

In this lecture, we simplify the space radiating current analysis to include the special (but very important) case of the general wire antenna. Concentrating on results for the half-wave dipole, we demonstrate how a radiator more realistic than the ideal Hertzian dipole operates. We close with a thorough summary of the most common types of wire antennas and their radiation and electrical parameters.

### Field Solution for the Half-Wave Dipole

Simplification for radiating line-currents:

$$\tilde{J}_z(x', y', z') = \tilde{I}(z') \delta(x') \delta(y')$$

$$\tilde{A}_z(\vec{r}) = \frac{\mu}{4\pi r} \exp(-jkr) \int_{-\infty}^{+\infty} \tilde{I}_z(z') \exp(+jkz' \cos\theta) dz'$$

Current on a Half-Wave Dipole

$$\tilde{I}_z(z') = I \cos(kz') u\left(\frac{\lambda}{4} - |z'|\right)$$

$$\tilde{A}_z(\vec{r}) = \frac{\mu}{4\pi r} \exp(-jkr) \int_{-\lambda/4}^{+\lambda/4} I \cos(kz') \exp(jkz' \cos\theta) dz'$$

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So let's simplify this expression for a case of current distribution  $I(z)$  that exists only on the  $z$ -axis. This corresponds to the case of a wire antenna, which is one of the most common instances in basic antennas. The most common of these common antennas is the half-wave dipole (HWDP), because it is a compact, efficient radiator with many different implementations in practice. It may be used by itself or as the radiative element in a reflector (dish) based antenna.

Note that we can start by defining the  $z$ -directed current density  $J_z$  in terms of the simpler 1-D current distribution  $I(z)$  with units of Amps by “collapsing” the current density onto the  $z$ -axis with two delta functions with respect to  $x$  and  $y$ . The simplified expression for magnetic potential is a single integration of this current with respect to a single complex exponent kernel. Here more than before is the very straightforward “Fourier Transform” relationship between current distribution and pattern.

For a HWDP, the current is non-zero over a  $\lambda/2$  region, where it is in-phase and sinusoidally-tapered in amplitude. This is basically the standing-wave current pattern at the end of an open-circuited transmission line whose last  $\lambda/4$  ends have been bent backwards.

### Field Solution for the Half-Wave Dipole

$$\int \cos(bx) \exp(cx) dx = \frac{\exp(cx)}{b^2 + c^2} [c \cos(bx) + b \sin(bx)]$$

field solution becomes

$$\tilde{A}_z(\vec{r}) = \frac{\mu}{4\pi r} \exp(-jkr) \int_{-\lambda/4}^{+\lambda/4} I \cos(kz') \exp(jkz' \cos\theta) dz'$$

$$= \frac{\mu}{4\pi r} \exp(-jkr) \frac{2I \cos\left(\frac{\pi}{2} \cos\theta\right)}{k \sin^2\theta}$$

$$\tilde{\vec{H}}(r, \phi, \theta) = \frac{jI}{2\pi r} \exp(-jkr) \frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta} \hat{\phi}$$

$$\tilde{\vec{E}}(r, \phi, \theta) = \frac{j\eta I}{2\pi r} \exp(-jkr) \frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta} \hat{\theta}$$

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Here is the solution for the HWDP electric and magnetic fields. Note the similarity to the Hertzian/ideal dipole radiator: the fields are at a maximum along the azimuth (theta = 90 degrees). The fields have a null along the z-axis (theta = 0 or 180 degrees). The antenna pattern is omnidirectional, having no dependence on azimuth angle, phi.

Note, however, that the overall elevation cut of the pattern is somewhat more “squinted” than the ideal dipole due to the  $\cos(\pi/2 \cos(\theta))$  term in the expressions. This slightly more complicated expression gives a half-power beamwidth of 78 degrees to the HWDP, as opposed to the 90 degrees for the ideal dipole.

