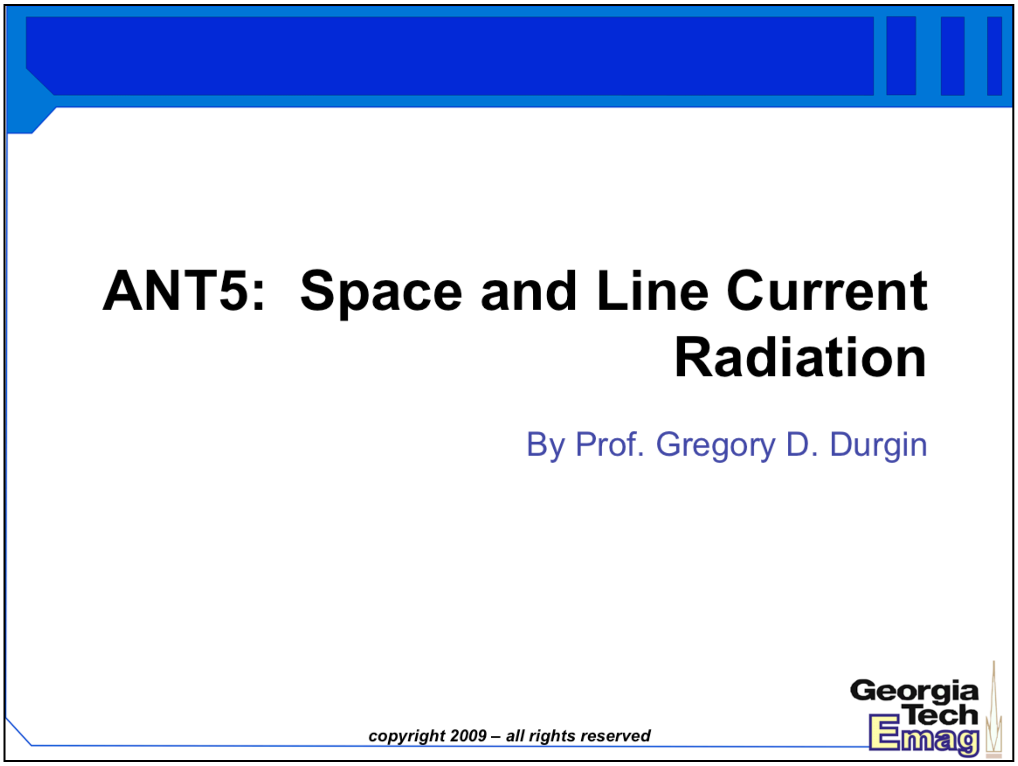
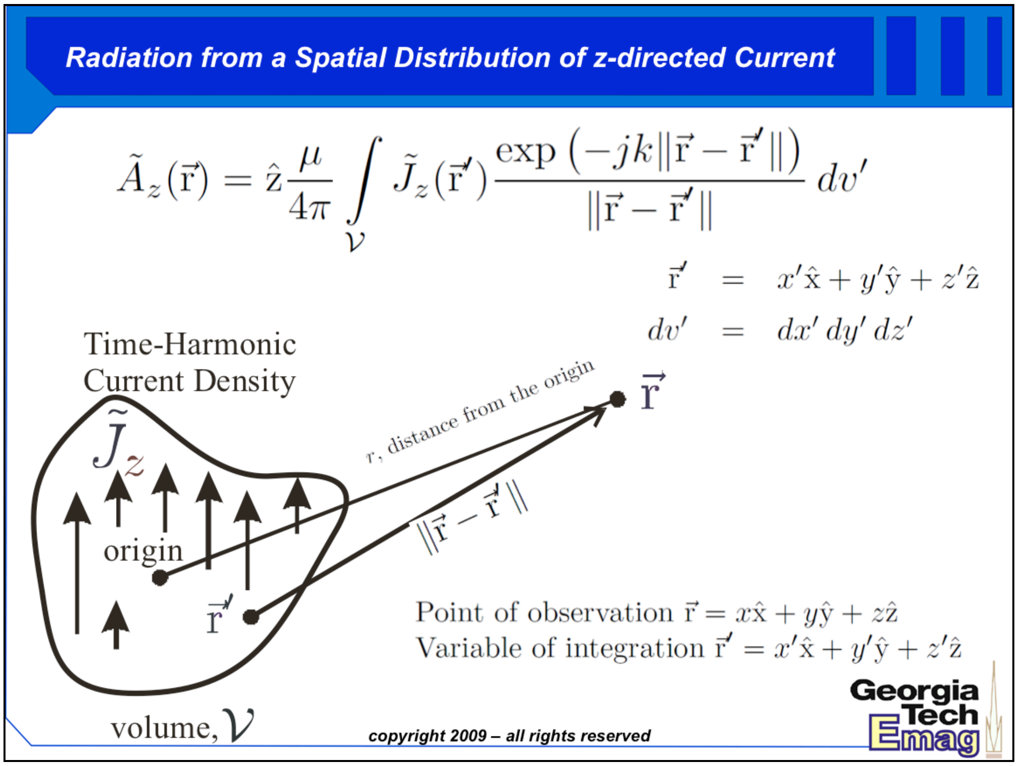


