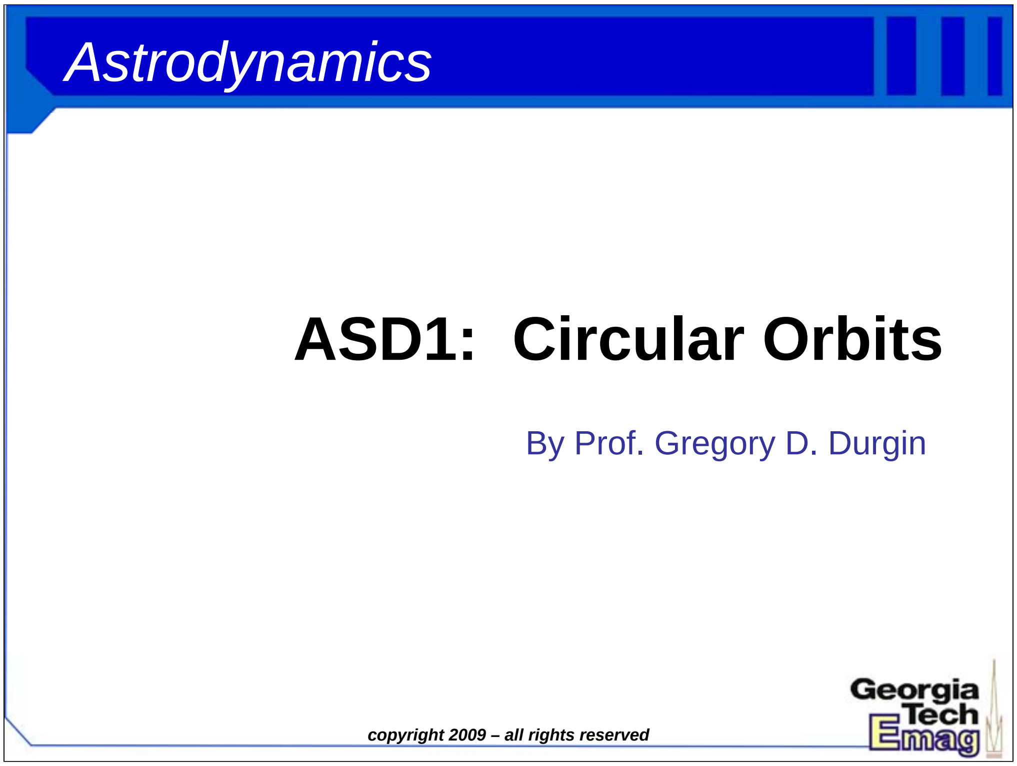
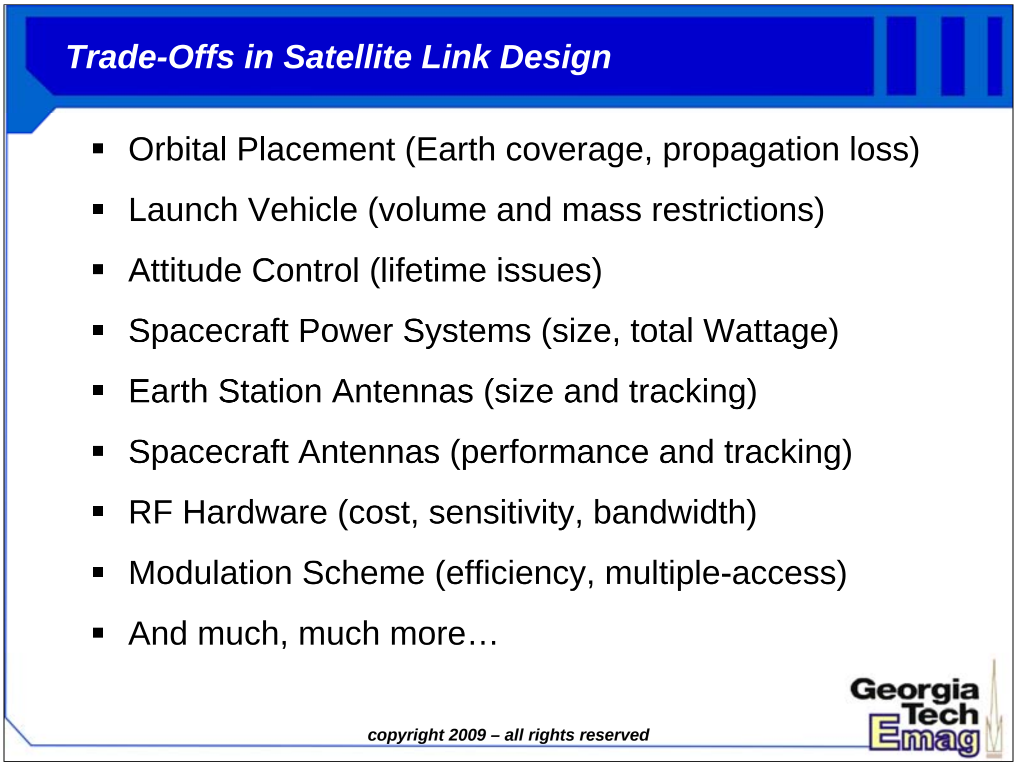



