In this module, we study the solution to the simplest radiative system – the Hertzian dipole. Once solved, this simplest of radiating systems is used to introduce the concept of directivity, gain, radiation impedance, and half-power beamwidth.
This simple solution for $A_z$ given a possible source at the origin can be used to synthesize a field from any $z$-directed spatial distribution of current. Thus, plug any given $J_z$ into the equation above and solve for $A_z$. Recall that the standard quantities of electromagnetics – E-field, H-field, and Power – may all be derived from $A_z$ after this integration is complete.
The simplest radiating system is the Hertzian dipole, resulting from an infinitesimal current element at the origin flowing in the z-direction. This can be described mathematically as a $J_z$ with three delta functions w.r.t. x, y, and z. Plugging this into our formula, we get the above expression for $A_z$. 

\[
\tilde{J}_z(x, y, z) = I \delta(x) \delta(y) \delta(z) = I \delta(\mathbf{r})
\]

This is a small, infinitesimal amount of current at the origin of length $dl$, which can equivalently be collapsed into delta functions.

\[
\tilde{A}_z(r) = \frac{\mu I \, dl}{4\pi} \frac{\exp(-jkr)}{r}
\]
Now it's just a matter of straightforward (but tedious) differential calculus to arrive at the solution for classical E and H field for the Hertzian dipole.

A few comments about the final solution:
1) It’s messy! Keep in mind that this is the *simplest possible* radiating system. This attribute of radiation is one reason why antenna engineering is difficult and expertise is often in short demand!
2) If we move our point of observation of the system more than a wavelength (r>\lambda) from the source/origin, then only the 1/r terms dominate. These represent propagating waves away from the source.
3) The 1/r^2 and 1/r^3 field terms dominate when observed for r<\lambda. These represent stored energy/circulating fields that swirl around the radiated system.
Let’s calculate the total amount of power radiated by the Hertzian Dipole. To do this, we calculate the Poynting vector (units of W/m²) from E and H. Then we integrate this quantity around 4π steradians. This gives us total power as a function of current magnitude, I (Amps), wavelength, and infinitesimal length Δl. Note how the Poynting vector is always pointing away from the origin, regardless of point of observation.
We can estimate the impedance of a very short dipole at the end of a transmission line by using this result from the Hertzian dipole. Say we take a tiny dl-length of line and bend it back. We know that this load would appear highly capacitive and can calculate the result from the Smith chart or the load transformation formula.

There will also be a small, real-valued resistance that represents power delivered to the antenna and radiated into space. This is called the radiation impedance. Assuming an ideal system where 100% of the power into the radiation resistance makes it into traveling waves, this I^2 R radiated power must be equal to the total power found on the previous slide. That actually gives us a method for solving the radiation resistance. See above that the radiation resistance is a function of (dl/lambda). As the length of the dipole increases, radiation resistance increases. Keep in mind that this expression is only valid for very small values of dl.
One example of a short dipole is the AM radio transmit tower. AM radio waves have very long wavelengths (> 300m), so that even the tallest towers can only support radiating wires that are a fraction of this length. In many cases, the radiator behaves effectively like a short dipole with large capacitive mismatches.

Besides matching the source discretely, one strategy is also to “mirror” the current on the ground plane so that the dipole looks effectively larger. This only works when the ground is conductive, which is a poor approximation for some regions on earth … especially those with dry and rocky soils.
Another quantity that we can calculate from the Hertzian dipole fields is the directivity. Directivity is defined as the ratio of power density radiated in a particular direction to the average power density radiated in the system. Under this definition, directivity will always integrate to 1 over the full 4π steradians of space. An example calculation for our Hertzian dipole is shown above.

The *gain* of a radiating system is equal to its directivity times an efficiency factor. This factor can account for additional power losses in the radiating system, such as Ohmic losses on the antenna’s conductive material, complex permittivity losses in the surrounding dielectric medium, and mismatch losses at the junction of the antenna. When all of these things are accounted for, the gain is technically “Realized Gain”, although standards bodies for antennas simply refer to this as “Gain”; this is the value reported on antenna spec sheets.

Note that under this definition, Gain pattern of a passive radiative system should *never* integrate to a value above 1 over 4π steradians. Gain > 1 is possible for *specific* directions, but this always comes at the cost of gain in other directions.
There are several useful values that come out of a gain pattern:

Maximum or Peak gain shows the maximum amount of power transferred when radiated power is observed from the direction of maximum pattern focusing. For the Hertzian dipole, this is anywhere along the horizon (theta=90°) and has a value of 3/2 (1.8 dBi in log scale).

Half-Power Beamwidth is the contiguous width, in degrees, of the radiation pattern about its peak corresponding to gains that do not drop below 50% of the peak gain. Thus, the Hertzian dipole pattern falls to a gain of ¾ at theta = 45 degrees and 135 degrees, corresponding to a half-power beamwidth in elevation of 90 degrees, centered at theta = 90 degrees. In azimuth, the half-power beamwidth is essentially 360 degrees since the pattern is omnidirectional about the horizon.