In this lecture, we study the general case of radiation from z-directed spatial currents. The far-field radiation equations that result from this treatment form some of the foundational principles of all antenna engineering. In fact, after this lecture, a student should be able to look at most types of antennas and, regardless of type or construction specifics, be able to infer the basic radiation pattern from the size and shape.



We start with the z-component of the vector magnetic potential's Greens formula, which was derived in ANT2. This time-harmonic equation relates a z-component of current density (Jz) to the z-component of Az. Note that in a general relationship, we would have 3 separable equations – one for z-components, one for y-components, and one for z-components. This divide-and-conquer approach is what makes the vector-magnetic-potential method much more straight-forward than other techniques for solving radiation problems.

Note that the integration occurs over 3-dimensions (x',y',z'), which are not to be confused with the point of observation (x,y,z). The integral is sliding around the mass of z-directed current, picking up the radiative contributions of each amplitude and phase of infinitessimal current elements. In this way, we view the spatial current distribution as simply the superposition of numerous Hertzian dipole elements.

Far-Field Approximations for Spatial Currents

Point of observation $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$
Variable of integration $\vec{r}' = x'\hat{x} + y'\hat{y} + z'\hat{z}$

Far-field Approximations:
Amplitude $\|\vec{r} - \vec{r}'\| \approx r$ (insensitive to small displacements)
Phase $\|\vec{r} - \vec{r}'\| \approx r - \hat{r} \cdot \vec{r}'$ (much more sensitive)

- assumes all vectors $\vec{r} - \vec{r}'$ are parallel
- valid for $r > \frac{D^2}{\lambda}$, where D is largest antenna dimension
- thus, far-field condition is $r > \max\left(\lambda, \frac{D^2}{\lambda}\right)$

Now we can simplify this integral if we assume that the point of observation is a significant distance from the spatial distribution of current, which should be roughly centered in the origin of our problem. Thus, for large *r*, we can make some simplifications to the integral in the previous page. Exactly what constitutes a large *r* will become evident after we make the approximations.

For amplitude terms, the magnitude of the difference between observation and integration vectors can be approximated as the distance from observation to the origin – or simply *r*. Amplitude terms in general tend to be insensitive to slight variations in this term.

Phase terms, however, are much more sensitive to approximation, largely because it's the modulus-2pi of the distance that contributes to a phase term – not the absolute, cumulative value of the observation distance. Making the same *r* approximation that we did in amplitude would be catastrophic, erasing all of the proper phase behavior that is critical to synthesizing a radiation pattern. Instead, we make the approximation that rays drawn from any point on the current distribution to the point of observation are parallel. This is true as long as the point of observation is greater than D^2/λ. Thus, when we talk about current distributions, there are actually two conditions that we need to define the far field. First, we must be greater than 1-wavelength away from the antenna because we used simplified far-field expressions in our superposition formula. But we also need to make sure that the observation distance is D^2/λ, which can be a much more stringent condition, particularly for electrically large antennas such as satellite dish antennas.