In this lecture, we study the basic mechanics of circular orbits, which are the simplest of orbits to analyze. We build on these concepts in the next lecture to handle the general case of elliptical orbits.

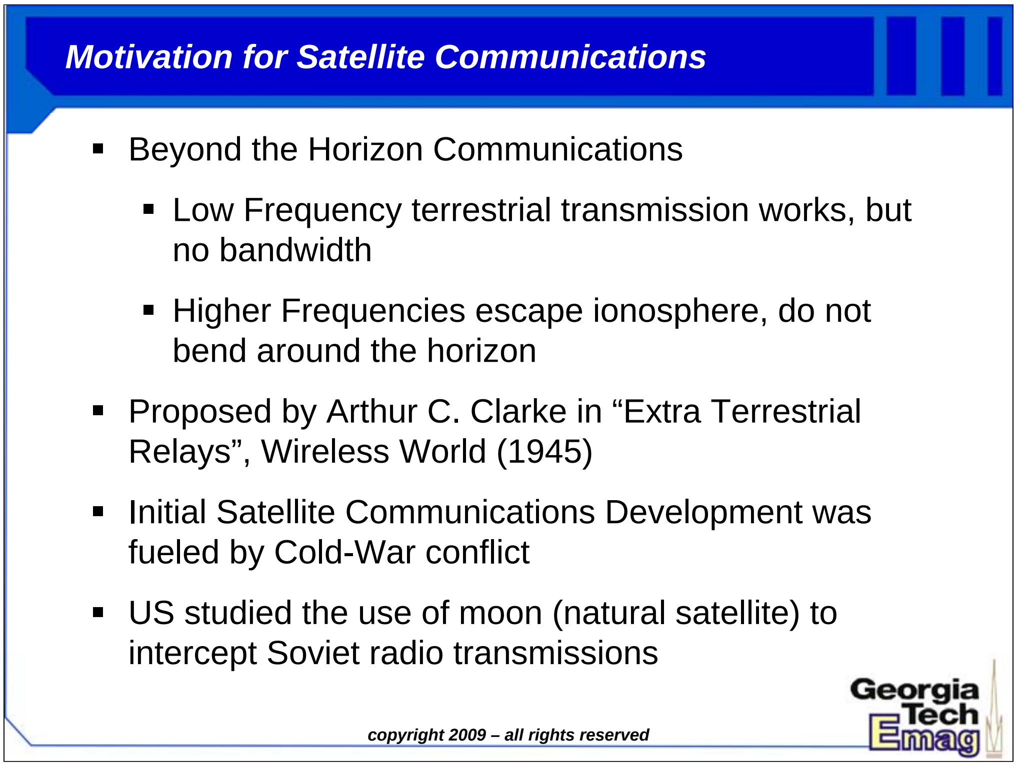
We should note that even though this is an Electrical Engineering course, we will still need to study the basic physics of orbital mechanics/astrodynamics. The fact is, a satellite's orbital trajectory vastly affects the design of its communication link. If we're cost-conscious at launch, we may only send the satellite a few hundred miles above the surface of the earth … but it would only cover a small region of the planet and would zip across the visible sky in just several minutes. We could push it further out, but this increases launch costs … and makes the link distance (the enemy of satellite distance) increase substantially. In both cases, we'd need to study the orbital mechanics to figure out how the earth station in our communications link needs to move its antenna to track the satellite transponder.

Unless we place the satellite at a "magical" distance away from the earth, in geostationary earth orbit where from earth the satellite appears to hang in the sky at the same point perpetually. Of course, a geostationary earth orbit is very far from the surface of the earth, making it expensive; care must be taken to specify antennas with enough gain at both the satellite and the earth station in order to overcome this long distance (and often an antenna pattern on earth that can hone in on a particular satellite and spatially remove nearby, in-band interferers also in GEO orbit – it's a crowded orbit nowadays!)




