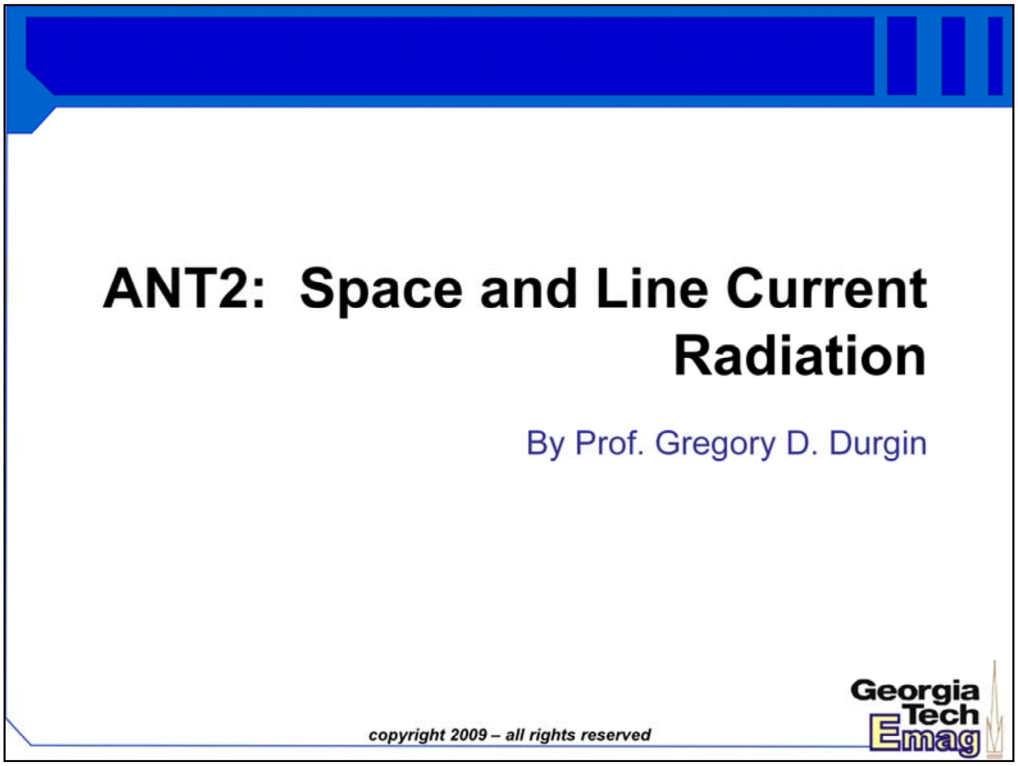
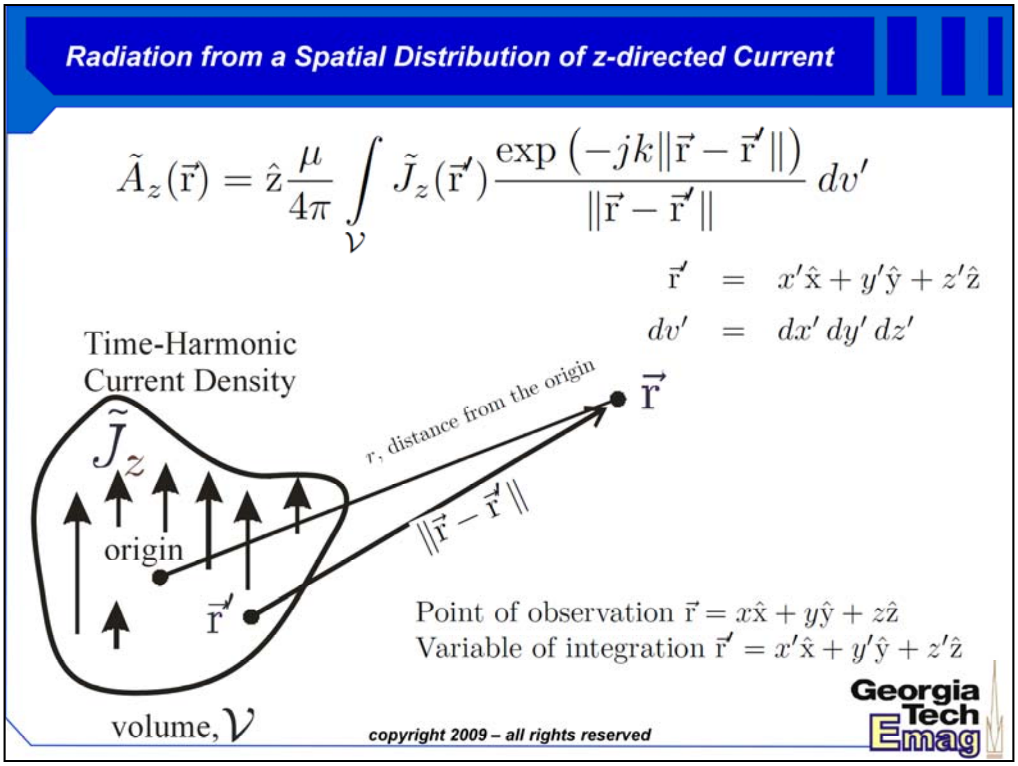