Vector Magnetic Potential Field Relationships

Far-field approximation for vector magnetic potential:

$$\begin{aligned}
\tilde{A}_z(\vec{r}) &\approx \frac{\mu}{4\pi}\int_{\mathcal{V}} \tilde{J}_z(\vec{r}')\frac{\exp(-jk\left[r-\hat{r}\cdot\vec{r}'\right])}{r}\,dv' \\
&\approx \frac{\mu}{4\pi r}\exp\left(-jkr\right)\int_{\mathcal{V}} \tilde{J}_z(\vec{r}')\exp\left(+jk\hat{r}\cdot\vec{r}'\right)\,dv' \\
&\approx \frac{\mu}{4\pi r}\exp\left(-jkr\right)\iiint_{-\infty}^{+\infty} \tilde{J}_z(x',y',z')\exp(+jk[x'\sin\theta\cos\phi + y'\sin\theta\sin\phi + z'\cos\theta])\,dx'\,dy'\,dz'
\end{aligned}$$

$$\hat{r} = \sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} + \cos\theta\hat{z}$$

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Here is the reduced formula for vector magnetic potential once all of the far-field approximations are inserted. Here we have also expanded out the unit vector-r that points toward the observer in terms of azimuth and elevation angles. When this is inserted into the expanded 3D integral, one of the more remarkable principles of antenna engineering becomes evident.

The radiation pattern of an antenna is effectively the Fourier transform of the spatial distribution of currents. As such, the pattern follows all of basic rules of a 3D Fourier Transform. Larger current distributions tend to result in smaller patterns (smaller half-power-beamwidths). Smaller current distributions tend to result in broader patterns (very small radiators are always near-omnidirectional in their patterns). Expanding the dimension of the current distribution in only one direction of space will only reduce the radiation pattern width in the corresponding plane. These basic principles allow antenna engineers to shape their radiation patterns by squeezing and stretching their radiative element in various dimensions.

One further consequence of this relationship: it is impossible to design an antenna pattern with an extended null region without resorting to an infinite current distribution in space. Why? In a Fourier relationship, a function in one domain with finite support (a non-singular region over which a function has value of zero) *must* have infinite support in its transformed domain (non-zero across the entire domain excepting singular points). This is a mathematical property, and since the physics follows this principle, a realistic antenna with finite support of Jz in the space domain *necessarily* has infinite support in the pattern domain. Synthesizing near-zero backlobes under these conditions is one of the most common and challenging tasks in antenna engineering.

