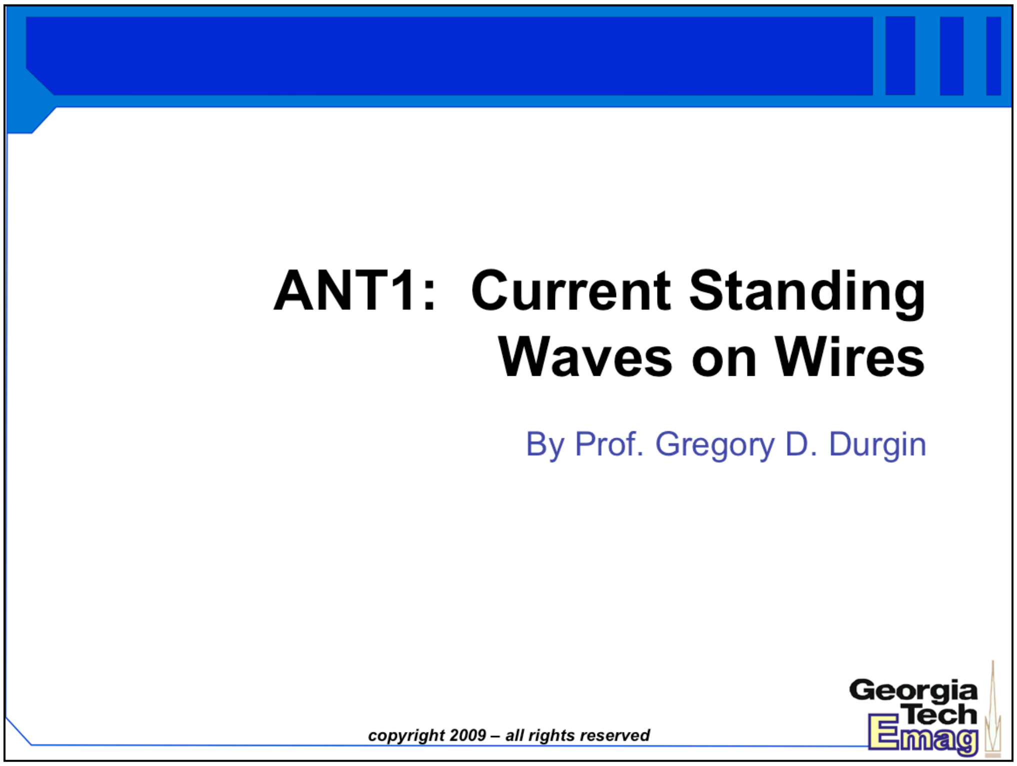
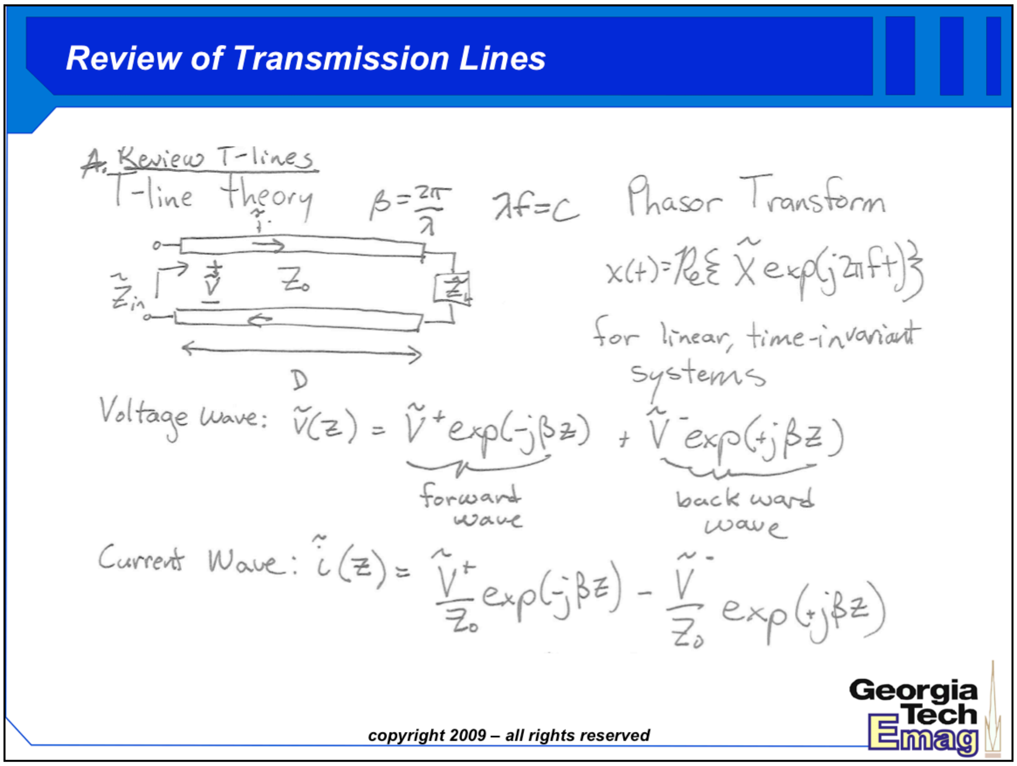