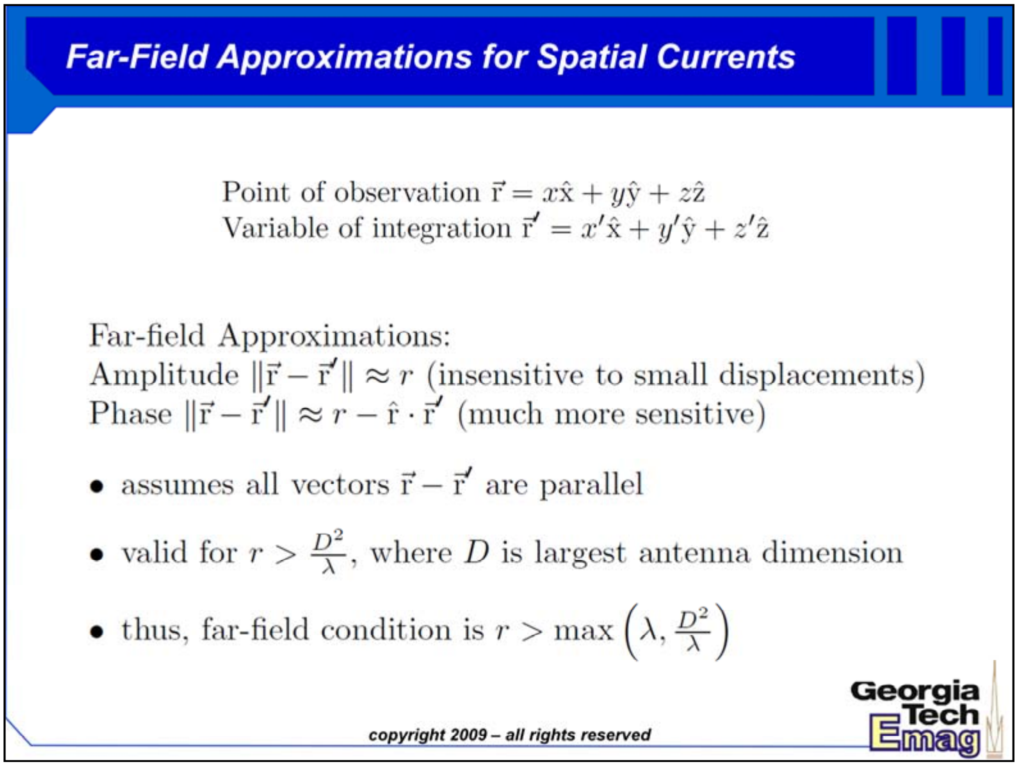
In this lecture, we study the general case of radiation from z-directed spatial currents. The far-field radiation equations that result from this treatment form some of the foundational principles of all antenna engineering. In fact, after this lecture, a student should be able to look at most types of antennas and, regardless of type or construction specifics, be able to infer the basic radiation pattern from the size and shape.

In the later section of the talk, we simplify the analysis to include the special (but very important) case of the general wire antenna. Concentrating on results for the half-wave dipole, we demonstrate how a radiator more realistic than the ideal Hertzian dipole operates. We close with a thorough summary of the most common types of wire antennas and their radiation and electrical parameters.



We start with the z-component of the vector magnetic potential's Greens formula, which was derived in ANT2. This time-harmonic equation relates a z-component of current density (Jz) to the z-component of Az. Note that in a general relationship, we would have 3 separable equations – one for z-components, one for y-components, and one for z-components. This divide-and-conquer approach is what makes the vector-magnetic-potential method much more straight-forward than other techniques for solving radiation problems.

Note that the integration occurs over 3-dimensions (x',y',z'), which are not to be confused with the point of observation (x,y,z). The integral is sliding around the mass of z-directed current, picking up the radiative contributions of each amplitude and phase of infinitessimal current elements. In this way, we view the spatial current distribution as simply the superposition of numerous Hertzian dipole elements.



Now we can simplify this integral if we assume that the point of observation is a significant distance from the spatial distribution of current, which should be roughly centered in the origin of our problem. Thus, for large *r*, we can make some simplifications to the integral in the previous page. Exactly what constitutes a large *r* will become evident after we make the approximations.

For amplitude terms, the magnitude of the difference between observation and integration vectors can be approximated as the distance from observation to the origin – or simply *r*. Amplitude terms in general tend to be insensitive to slight variations in this term.

Phase terms, however, are much more sensitive to approximation, largely because it's the modulus-2pi of the distance that contributes to a phase term – not the absolute, cumulative value of the observation distance. Making the same *r* approximation that we did in amplitude would be catastrophic, erasing all of the proper phase behavior that is critical to synthesizing a radiation pattern. Instead, we make the approximation that rays drawn from any point on the current distribution to the point of observation are parallel. This is true as long as the point of observation is greater than D^2/λ. Thus, when we talk about current distributions, there are actually two conditions that we need to define the far field. First, we must be greater than 1-wavelength away from the antenna because we used simplified far-field expressions in our superposition formula. But we also need to make sure that the observation distance is D^2/λ, which can be a much more stringent condition, particularly for electrically large antennas such as satellite dish antennas.