Various Types of Line Antennas				
Antenna	Hertzian Dipole	Short Dipole	$\frac{\lambda}{2}$ Dipole	$\frac{\lambda}{4}$ Monopole
Current Distribution	 $I dl \delta(z)$	 $I \left(1 - \frac{2 z }{dl}\right) u\left(\frac{dl}{2} -  z \right)$	 $I \cos(kz) u\left(\frac{\lambda}{4} -  z \right)$	 $I \cos(kz) \quad 0 \leq z \leq \frac{\lambda}{4}$ PEC $z < 0$
Directivity	$\frac{3}{2} \sin^2 \theta$	$\frac{3}{2} \sin^2 \theta$	$\frac{\cos^2\left(\frac{\pi}{2} \cos \theta\right)}{\sin^2 \theta}$	$\frac{\cos^2\left(\frac{\pi}{2} \cos \theta\right)}{\sin^2 \theta}$ $\theta \leq 90^\circ$
Peak Gain	1.8 dBi	1.8 dBi	2.2 dBi	5.2 dBi
Radiation Resistance	$80\pi^2 \left(\frac{dl}{\lambda}\right)^2$	$20\pi^2 \left(\frac{dl}{\lambda}\right)^2$	$73\Omega$	$36\Omega$
Half-Power Beamwidth, $\theta_{3dB}$	$90^\circ$	$90^\circ$	$78^\circ$	$39^\circ$

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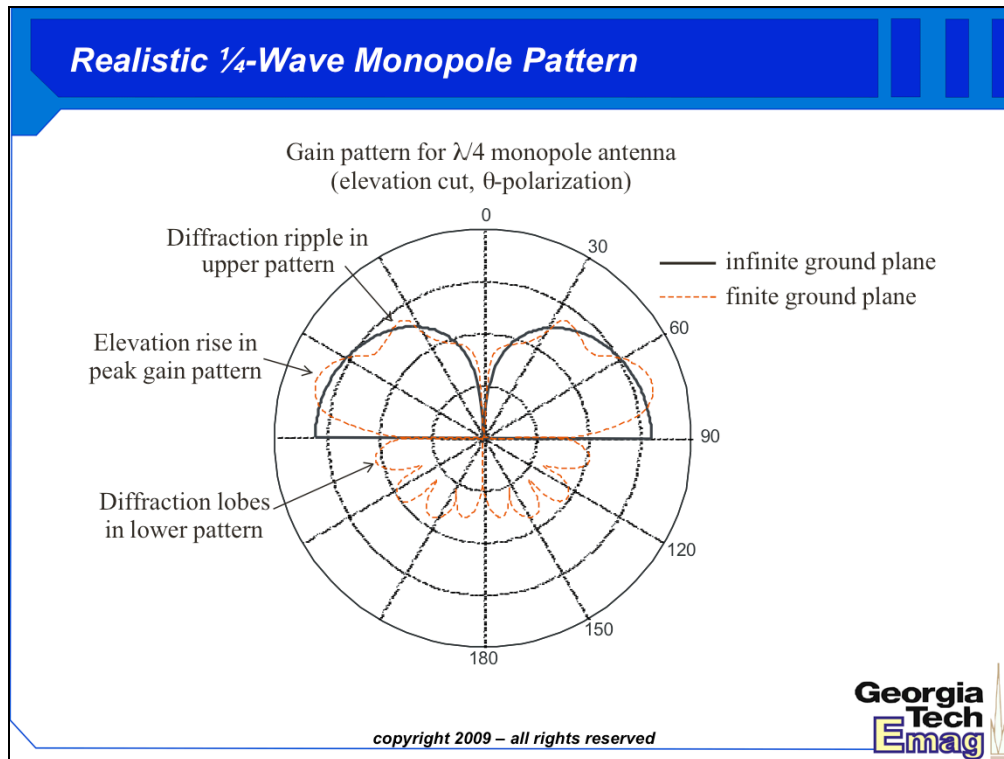
Here is an overview of common types of line antennas and their electrical parameters, many of which may be derived using the same techniques discussed in this lecture.

**Ideal Dipole** – the Hertzian dipole, which is an extremely small length of radiator with length  $dl$ . The fundamental radiator for superposition integrals.

**Short Dipole** – a very small dipole similar to the Hertzian dipole, however with a current distribution that has a triangular taper. The radiation pattern is identical, but the impedance drops compared to the ideal dipole. More realistic current distribution for a small wire radiator where an open circuit transmission line has bent back to make an antenna.