So those are some of the trade-off's in a nut-shell. Orbits affect launch costs and propagation, which affect antennas and RF hardware. These, in turn, place some fundamental limits on communication performance, how many users can communicate with a fixed amount of radio resources. And then there is how communication hardware trades-off cost and performance, limiting what can realistically be implemented at an earth station (or mobile handset, or surface rover, etc.) And satellites, unlike many other types of communication nodes, are stranded in space in a power-limited environment – where it cannot even be easily serviced or upgraded. All of these design attributes and more trade-off against one another, so it is important that the engineer be trained to recognize the interrelationships.

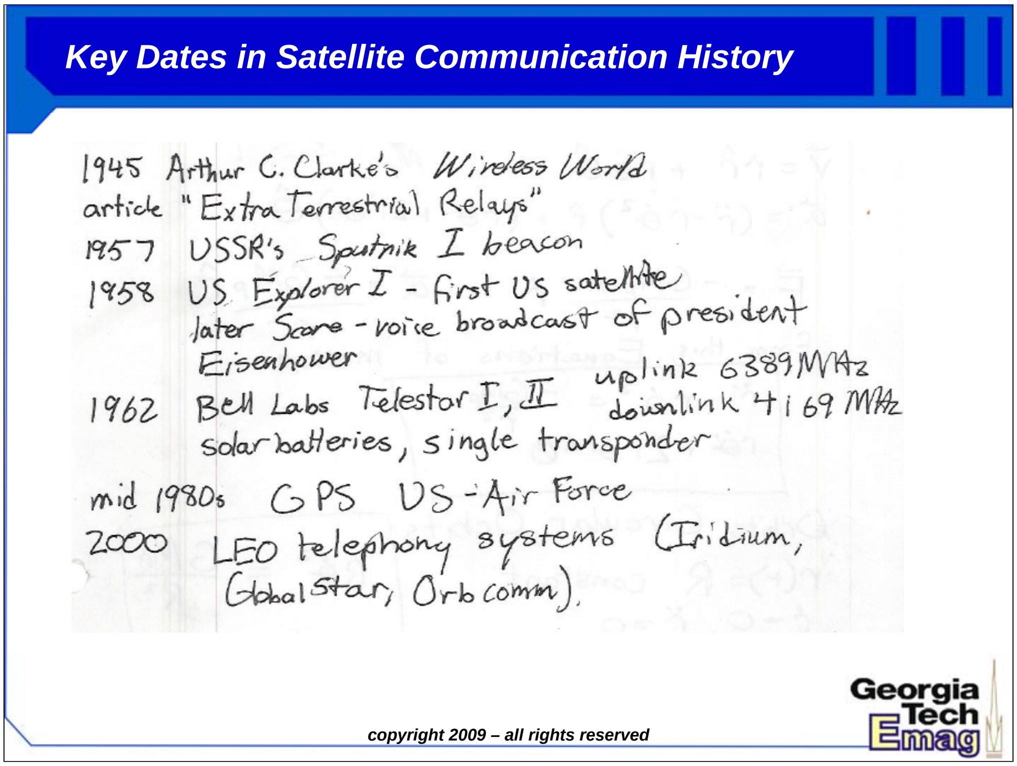
At times, it may even feel like pounding out lumps in a carpet. Just when you think you've hammered down one lump, you realize a spot popped up somewhere else in the room. "Oh, I don't have enough carrier-to-noise ratio in my space-to-earth link." Well, I'll just transmit extra power. But that would require more spacecraft energy and more solar cells, which would add weight and it looks like I'm already at my mass and volume limit for the rocket that I've chosen to launch my craft (the next rocket size up is millions of dollars more … a non-starter). I could purchase a more sensitive low-noise amplifier (LNA) at the receiver, but a quick calculation shows that the current one is pretty good – the best-on-the-market amplifier only buys me a fraction of a dB in the link. Well, I could increase my dish size on earth, which turns out to be a nice solution on paper, but the dish becomes so directional that point issues arise. Well, I could increase the order of modulation (use 64-QAM instead of QPSK) and add extra error correction code. Hey, this gives me the proper link performance, but now I need to buy much beefier DSP hardware for the satellite to process the signal – and the cost of high-performance, space-hardened electronics is not trivial.