Vector Magnetic Potential Field Relationships

Far-field approximation for vector magnetic potential:

$$\begin{aligned}\tilde{A}_z(\vec{\mathrm{r}}) &\approx \frac{\mu}{4\pi}\int_{\mathcal{V}} \tilde{J}_z(\vec{\mathrm{r}}')\frac{\exp(-jk\left[r-\hat{\mathrm{r}}\cdot\vec{\mathrm{r}}'\right])}{r}\,dv' \\ &\approx \frac{\mu}{4\pi r}\exp\left(-jkr\right)\int_{\mathcal{V}} \tilde{J}_z(\vec{\mathrm{r}}')\exp\left(+jk\hat{\mathrm{r}}\cdot\vec{\mathrm{r}}'\right)\,dv' \\ &\approx \frac{\mu}{4\pi r}\exp\left(-jkr\right)\iiint_{-\infty}^{+\infty}\tilde{J}_z(x',y',z')\exp(+jk[x'\sin\theta\cos\phi + y'\sin\theta\sin\phi + z'\cos\theta])\,dx'\,dy'\,dz'\end{aligned}$$

$$\hat{\mathrm{r}} = \sin\theta\cos\phi\hat{\mathrm{x}} + \sin\theta\sin\phi\hat{\mathrm{y}} + \cos\theta\hat{\mathrm{z}}$$

copyright 2009 – all rights reserved

Georgia Tech Emag

Here is the reduced formula for vector magnetic potential once all of the far-field approximations are inserted. Here we have also expanded out the unit vector-r that points toward the observer in terms of azimuth and elevation angles. When this is inserted into the expanded 3D integral, one of the more remarkable principles of antenna engineering becomes evident.

The radiation pattern of an antenna is effectively the Fourier transform of the spatial distribution of currents. As such, the pattern follows all of basic rules of a 3D Fourier Transform. Larger current distributions tend to result in smaller patterns (smaller half-power-beamwidths). Smaller current distributions tend to result in broader patterns (very small radiators are always near-omnidirectional in their patterns). Expanding the dimension of the current distribution in only one direction of space will only reduce the radiation pattern width in the corresponding plane. These basic principles allow antenna engineers to shape their radiation patterns by squeezing and stretching their radiative element in various dimensions.

One further consequence of this relationship: it is impossible to design an antenna pattern with an extended null region without resorting to an infinite current distribution in space. Why? In a Fourier relationship, a function in one domain with finite support (a non-singular region over which a function has value of zero) *must* have infinite support in its transformed domain (non-zero across the entire domain excepting singular points). This is a mathematical property, and since the physics follows this principle, a realistic antenna with finite support of Jz in the space domain *necessarily* has infinite support in the pattern domain. Synthesizing near-zero backlobes under these conditions is one of the most common and challenging tasks in antenna engineering.

Far-field Field Equations for Spatial Currents

Next Step: Calculate $\vec{\mathrm{E}}$ and $\vec{\mathrm{H}}$

$$\tilde{\vec{\mathrm{H}}}(r,\phi,\theta) = \frac{1}{\mu}\nabla \times (\tilde{A}_z\hat{\mathrm{z}})$$

$$\approx \frac{jk\sin\theta}{\mu}\tilde{A}_z(r,\phi,\theta)\hat{\phi}$$

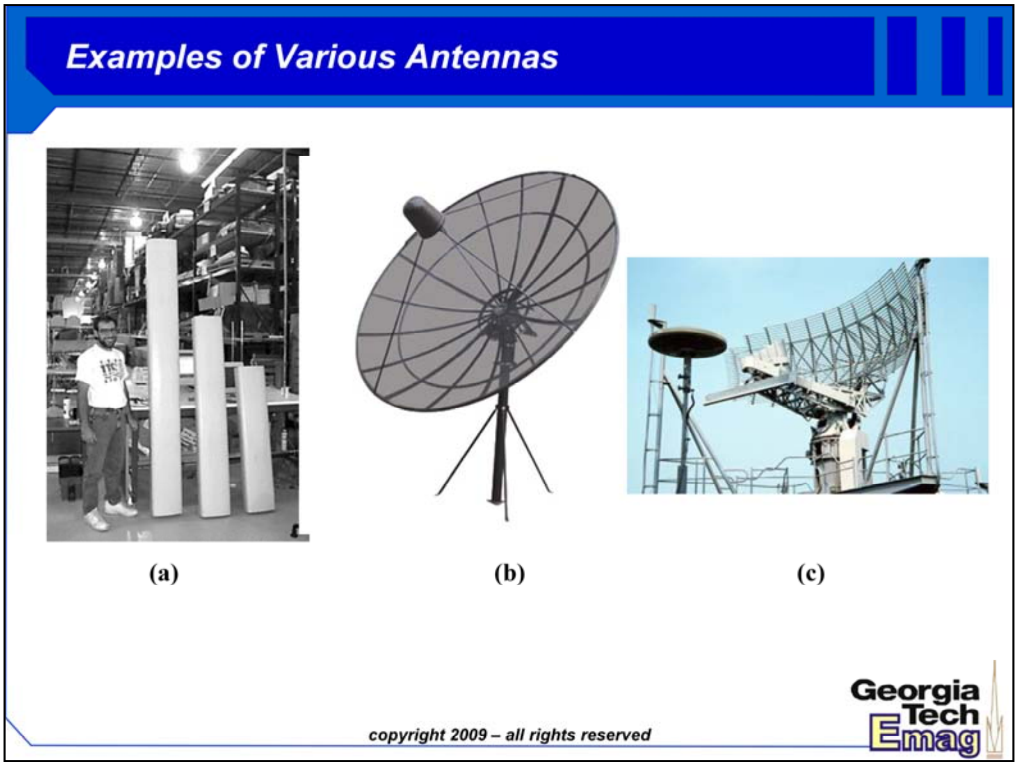
$$\tilde{\vec{\mathrm{E}}}(r,\phi,\theta) = \frac{1}{j2\pi f\mu\epsilon}\nabla \times \nabla \times (\tilde{A}_z\hat{\mathrm{z}})$$