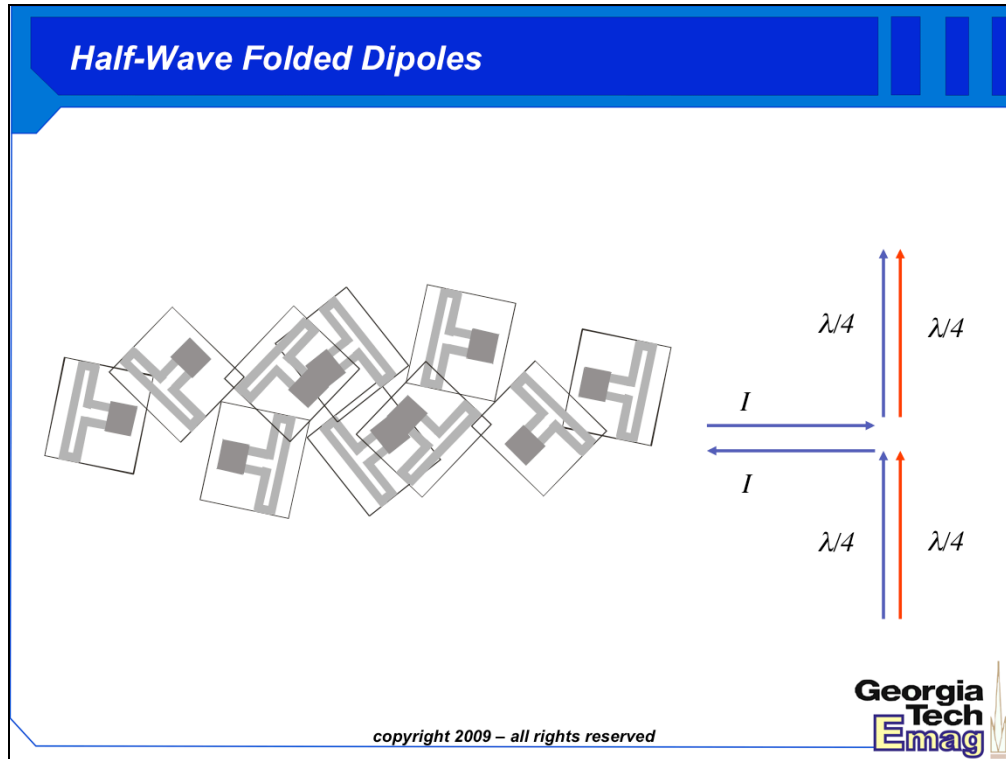
**HWDP** – an efficient radiator with a large impedance at resonance (70 Ohms). Slightly more gain than the smaller radiating elements. Often, gain reported on specification sheets is referenced against the peak gain of a HWDP (dBd) instead of an idealized isotropic radiator (dBi), resulting in values that are 2.1 dB lower.

**Quarter-Wavelength Monopole** – this antenna's radiation pattern, by virtue of the image current in the infinite ground plane, behaves identically to the HWDP in the upper hemisphere ( $\theta < 90$  degrees). Because there is no radiation (for infinite ground plane) in the lower hemisphere, the result is a +3 dB gain to the upper radiation pattern, which otherwise has the same shape as the HWDP. The impedance drops by a factor of 2 for this antenna as well.



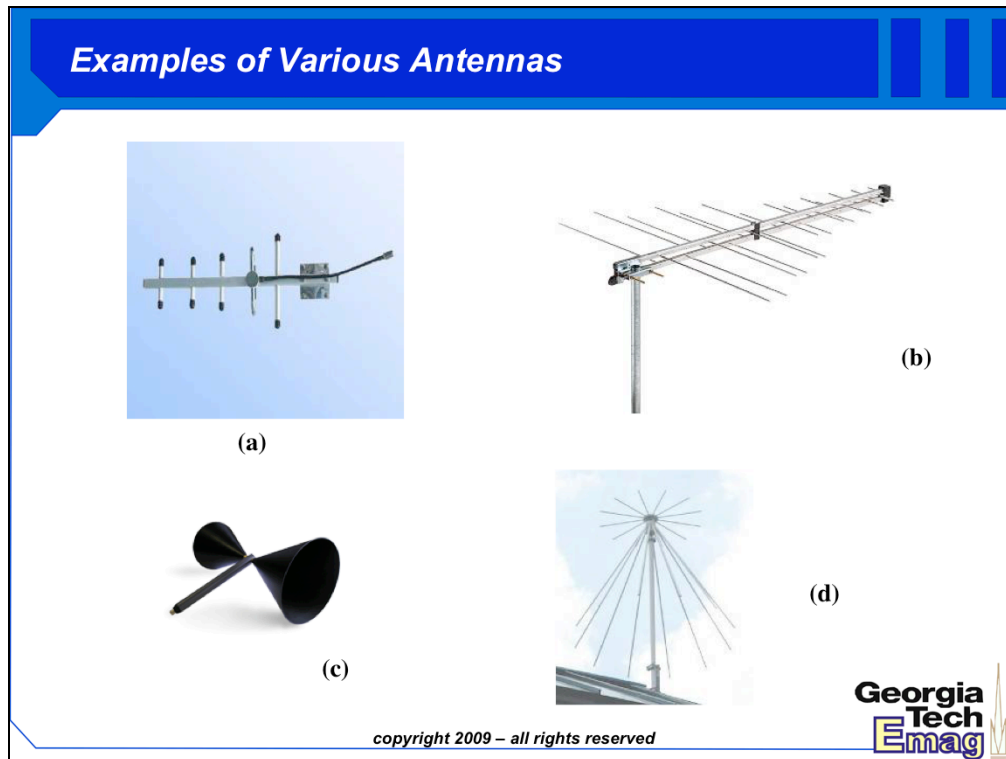
The quarter-wave monopole is one of the most useful forms of antennas, as it radiates efficiently and requires only a ground plane to beat against. What's more, any ground plane will usually work (after some tuning). For example, a car will place the quarter-wavelength monopole on the hood of the vehicle, which becomes the ground plane. For a cellular phone, a "pig-tail" antenna sticks out of the casing and uses the phone's metal housing as the ground plane. A broadcast antenna tower will radiate against the ground it sits on.