There are many types of useful earth communication links in satellite radio, but they all typically have one common element: the ability to communicate between two places on earth that lie mutually beyond the horizon from one another. Before satellites, there was no effective way to reliably accomplish this (at high information rates) with radio communications. You could do some short-wave communications, using the fact that the ionosphere serves as a sort of the upper plate of a conductive waveguide (the ground is the bottom plate) and allows radio waves under 1 MHz to either bend around the curvature of the earth or ricochet beyond the horizon. But there’s not much bandwidth available here for communications.

In a 1945 volume of Wireless World magazine , Arthur C. Clarke published an article entitled “Extra Terrestrial Relays”, with this idea of using devices in space to relay lots of data (in his day, analog voice and video) across large stretches of the earth. This was a revolutionary idea. The cold war was beginning to heat up and America had an adversary that it wished to monitor, the Soviet Union, on the other side of the world. It was cold war competition that would drive the earliest development of satellite communications.

In fact, during the 1950s, the US seriously looked at spying on the Soviets using the earths natural satellite – the Moon – to do the communications relaying. The idea was that, under certain times of day, Soviet radio signals would bounce off the moon, back to a US-friendly location where the link could be monitored. This problem resembled a radar problem and, although the moon has a large RCS, it is extraordinarily far away (>100,000 miles). Furthermore, it has a rough surface and is in motion, making for a tricky problem to characterize – even if you could build an earth station with receiver hardware sensitive enough with enough collecting area to reign in the signal.

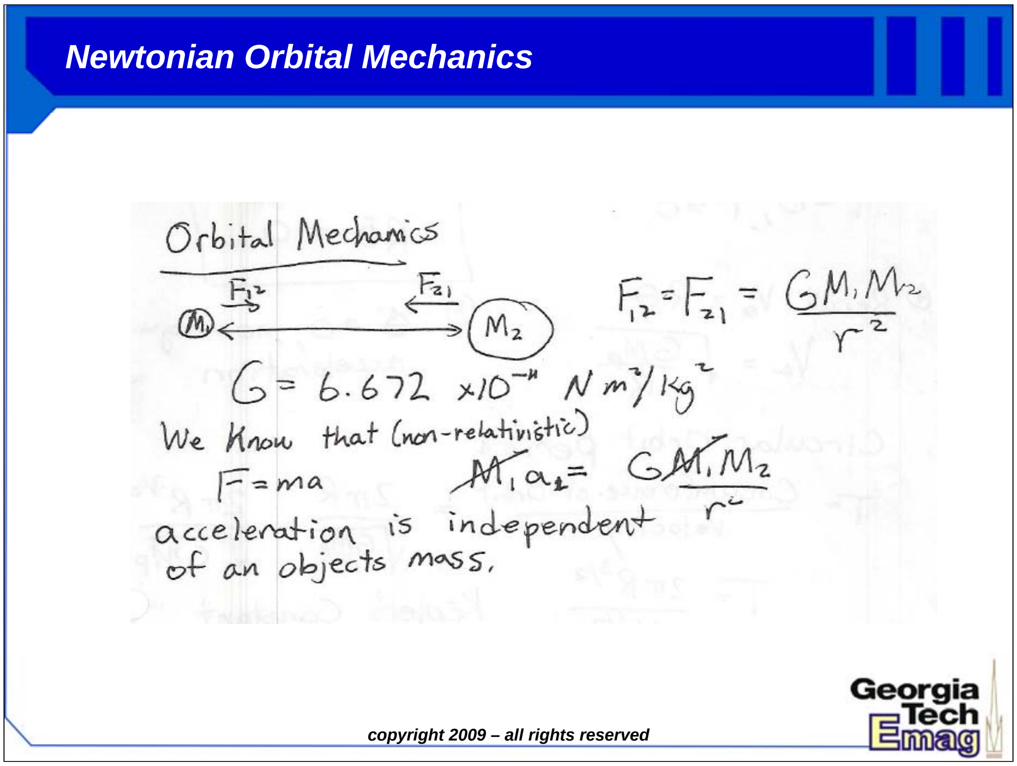



Then came the great surprise of 1957, when the Soviets launched Sputnik I – the first man-made satellite, a battery-driven radio beacon. It is difficult now to appreciate the psychological impact this had on the world and the US in particular. Our greatest adversary had this *thing* hovering over our country every hour or so, radiating *Russian radio waves* down on us. And there wasn't a thing we could do to stop it.