In this module, we review the standing wave current and voltage solutions that exist on an open-circuited transmission line. We discuss how transmission lines, in general, confine and transport fields. By modifying the geometry of an open circuited transmission line, we demonstrate how currents can constructively lead to radiated fields.



Recall transmission line theory, which we use to model signal transport in circuits over distances that are electromagnetically significant (with respect to a wavelength). The above diagram is the standard "two bar" representation of a transmission line in our circuit diagrams. This abstraction represents two conductive paths for a voltage drop and current waveform to traverse. That path could be a coaxial cable (with inner and outer conductor), a microstrip line above a ground plane on a printed circuit board, or it could literally be two bars of metal.

Signals on the line are characterized by a total voltage and current as a function of observation distance, z, down the length of the line. These total voltage and current functions of position consist of two parts: a forward-propagating and a backward-propagating waveform. The mathematical forms of these waveforms are shown above. The V+ is the phasor (amplitude and phase) of the forward propagating wave in units of volts and the V- is the amplitude and phase of the backward propagating wave. The beta is the wavenumber of the time harmonic system – 2pi/wavelength (rad/m) – and Z_0 is the intrinsic impedance of the line (Ohms).

The only other parameter we need to know to characterize the line is its physical length, D (m).

Basics of Reflection

Boundary Condition: $\tilde{Z}_L = \left.\frac{\tilde{V}(z)}{\tilde{I}(z)}\right|_{z=\text{end}}$ $\qquad \tilde{\Gamma}_L = \frac{\tilde{Z}_L - Z_0}{\tilde{Z}_L + Z_0}$

Load transformation

$$\tilde{Z}_{in} = Z_0 \frac{\tilde{Z}_L + jZ_0 \tan\beta D}{Z_0 + j\tilde{Z}_L \tan\beta D}$$