In the above plot from Stutzman and Thiele, we can see the non-ideal effects of a finite-sized ground plane. The graph is the elevation-cut pattern of a dipole across the full 0 to 180 degrees of elevation angle (90-degrees corresponds to azimuth). The full line is the radiation pattern of a monopole on a 6-wavelength diameter circular ground plane. The dotted line is the radiation pattern of a monopole on a 6-wavelength square ground plane. Note two things: 1) the realistic groundplanes move the peak gain point in the radiation pattern from azimuth (theta = 90 degrees) upward to a new point of theta = 70 degrees. 2) there is non-zero radiation into the hemisphere of space below the ground plane. Both of these irregularities can be explained by diffracted waves that launch off the edges of the finite groundwave. In fact, we can see the interference pattern of two diffracted waves (on opposite ends of the dipole) constructively and destructively adding for different observation angles greater than 90 degrees.



Half-wave folded dipoles are similar in operation to the HWDP, with identical radiation patterns and sizes. The key difference lies in the shorting bar, which places a second line of sinusoidally-tapered currents alongside the original HWDP radiator currents. Interestingly, this doubling of current distribution results in a factor-of-four increase in the radiation impedance of a half-wave folded dipole. The actual resonant result depends on the gage of the wire used for construction, but the nominal impedance is typically around 280 Ohms.

Because of their high impedance, folded dipole antennas and their meandering variations are favorites of RFID tag designers. Why? Tag antennas behave well in free space, but typically experience significant drops in radiation impedance when brought close to a dielectric or conductive objects. The antenna still radiates in this condition, but it becomes difficult to couple power into and out of the radiator due to impedance mismatches. In extreme cases, on metal impedance of an antenna can drop below a single Ohm! One way to mediate this problem is to start with an antenna that has a very large radiation impedance to start with – such as the folded dipole!



- (a) Yagi-Uda Array – Often called a “Yagi” (Yagi was the Japanese advisor, Uda was the grad student that did all the work), this antenna has a dipole radiator. All of the other elements are dead or “parasitic”. The larger element is the “reflector” and the smaller elements on the other side of the HWDP are “directors”. Adding a single reflector and director shifts the gain pattern (leftward in the picture). Gain increases with additional directors. Here is an example where the Fourier Transform rule-of-thumb for currents does not work as intuitively – this is a “traveling wave antenna”.
- (b) Log Periodic – A really broadband, directional form of dipole. Why is it broadband? Because the diminished (logarithmic) scaling-down of radiating arms that are all tied together, there should be somewhere along the left or right side a pair of arms that resonates like a half-wave dipole; the other elements do not radiate as effectively and do not contribute to the gain pattern. Thus, as frequency is swept higher and higher, the “active arms” of the antenna switch further and further down the antenna.
- (c) Bicone Antenna – Think of this as a fat HWDP. The benefit is that the bandwidth of operation becomes much larger than the typical 2-3% of carrier frequency experienced by a wire-based HWDP.
- (d) Discone Antenna – Bicones are to HWDP what Discones are to quarter-wavelength monopoles. In this design, one of the conical elements of the bicone is replaced with a ground plane. The picture of the discone antenna above is particularly interesting because it is mechanically large (low frequency) and saves construction material and weight by replacing the solid sheet of conical and ground plane surfaces with closely-spaced (electromagnetically speaking) conductive rods. Beyond a certain frequency, the “look” the same as solid metal sheets to a radio wave.