$$\approx \frac{jk\eta\sin\theta}{\mu}\tilde{A}_z(r,\phi,\theta)\hat{\theta}$$

copyright 2009 – all rights reserved

Georgia Tech Emag

Now, once we have made the calculation for Az, it becomes very straightforward to calculate magnetic and electric fields. Note that the curl operators in the far-field expressions can be greatly simplified. Z-directed current elements will only result in E-fields that are vertically oriented (in the elevation direction) and H-fields that are horizontally oriented (in the azimuth direction).



(a) Cellular Base Station Antenna – composed of a vertical array of dipoles or cross-polar elements. Provides widebeam coverage in azimuth (where the current distribution is spatially thin on the antenna) and narrow-beam coverage in elevation (where the current distribution is long on the antenna). This increases gain along the horizon, which is where most of the cellular handsets/users should be.

(b) Satellite Dish Antenna – In antenna engineering, it is possible to use Huygens principle to equate field distribution with a current distribution (recall that Huygens principle says that a wavefront acts just a like a collection of point sources). Thus, if we think of the circular aperture of a satellite dish antenna as having an equivalent "source current", we can apply the same rules for synthesizing the antenna's radiation pattern. Thus, this circular dish antenna produces a very small half-power beamwidth in both azimuth and elevation.

(c) Radar Reflector Antenna – Note that this reflector is much larger in the horizontal direction than in the vertical direction. Thus, the azimuth pattern should be much narrower than the elevation pattern. Incidentally, this is exactly what you want in a radar antenna. For a given mechanically positioned direction, only a small sector of the horizon will be excited (and also received), allowing a scattering target's bearing angle to be known precisely. This antenna does not discriminate nearly as much in terms of elevation, but that is an ancillary piece of information for most RADAR applications.

Also, note that the reflector is made of horizontal metal bars, which will only work for horizontal e-field polarization. This minimizes the grazing terrain scatter return, the most common form of "clutter" degradation in a RADAR measurement.

Field Solution for the Half-Wave Dipole

Simplification for radiating line-currents:

$$\tilde{J}_z(x', y', z') = \tilde{I}(z')\,\delta(x')\delta(y')$$

$$\tilde{A}_z(\vec{r}) \;=\; \frac{\mu}{4\pi r}\exp\left(-jkr\right)\int_{-\infty}^{+\infty}\tilde{I}_z(z')\exp(+jkz'\cos\theta)\;dz'$$

Current on a Half-Wave Dipole

$$\tilde{I}_z(z') = I\cos\left(kz'\right)\;\mathrm{u}\left(\frac{\lambda}{4} - |z'|\right)$$