Far-field Field Equations for Spatial Currents

Next Step: Calculate $\vec{\mathrm{E}}$ and $\vec{\mathrm{H}}$

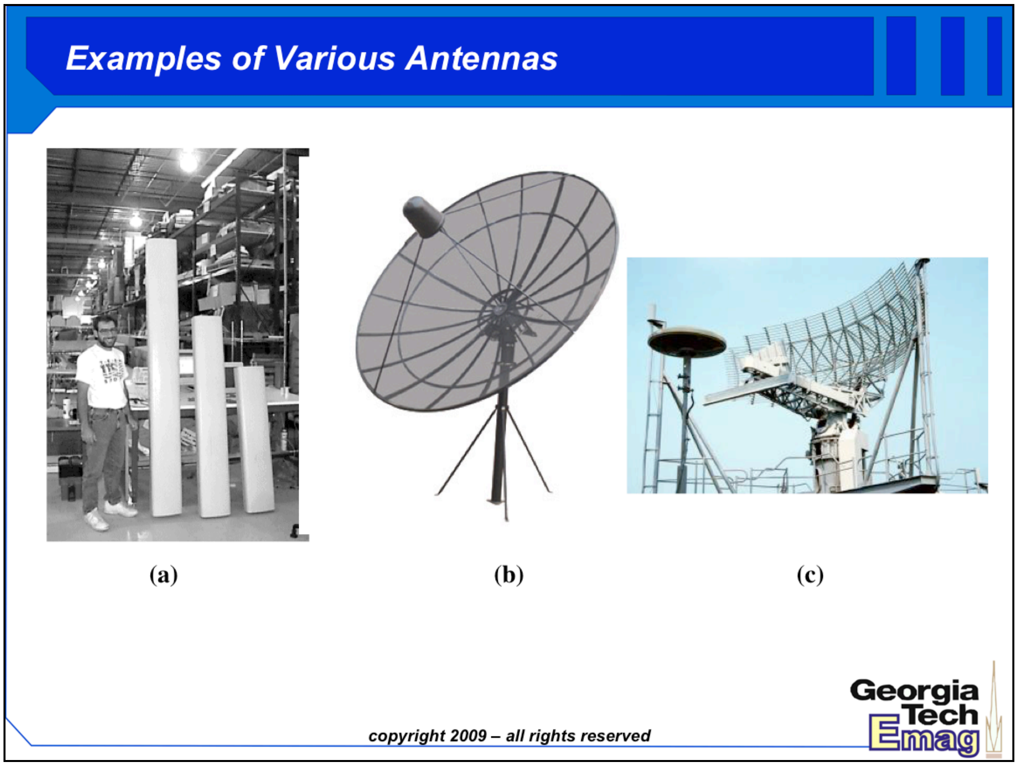
$$\tilde{\vec{\mathrm{H}}}(r,\phi,\theta) = \frac{1}{\mu}\nabla \times (\tilde{A}_z \hat{\mathrm{z}})$$

$$\approx \frac{jk\sin\theta}{\mu}\tilde{A}_z(r,\phi,\theta)\hat{\phi}$$

$$\tilde{\vec{\mathrm{E}}}(r,\phi,\theta) = \frac{1}{j2\pi f\mu\epsilon}\nabla \times \nabla \times (\tilde{A}_z \hat{\mathrm{z}})$$

$$\approx \frac{jk\eta\sin\theta}{\mu}\tilde{A}_z(r,\phi,\theta)\hat{\theta}$$

Now, once we have made the calculation for Az, it becomes very straightforward to calculate magnetic and electric fields. Note that the curl operators in the far-field expressions can be greatly simplified. Z-directed current elements will only result in E-fields that are vertically oriented (in the elevation direction) and H-fields that are horizontally oriented (in the azimuth direction).



(a) Cellular Base Station Antenna – composed of a vertical array of dipoles or cross-polar elements. Provides widebeam coverage in azimuth (where the current distribution is spatially thin on the antenna) and narrow-beam coverage in elevation (where the current distribution is long on the antenna). This increases gain along the horizon, which is where most of the cellular handsets/users should be.

(b) Satellite Dish Antenna – In antenna engineering, it is possible to use Huygens principle to equate field distribution with a current distribution (recall that Huygens principle says that a wavefront acts just a like a collection of point sources). Thus, if we think of the circular aperture of a satellite dish antenna as having an equivalent "source current", we can apply the same rules for synthesizing the antenna's radiation pattern. Thus, this circular dish antenna produces a very small half-power beamwidth in both azimuth and elevation.

(c) Radar Reflector Antenna – Note that this reflector is much larger in the horizontal direction than in the vertical direction. Thus, the azimuth pattern should be much narrower than the elevation pattern. Incidentally, this is exactly what you want in a radar antenna. For a given mechanically positioned direction, only a small sector of the horizon will be excited (and also received), allowing a scattering target's bearing angle to be known precisely. This antenna does not discriminate nearly as much in terms of elevation, but that is an ancillary piece of information for most RADAR applications.

Also, note that the reflector is made of horizontal metal bars, which will only work for horizontal e-field polarization. This minimizes the grazing terrain scatter return, the most common form of "clutter" degradation in a RADAR measurement.