Sanity check

$$\left.\tilde{Z}_{in}\right|_{D=0} = \tilde{Z}_L$$

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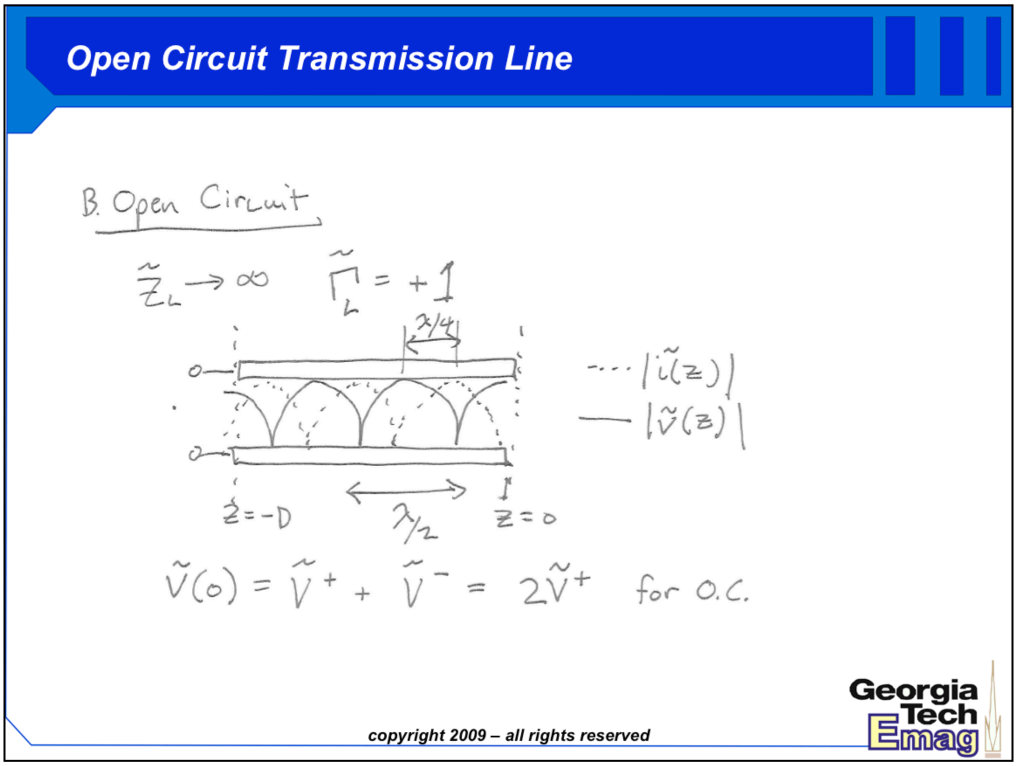
Transmission lines with forward propagating waves will spawn backwards propagating waves if they are terminated in loads that do not have the same impedance as Z_0, i.e. loads that accept voltage and current in a different ratio than the natural ratio supported by the geometry and material of the transmission line. The load reflection coefficient Gamma_L is given above.

One immediate consequence of reflection on the transmission line is load transformation. A load Z_L terminating a transmission line will no longer look like Z_L if observed at a point D from the end of the line. The equation above allows an engineer to calculate the Thevenin equivalent impedance, Z_in, for a load, Z_L, as seen from a transmission line of intrinsic impedance Z_0 at a distance D from the load. As the electrical length D increases, loads vacillate between complex values that are capacitive, real, inductive, and real again.



To introduce the concept of a radiative system, we start with the open circuit, in which the reflection coefficient is +1; all voltage is reflected from the open-circuited transmission line. This makes perfect sense since there is no other place for the power to go.

One consequence of a +1 reflection coefficient is that the two opposite-traveling, equal-amplitude voltage and current waves on the line constructively and destructively interfere with one another, creating a regular pattern of constructive peaks and destructive nulls that repeats itself every half-wavelength. The first peak of the total voltage occurs at the end of the line while the first *null* of the total current occurs at the end of the line; the constructive-destructive interference pattern for current and voltage are identical, just a quarter-wavelength offset in phase.

Standing Waves

$$\tilde{V}(z) = \tilde{V}^+ \exp(-j\beta z) + \tilde{V}^+ \exp(+j\beta z)$$

$$= 2\tilde{V}^+ \left[\underbrace{\frac{\exp(+j\beta z) + \exp(-j\beta z)}{2}}_{\cos\beta z} \right]$$

$$= 2\tilde{V}^+ \cos\beta z$$

standing wave

$$\tilde{i}(z) = \frac{\tilde{V}^+}{Z_o} \exp(-j\beta z) - \frac{\tilde{V}^+}{Z_o} \exp(+j\beta z)$$

$$= -j\frac{2V^+}{Z_o} \left[\underbrace{\frac{\exp(+j\beta z) - \exp(-j\beta z)}{j2}}_{\sin\beta z} \right]$$

$$= -j\frac{2V^+}{Z_o} \sin\beta z$$

2

Here we derive the actual mathematical descriptions of total voltage and current waveforms on an open-circuited transmission line. Note that both solutions result in standing waves with amplitudes that follow sinusoids. The only difference between current and voltage is that total current follows a sine wave and total voltage follows a cosine wave.