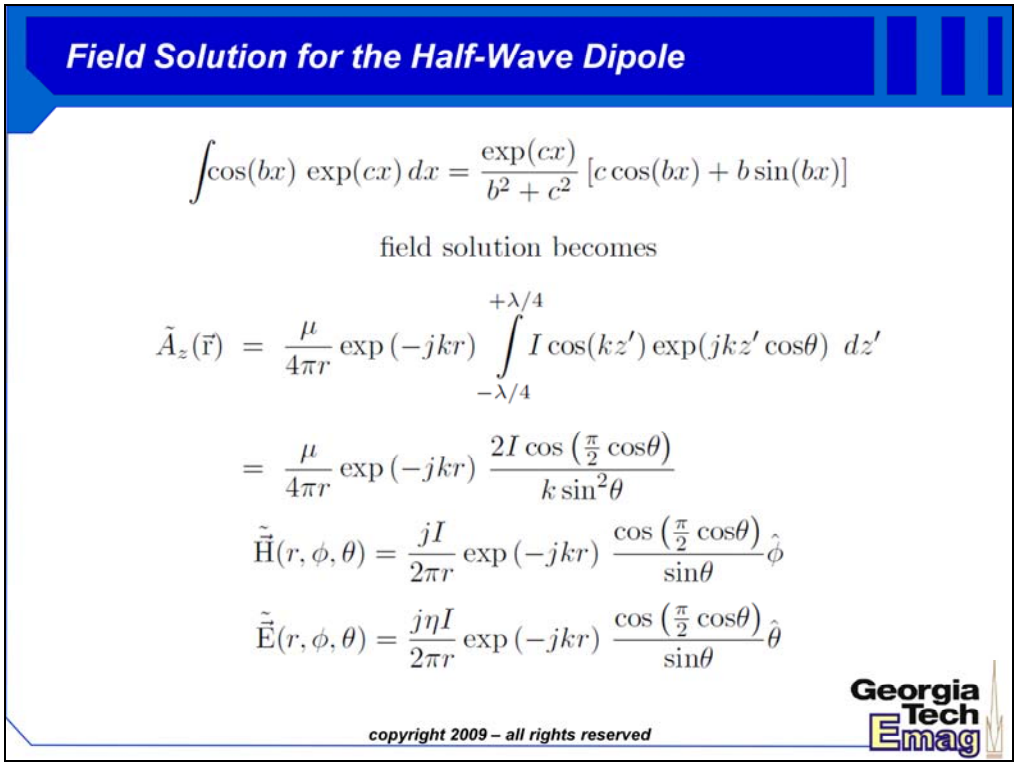
$$\tilde{A}_z(\vec{r}) \;=\; \frac{\mu}{4\pi r}\exp\left(-jkr\right)\int_{-\lambda/4}^{+\lambda/4} I\cos(kz')\exp(jkz'\cos\theta)\;dz'$$

copyright 2009 – all rights reserved

So let's simplify this expression for a case of current distribution I(z) that exists only on the z-axis. This corresponds to the case of a wire antenna, which is one of the most common instances in basic antennas. The most common of these common antennas is the half-wave dipole (HWDP), because it is a compact, efficient radiator with many different implementations in practice. It may be used by itself or as the radiative element in a reflector (dish) based antenna.

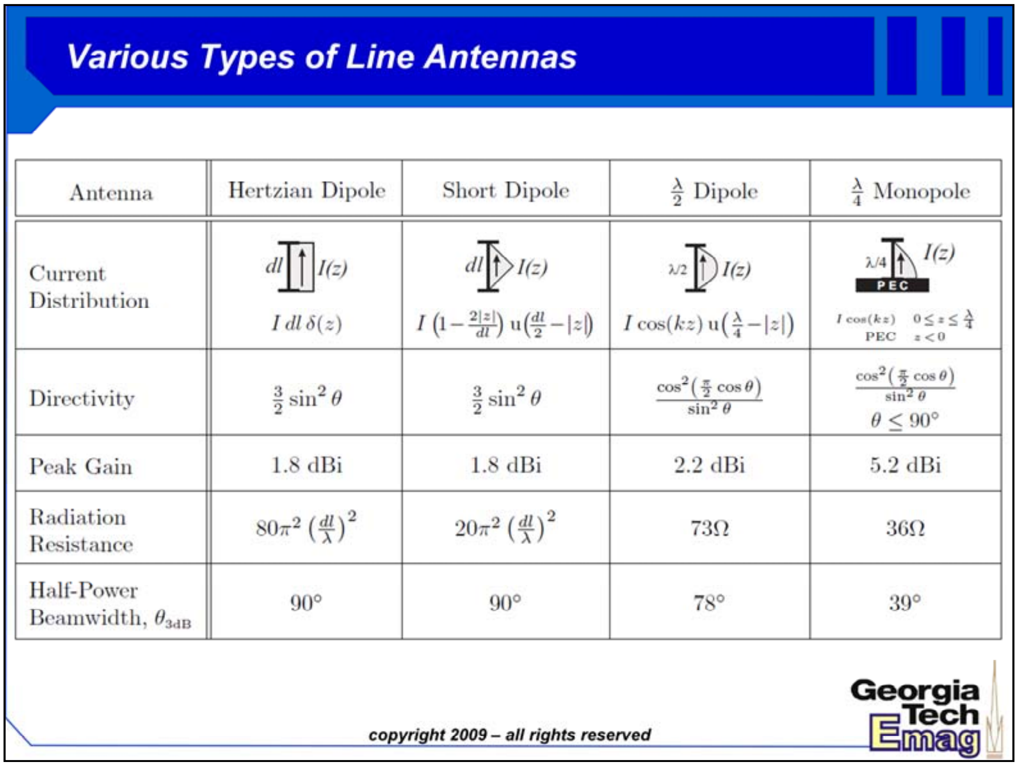
Note that we can start by defining the z-directed current density Jz in terms of the simpler 1-D current distribution I(z) with units of Amps by "collapsing" the current density onto the z-axis with two delta functions with respect to x and y. The simplified expression for magnetic potential is a single integration of this current with respect to a single complex exponent kernal. Here more than before is the very straightforward "Fourier Transform" relationship between current distribution and pattern.

