In this lecture, we study the general case of radiation from z-directed spatial currents. The far-field radiation equations that result from this treatment form some of the foundational principles of all antenna engineering. In fact, after this lecture, a student should be able to look at most types of antennas and, regardless of type or construction specifics, be able to infer the basic radiation pattern from the size and shape.

In the later section of the talk, we simplify the analysis to include the special (but very important) case of the general wire antenna. Concentrating on results for the half-wave dipole, we demonstrate how a radiator more realistic than the ideal Hertzian dipole operates. We close with a thorough summary of the most common types of wire antennas and their radiation and electrical parameters.
Example Analysis: Cellular Base Station Antenna

Cellular base station antennas tend to focus power along the horizon where all the paying customers operate. To do this, they manufacture antennas with stacked, in-phase half-wave dipoles (see the 5-element example in the figure to the right) such that the total radiating current may be represented by

\[ \tilde{I}(z) = I_0 \left| \cos \left( \frac{2\pi z}{\lambda} \right) \right| u \left( \frac{N \lambda}{4} - |z| \right) \]

Where \( N \) is the integer number of stacked dipoles and \( u() \) is the unit step function. Assuming ideal efficiency, make a dB-polar plot of the 0-pol elevation-cut gain pattern of a base station antenna for the cases of \( N = 4, 6, 8 \) and 10. Graphically estimate the peak gain and half-power beamwidth in \( \theta \) for each case. Multiply the linear value of the peak gain and the HPBW angle for each case. What do you notice?
So let’s simplify this expression for a case of current distribution $I(z)$ that exists only on the $z$-axis. This corresponds to the case of a wire antenna, which is one of the most common instances in basic antennas. The most common of these common antennas is the half-wave dipole (HWDP), because it is a compact, efficient radiator with many different implementations in practice. It may be used by itself or as the radiative element in a reflector (dish) based antenna.

Note that we can start by defining the $z$-directed current density $J_z$ in terms of the simpler 1-D current distribution $I(z)$ with units of Amps by “collapsing” the current density onto the $z$-axis with two delta functions with respect to $x$ and $y$. The simplified expression for magnetic potential is a single integration of this current with respect to a single complex exponent kernal. Here more than before is the very straightforward “Fourier Transform” relationship between current distribution and pattern.

For a HWDP, the current is non-zero over a $\lambda/2$ region, where it is in-phase and sinusoidally-tapered in amplitude. This is basically the standing-wave current pattern at the end of an open-circuited transmission line whose last $\lambda/4$ ends have been bent backwards.
Here is the solution for the HWDP electric and magnetic fields. Note the similarity to the Hertzian/ideal dipole radiator: the fields are at a maximum along the azimuth (\(\theta = 90\) degrees). The fields have a null along the z-axis (\(\theta = 0\) or 180 degrees). The antenna pattern is omnidirectional, having no dependence on azimuth angle, \(\phi\).

Note, however, that the overall elevation cut of the pattern is somewhat more “squinted” than the ideal dipole due to the \(\cos(pi/2 \cos(\theta))\) term in the expressions. This slightly more complicated expression gives a half-power beamwidth of 78 degrees to the HWDP, as opposed to the 90 degrees for the ideal dipole.
Potential for Array of N Half-wave Dipoles

\[
|\vec{A}_z(\vec{r})| = \frac{\mu}{4\pi r} \left| \int_{-\lambda/4}^{(2N-1)\lambda/4} I |\cos(kz')| \exp(jk'z' \cos\theta) \, dz' \right|
\]

\[= \frac{\mu}{4\pi r} \sum_{n=0}^{N-1} (-1)^n \int_{(2n-1)\lambda/4}^{(2n+1)\lambda/4} I \cos(kz') \exp(jk'z' \cos\theta) \, dz' \]

\[= \frac{\mu}{4\pi r} \sum_{n=0}^{N-1} \frac{2I \cos\left(\frac{\pi}{2} \cos\theta\right)}{k \sin^2\theta} \exp(jn\pi \cos\theta) \]

\[= \frac{\mu}{2\pi r} \frac{I \cos\left(\frac{\pi}{2} \cos\theta\right)}{k \sin^2\theta} \left[ \sum_{n=0}^{N-1} \exp(jn\pi \cos\theta) \right] \frac{\sin(N\pi/2 \cos\theta)}{\sin(\pi/2 \cos\theta)} \]

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Now calculate the total power pattern for the radiating system:

Power Density (W/m²):  \[ \frac{k^2 \eta \sin^2 \theta}{2\mu^2} |\tilde{A}_z|^2 \]

\[ = \frac{I^2 \eta}{8\pi^2 r^2} \left( \frac{\cos^2 \left( \frac{\pi}{2} \cos \theta \right)}{\sin^2 \theta} - \frac{\sin^2 \left( \frac{\pi}{2} \cos \theta \right)}{\sin^2 \theta} \right) \]

\[ = \text{dipole gain pattern} - \text{array pattern} \]
Half-Wave Dipole Analysis

Directivity of a λ/2 dipole

Half-wave dipole array of 1 element(s)
Element input current 1.0 A

-----------------------------------------------
Total radiated power: 36.4 W/m² (15.6 dBi)
Peak gain: 1.4 (2.2 dBi)
Half-power beamwidth: 78.3 deg
Side-lobe level: InE dB
Radiation resistance: 72.7 Ohms
Vertical Array of 4 \( \lambda/2 \)-Dipoles

Sample text here...