So this was a wake-up call and the US quickly launched its own satellite, Explorer I, in 1958; later that same year, the Score satellite launched and broadcast the President Eisenhower's State of the Union address.

In 1962, Bell Labs launched Telestar I and II, which are considered the fist "classic" telecommunications satellite. They had 6389 MHz uplinks and 4169 MHz downlinks and solar batteries (longer lifetime … early satellites like Sputnik and Explorer had purely battery-driven systems and died less than a year after launch). The Telestars had only one transponder (channel for relaying) but they became the archetypes for later voice and video relays. If you've seen some of the old-style 6-12 foot satellite dishes, these operate in the 4 GHz band.

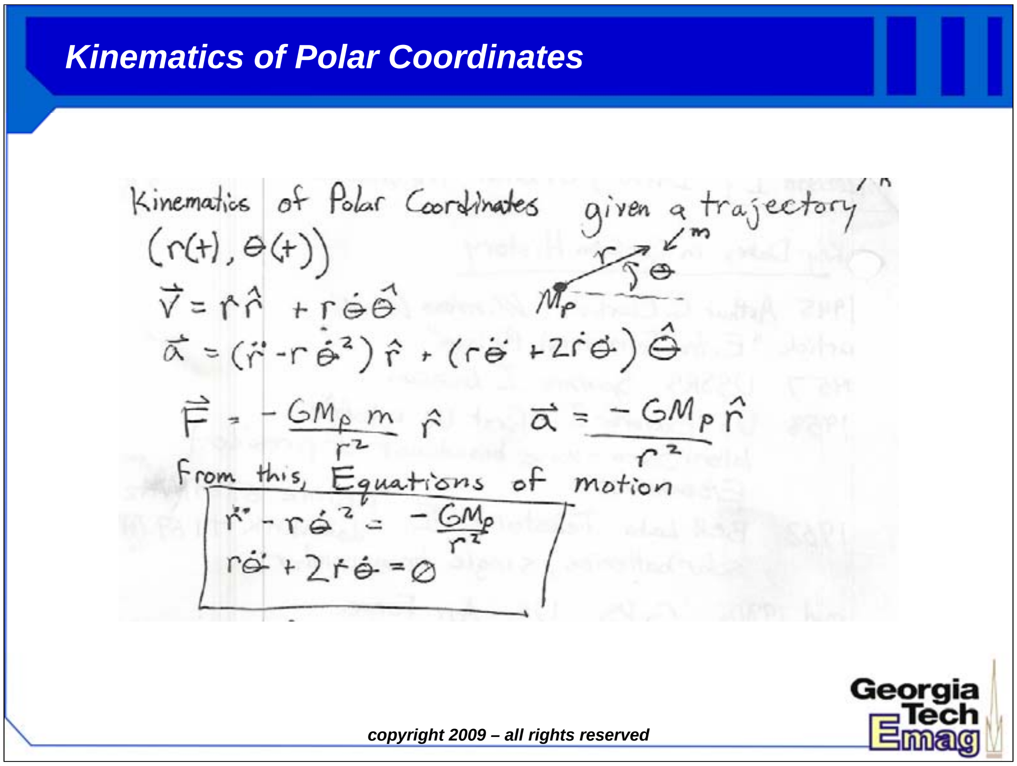
In the mid 1980s, GPS came online publicly. Operated by the US Air Force, it was the first globally available positioning system. Developed in the 70s with the initial impetus to guide ICBMs (why does one need 1m positioning accuracy to drop a nuclear weapon?), GPS quickly revolutionized navigation for shipping and aviation; and now enjoys quite a bit of commercial private use. In 2000, there was a proliferation of low-earth orbit communications systems. These networks (Iridium, Globalstar, Orbcomm) were great engineering successes, demonstrating mobile communications across the globe. Their business cases failed miserably.




Let's start by reviewing the basics of gravitation, using this to derive our orbital mechanics descriptions from first principles. Given two masses, M1 and M2, separated by a distance r, there is a mutually attractive force on these two bodies, equal in magnitude, opposite in direction. The formula describing the magnitude of this force is the crown-gem of Newtonian gravitation theory, predicting that the force exerted on each mass falls off inversely proportional to the square of the separation distance. The gravitation constant in the formula, G, is extremely small, such that at least one of the masses must be extremely massive to build up any appreciable force over distance.