For a HWDP, the current is non-zero over a $\lambda/2$ region, where it is in-phase and sinusoidally-tapered in amplitude. This is basically the standing-wave current pattern at the end of an open-circuited transmission line whose last $\lambda/4$ ends have been bent backwards.



Here is the solution for the HWDP electric and magnetic fields. Note the similarity to the Hertzian/ideal dipole radiator: the fields are at a maximum along the azimuth (theta = 90 degrees). The fields have a null along the z-axis (theta = 0 or 180 degrees). The antenna pattern is omnidirectional, having no dependence on azimuth angle, phi.

Note, however, that the overall elevation cut of the pattern is somewhat more “squinted” than the ideal dipole due to the cos(pi/2cos(theta) term in the expressions. This slightly more complicated expression gives a half-power beamwidth of 78 degrees to the HWDP, as opposed to the 90 degrees for the ideal dipole.



Various Types of Line Antennas

Antenna	Hertzian Dipole	Short Dipole	$\frac{\lambda}{2}$ Dipole	$\frac{\lambda}{4}$ Monopole
Current Distribution	$I\,dl\,\delta(z)$	$I\left(1-\frac{2\lvert z\rvert}{dl}\right)\mathrm{u}\left(\frac{dl}{2}-\lvert z\rvert\right)$	$I\cos(kz)\,\mathrm{u}\left(\frac{\lambda}{4}-\lvert z\rvert\right)$	$I\cos(kz)$ $0\le z\le\frac{\lambda}{4}$; PEC $z<0$
Directivity	$\frac{3}{2}\sin^2\theta$	$\frac{3}{2}\sin^2\theta$	$\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$	$\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$ $\theta\le 90^\circ$
Peak Gain	1.8 dBi	1.8 dBi	2.2 dBi	5.2 dBi
Radiation Resistance	$80\pi^2\left(\frac{dl}{\lambda}\right)^2$	$20\pi^2\left(\frac{dl}{\lambda}\right)^2$	73Ω	36Ω
Half-Power Beamwidth, θ_{3dB}	90°	90°	78°	39°

copyright 2009 – all rights reserved

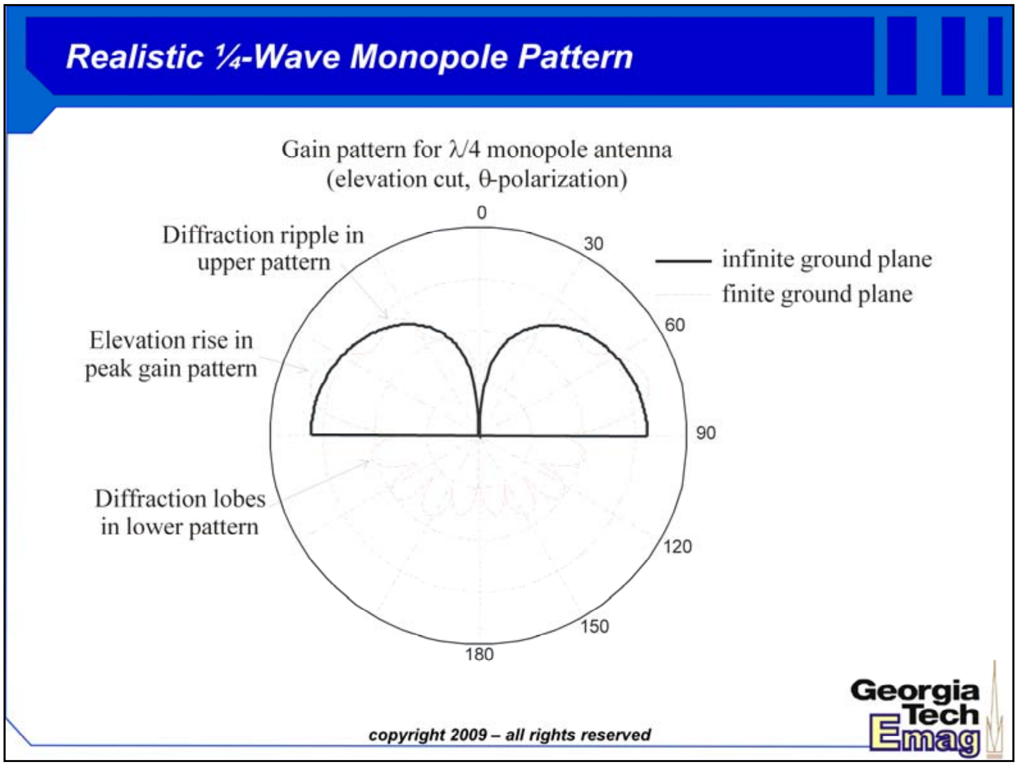
Here is an overview of common types of line antennas and their electrical parameters, many of which may be derived using the same techniques discussed in this lecture.

Ideal Dipole – the Hertzian dipole, which is an extremely small length of radiator with length dl. The fundamental radiator for superposition integrals.