Half-wave dipole array of 4 element(s)
Element input current 4.0 A

-----------------------------
Total radiated power: 100.9 W/m² (21.2 dBm)
Peak gain: 7.3 (8.7 dBi)
Half-power beamwidth: 13.5 deg
Side-lobe level: 11 dB
Radiation resistance: 16.4 Ohms

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Vertical Array of 6 $\lambda$/2-Dipoles

Half-wave dipole array of 6 element(s)
Element input current 6.0 A

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total radiated power</td>
<td>192.0 W/m² (22.8 dBm)</td>
</tr>
<tr>
<td>Peak gain</td>
<td>11.2 (10.5 dBi)</td>
</tr>
<tr>
<td>Half-power beaidth</td>
<td>8.1 deg</td>
</tr>
<tr>
<td>Side-lobe level</td>
<td>12.9 dB</td>
</tr>
<tr>
<td>Radiation resistance</td>
<td>10.7 Ohms</td>
</tr>
</tbody>
</table>
Vertical Array of 8 $\lambda/2$-Dipoles

- Directivity of 8 stacked $\lambda/2$ dipoles
- Half-wave dipole array of 8 element(s)
- Element input current 8.0 A

- Total radiated power: 252.8 W/m² (24.0 dBi)
- Peak gain: 15.2 (11.8 dBi)
- Half-power beamwidth: 6.1 deg
- Side-lobe level: 13.2 dB
- Radiation resistance: 7.9 Ohms

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**Vertical Array of 10 λ/2-Dipoles**

- Directivity of 10 stacked λ/2 dipoles

**Technical Specifications**

- Half-wave dipole array of 10 element(s)
- Element input current 10.0 A

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total radiated power</td>
<td>313.3 W/m²</td>
</tr>
<tr>
<td>Peak gain</td>
<td>19.1 (15.4 dBi)</td>
</tr>
<tr>
<td>Half-power beamwidth</td>
<td>4.5 deg</td>
</tr>
<tr>
<td>Side-lobe level</td>
<td>15.1 dB</td>
</tr>
<tr>
<td>Radiation resistance</td>
<td>6.3 Ohms</td>
</tr>
</tbody>
</table>

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Example: Base Station Antenna Spec Sheet

Stella Doradus Ireland Ltd.

24 12098 2.4GHz Base Station antenna

Electrical Specifications:
- Gain: 15dBi
- 3dB Beam Pattern: 120° x 90°
- Bandwidth: 2.4-2.485GHz
- VSWR: 1.8 : 1
- Front to Back Ratio: 35dB
- Polarization: Vertical
- Power Rating: 50W
- Impedance: 50 Ohms
- Termination: N-terminated
- Cross Pol. Discrimination: 25dB
- Surge Protection: In Built

Mechanical Specifications:
- Length: 120cm
- Width: 17cm
- Depth: 10 cm
- Weight: 29kg
- Windspeed: 210km/hr
- Mechanical Tilt: 0-5 degrees
- Mounting Pipe: 5 cm pipe

Materials:
- Radiating Element: Beam forming PCB patch array
- Radome (Feed): ABS Grey
- Champs: HDG steel + galvanised steel bolts

http://www.stelladoradus.com/pdfs/2.4Base/24%202008%2005-08.pdf

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Example: Base Station Antenna Spec Sheet

24 12308 Azimuth Pattern

24 12308 Elevation Pattern

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**Example: Base Station Antenna Spec Sheet**

MT-243015/NH
902-928 MHz 10dBi 180°
Horizontal Pol.
Base Station Antenna

<table>
<thead>
<tr>
<th>Specifications</th>
<th>MT: 243015/NH</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ELECTRICAL</strong></td>
<td></td>
</tr>
<tr>
<td>Regulatory Compliance</td>
<td>RoHS, CE (EMI)</td>
</tr>
<tr>
<td>Frequency Range</td>
<td>902-928 MHz</td>
</tr>
<tr>
<td>Gain</td>
<td>10 dB (typ)</td>
</tr>
<tr>
<td>SWR</td>
<td>1.7:1 (typ), 2:1 (max)</td>
</tr>
<tr>
<td>Azimuth Beamwidth</td>
<td>180° (typ)</td>
</tr>
<tr>
<td>Polarization</td>
<td>Linear Horizontal</td>
</tr>
<tr>
<td>Cross Polarization</td>
<td>-16 dB (max), -19 dB (typ)</td>
</tr>
<tr>
<td>VSWR</td>
<td>2.8 (typ)</td>
</tr>
<tr>
<td>Input Impedance</td>
<td>50 Ω (min)</td>
</tr>
<tr>
<td>Input Power</td>
<td>30 W (max)</td>
</tr>
<tr>
<td>Lightning Protection</td>
<td>IV, Grounded</td>
</tr>
</tbody>
</table>

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Example: Base Station Antenna Spec Sheet

902-928 MHz 10dBi 180° Horizontal Pol. BTS Antenna

<table>
<thead>
<tr>
<th>Azimuth Radiation Pattern</th>
<th>Dimensions [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Elevation Radiation Pattern</th>
<th>Midband Freq. 915 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total antenna height is 1.3 meters (4 wavelengths)