdf~~ experimental outcomes that take on the shape of a PDF.

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Model for Rayleigh Fading

Back to Original problem

$$\tilde{V}(\vec{r}) = \sum_{i=1}^{N} V_i \exp(j\Phi_i)$$

Law of Large Numbers

$$= \underbrace{\sum_{i=1}^{N} V_i \cos\Phi_i}_{\text{zero-mean Gausian, } \sigma} + j \underbrace{\sum_{i=1}^{N} V_i \sin\Phi_i}_{\text{zero-mean Gausian, } \sigma}$$

In envelope

$$f_R(\rho) = \frac{\rho}{\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) u(\rho)$$

$$f_P(p) = \frac{1}{P_{av}} \exp\left(\frac{-p}{P_{av}}\right) u(p)$$

Rayleigh Fading

Is this a PDF?

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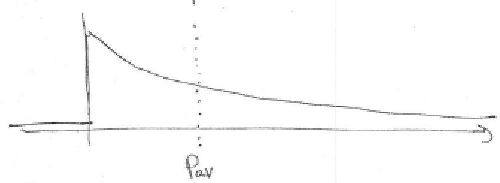
Rayleigh PDF

$$f_\rho(\rho) = \frac{1}{P_{av}} \exp\left(\frac{-\rho}{P_{av}}\right) u(\rho)$$

Rayleigh Fading

Is this a PDF?



✓ always positive

$$\frac{1}{P_{av}}\int_0^\infty \exp\left(\frac{-\rho}{P_{av}}\right) d\rho = -\exp\left(\frac{-\rho}{P_{av}}\right)\Big|_0^\infty = [0-(-1)]$$

$$= 1 \checkmark$$

Example: Cellular Fading

~~sample~~

A digital comm link operates in a local area of $P_{av} = -86$ dBm. The average noise + interference in this area does not fade and has $P_N = -108$ dBm. If the phone requires an SINR of at least 14 dB, how often would one lose the link?

$$P_{av} = 10^{\frac{-86}{10}} = 2.5\times 10^{-12}\ \mathrm{W}$$

Solution Using Rayleigh PDF

one lose the link? $P_{av} = 10^{\frac{-86}{10}} = 2.5\times10^{-12}$ W

$$f_p(p) = \frac{1}{2.5\times10^{-12}} \exp\left(\frac{-p}{2.5\times10^{-12}}\right)$$

$$\Pr\left[P < 4.0\times10^{-13}\,W\right] = \int_0^{4\times10^{-13}} f_p(p)\,dp$$

$$P_{Th} = -108\text{ dBm} + 14\text{ dB} = -94\text{ dBm}$$

$$= -\exp\left(\frac{-p}{2.5\times10^{-12}}\right)\Bigg|_0^{4\times10^{-13}}$$

$$= 0.148$$

or

14.8% of the time

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