Short Dipole – a very small dipole similar to the Hertzian dipole, however with a current distribution that has a triangular taper. The radiation pattern is identical, but the impedance drops compared to the ideal dipole. More realistic current distribution for a small wire radiator where an open circuit transmission line has bent back to make an antenna.

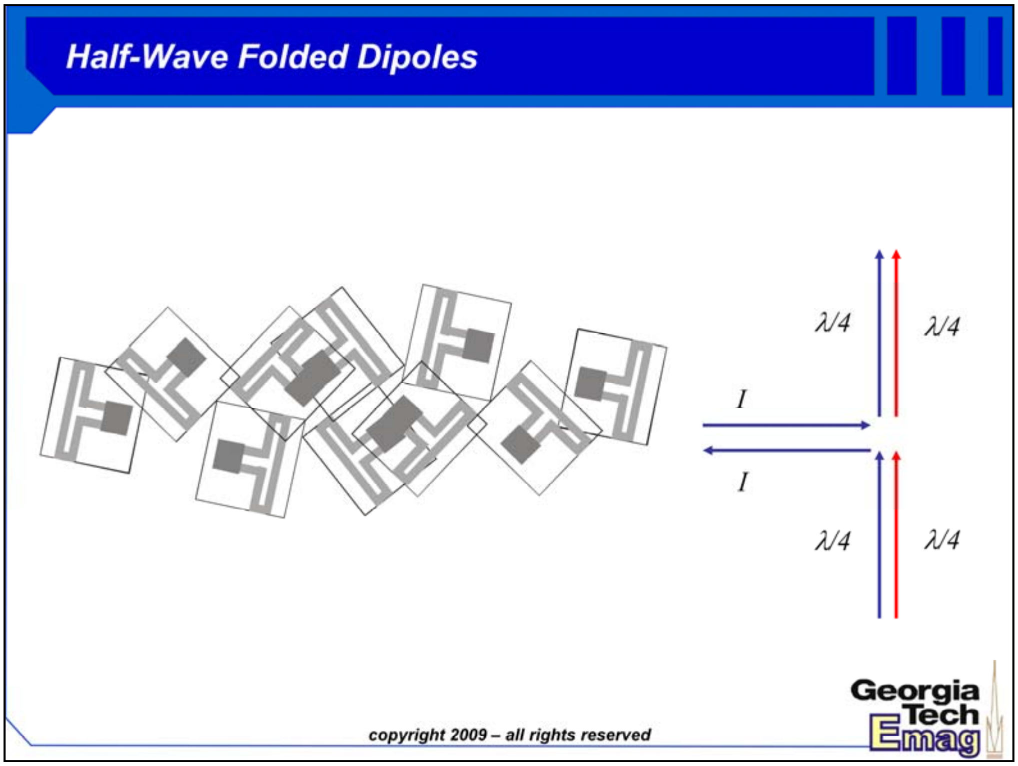
HWDP – an efficient radiator with a large impedance at resonance (70 Ohms). Slightly more gain than the smaller radiating elements. Often, gain reported on specification sheets is referenced against the peak gain of a HWDP (dBd) instead of an idealized isotropic radiator (dBi), resulting in values that are 2.1 dB lower.

Quarter-Wavelength Monopole – this antenna's radiation pattern, by virtue of the image current in the infinite ground plane, behaves identically to the HWDP in the upper hemisphere (theta < 90 degrees). Because there is no radiation (for infinite ground plane) in the lower hemisphere, the result is a +3 dB gain to the upper radiation pattern, which otherwise has the same shape as the HWDP. The impedance drops by a factor of 2 for this antenna as well.