Voltage Standing Wave Ratio

VSWR = Voltage Standing Wave Ratio

$$= \frac{V_{max}}{V_{min}} = \frac{1 + |\tilde{\Gamma}_L|}{1 - |\tilde{\Gamma}_L|}$$

$= \infty$ (dB or linear) for open circuit.

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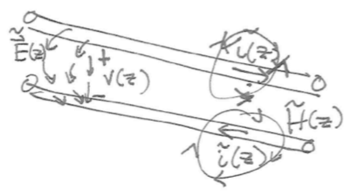
One measure of a transmission line reflectance is the voltage-standing wave ratio, VSWR, which is the ration of the total voltage (or current) maximum to its minimum. It may be expressed in terms of reflection coefficients using the expression above. Of course, for the open-circuited system described above, the VSWR is infinity – in linear or log scale.

Field Theory of Transmission Lines

C. Field Theory

T-lines are abstractions, but say ~~we~~ really do have two wires ending in an OC.



There are corresponding 3D $\tilde{E}$ and $\tilde{H}$ fields around any given T-line topology.

Note: Good T-lines confine fields

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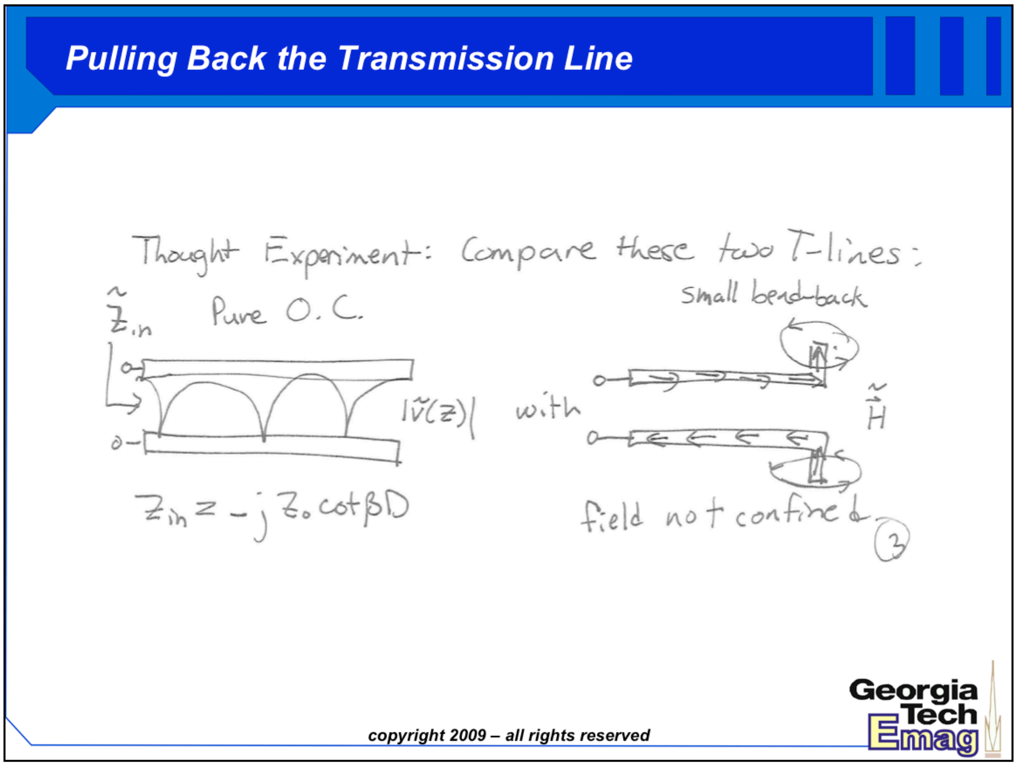
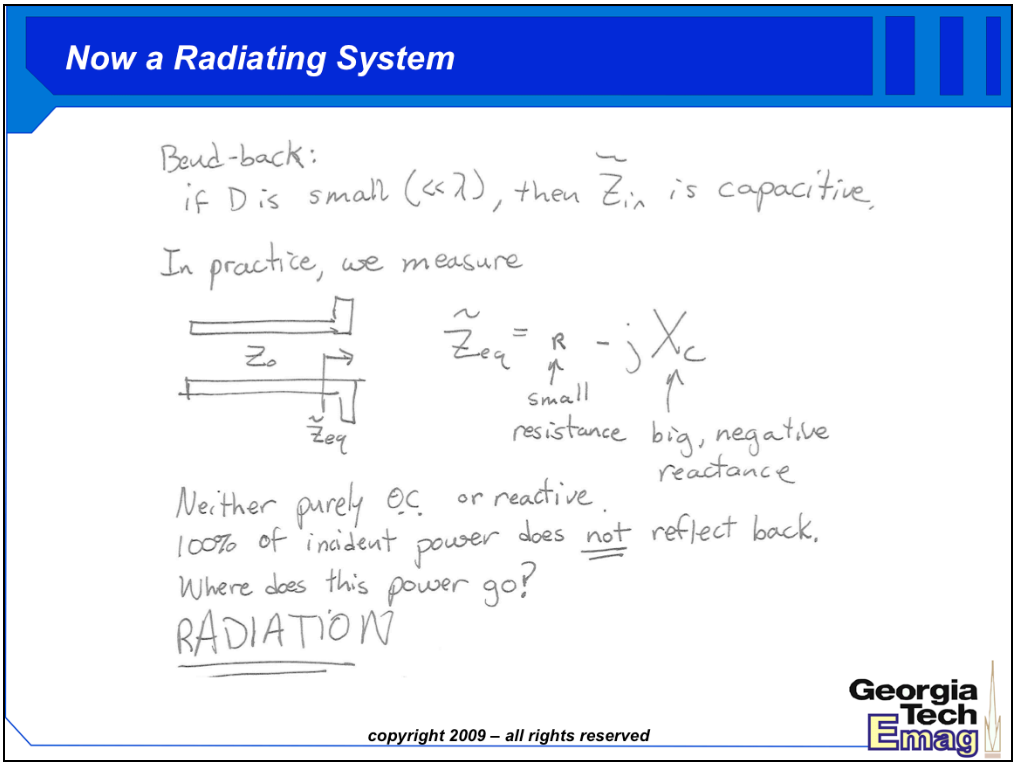
We always need to recognize that voltages and currents are abstractions of more fundamental field quantities that describe the physics of charge motion on a transmission line. Each voltage drop is really a collection of electric fields that surrounds the wires of a two-conductor transmission line. Each current is really also a collection of magnetic fields that circulate around the wires of the transmission line. These fields are strongest between the conductive pathways and much weaker outside the system. That is, after all, what a transmission line is *supposed* to do – confine fields so that the transport of a signal neither makes electrical interference for other circuits nor accepts interference from other sources.



Now consider a two-wire transmission line that we bend back the very end of the two conductors as shown above. If the same current pattern is preserved, then there will be a pair of small, tapered currents on the end of the line that no longer confine magnetic fields. Rather, the magnetic fields resulting from these vertically-aligned currents now *constructively* interfere to the left and right (and all about the azimuth) in the diagram above. This leads to radiation.



Normally, if this small bend-back were not radiating, we would expect to measure a purely reactive (capacitive) load at the end of the transmission line – a large X_c value. However, in practices, there would be a small resistive portion as well. This resistance does not represent (entirely) Ohmic losses. Rather, the resistance represents power lost to traveling waves as the short, bent-back section of transmission line tries to complete the circuit with displacement currents (changes in electric field).