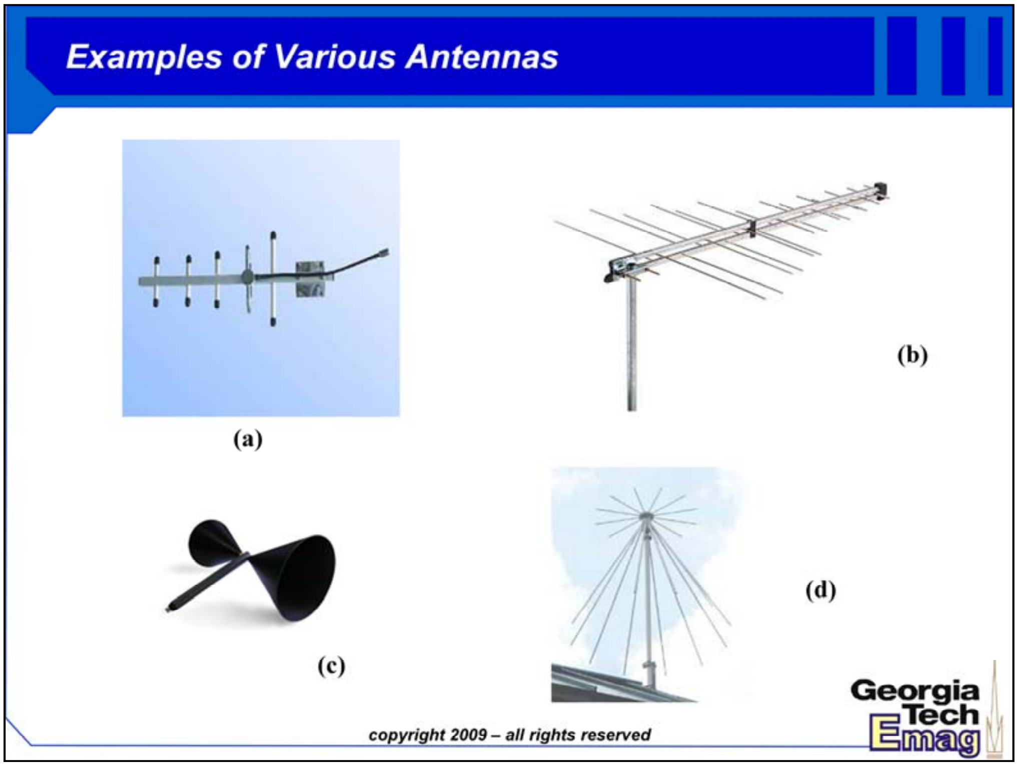
The quarter-wave monopole is one of the most useful forms of antennas, as it radiates efficiently and requires only a ground plane to beat against. What's more, any ground plane will usually work (after some tuning). For example, a car will place the quarter-wavelength monopole on the hood of the vehicle, which becomes the ground plane. For a cellular phone, a "pig-tail" antenna sticks out of the casing and uses the phone's metal housing as the ground plane. A broadcast antenna tower will radiate against the ground it sits on.

In the above plot from Stutzman and Thiele, we can see the non-ideal effects of a finite-sized ground plane. The graph is the elevation-cut pattern of a dipole across the full 0 to 180 degrees of elevation angle (90-degrees corresponds to azimuth). The full line is the radiation pattern of a monopole on a 6-wavelength diameter circular ground plane. The dotted line is the radiation pattern of a monopole on a 6-wavelength square ground plane. Note two things: 1) the realistic groundplanes move the peak gain point in the radiation pattern from azimuth (theta = 90 degrees) upward to a new point of theta = 70 degrees. 2) there is non-zero radiation into the hemisphere of space below the ground plane. Both of these irregularities can be explained by diffracted waves that launch off the edges of the finite groundwave. In fact, we can see the interference pattern of two diffracted waves (on opposite ends of the dipole) constructively and destructively adding for different observation angles greater than 90 degrees.



Half-wave folded dipoles are similar in operation to the HWDP, with identical radiation patterns and sizes. The key difference lies in the shorting bar, which places a second line of sinusoidally-tapered currents alongside the original HWDP radiator currents. Interestingly, this doubling of current distribution results in a factor-of-four increase in the radiation impedance of a half-wave folded dipole. The actual resonant result depends on the gage of the wire used for construction, but the nominal impedance is typically around 280 Ohms.

Because of their high impedance, folded dipole antennas and their meandering variations are favorites of RFID tag designers. Why? Tag antennas behave well in free space, but typically experience significant drops in radiation impedance when brought close to a dielectric or conductive objects. The antenna still radiates in this condition, but it becomes difficult to couple power into and out of the radiator due to impedance mismatches. In extreme cases, on metal impedance of an antenna can drop below a single Ohm! One way to mediate this problem is to start with an antenna that has a very large radiation impedance to start with – such as the folded dipole!



(a) Yagi-Uda Array – Often called a "Yagi" (Yagi was the Japanese advisor, Uda was the grad student that did all the work), this antenna has a dipole radiator. All of the other elements are dead or "parasitic". The larger element is the "reflector" and the smaller elements on the other side of the HWDP are "directors". Adding a single reflector and director shifts the gain pattern (leftward in the picture). Gain increases with additional directors. Here is an example where the Fourier Transform rule-of-thumb for currents does not work as intuitively – this is a "traveling wave antenna".

(b) Log Periodic – A really broadband, directional form of dipole. Why is it broadband? Because the diminished (logarithmic) scaling-down of radiating arms that are all tied together, there should be somewhere along the left or right side a pair of arms that resonates like a half-wave dipole; the other elements do not radiate as effectively and do not contribute to the gain pattern. Thus, as frequency is swept higher and higher, the "active arms" of the antenna switch further and further down the antenna.

(c) Bicone Antenna – Think of this as a fat HWDP. The benefit is that the bandwidth of operation becomes much larger than the typical 2-3% of carrier frequency experienced by a wire-based HWDP.

(d) Discone Antenna – Bicones are to HWDP what Discones are to quarter-wavelength monopoles. In this design, one of the conical elements of the bicone is replaced with a ground plane. The picture of the discone antenna above is particularly interesting because it is mechanically large (low frequency) and saves construction material and weight by replacing the solid sheet of conical and ground plane surfaces with closely-spaced (electromagnetically speaking) conductive rods. Beyond a certain frequency, the "look" the same as solid metal sheets to a radio wave.