An interesting thing happens when we bring in the Newtonian relationship of force and acceleration into the picture: F=ma. If the force, F, is solely due to gravitation, then we may equate the two formulas and solve for how one object accelerates in the presence of another. Interestingly, two masses will cancel one another and lead to an acceleration of an object that only depends on the mass of the other acting upon it.

As a side note, this mass-equivalence principle – the fact that gravitation-causing mass of an object is the same intrinsic quantity as the objects resistance to change in momentum – puzzled physicists for the longest time, because it didn't have to work out that way. In fact, it was this property of gravitation, along with a few other observations, that led Einstein to formulate his theory of general relativity. Since an objects motion through space is independent of any intrinsic property (its own mass), then its trajectory appears to be a property of the space it occupies; when he considered that light would bend in a gravitation field with similar properties, he then brilliantly postulated that gravitation was a local property of space, which was "curved" by the presence of other masses. Thus, it was more correct to characterize the attributes of a region of space and study light and matter mechanics in this region of space than to force everything into a "force-over-distance" field framework. Our class unabashedly uses the older Newtonian model because it is just so dang simple and useful.




So we know what the acceleration will be for an object in the presence of a mass (let's use Mp as mass of planet from now on). From this, we can derive the basic differential equations of motion in two dimensions. Note that our 2D dimensional treatment can be extended to 3D without loss of generality, since an orbital trajectory in a 2-body system can always be reduced to an "orbital plane" with which we can transform the mechanics into a 2D description.

Now, it is easiest to place the massive planet at the origin and work the problem in cylindrical coordinates. Here, I am using the mathematical/physics convention of marking radial distance from the origin with r and angle of position as theta. When we do some electromagnetic theory, I'll switch back into the EM/engineering convention when r becomes rho and theta becomes phi. But we'll make the physicists happy in our analysis of astrodynamics.

Now, proper positions, velocities, and accelerations are all vector quantities. The position vector (r,theta) describes the point of our satellite, orbiting about the planetary origin; both r and theta are functions of time. If we open up the back of our dynamics textbook, we can find the basic definitions of velocity and acceleration in the polar coordinate system based on these positional functions (they're not nearly as straightforward as in the Cartesian coordinate system). From the previous slide, we know that there will always be an acceleration towards the origin (no radial acceleration); equating this result to our kinematic definition of acceleration produces two sets of second-order differential equations. These our the general dynamic equations of orbiting bodies. We'll start with the simplest possible case of circular orbits and come back to this general system to extend our analysis.

## Circular Orbits

Orbits, Circular Orbits

$r(t) = R$ constant

$\dot{r} = 0, \ \ddot{r} = 0$

$$\boxed{\begin{aligned} R\dot{\theta}^2 &= \frac{GM_p}{R^2} \\ R\ddot{\theta} &= 0 \end{aligned}}$$

① Recall $V_\theta = R\dot{\theta}$

$$V_\theta = \sqrt{\frac{GM_p}{R}}$$

② $\ddot{\theta} = 0$, no angular acceleration

Circular Orbit period

$$T = \frac{\text{Circumference of Orbit}}{\text{Velocity}} = \frac{2\pi R}{\sqrt{\frac{GM_p}{R}}} = \frac{2\pi R^{3/2}}{\sqrt{GM_p}}$$

$$T = \frac{2\pi R^{3/2}}{\mu^{1/2}}$$

Kepler's Constant

$$\mu = GM_p$$

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The conditions for circular orbits are fairly straightforward to express and allow us to simplify the set of differential equations of motion. First, the satellite must stay a fixed distance, R, from the center. It has to be circular, after all. This implies that the radial component of the satellites position vector will be constant (r(t)=R) and that its first and second time derivatives must be 0 (if it's constant, the radial trajectory can't change).

This results in much simpler equations. One immediate consequence is that the second-derivative of the radial component of trajectory *must* be zero. In other words, a satellite will never corkscrew around the planet at a faster and faster (or slower and slower) rate; instead, the satellite in circular orbit will sweep out an unchanging radial velocity around the planet. Since R times theta-dot is the satellite's tangential velocity around the circular orbit, tangential velocity will be constant. Furthermore, the second equation implies that this constant velocity will depend on the distance from the planet. As a satellite moves further from the planet, it travels slower to stay in a stable circular orbit.

Satellites speed up as they are established in circular orbits closer to the planet. Furthermore, the total distance they must travel to complete one orbital period (the circumference of the circle) decreases as well. As a result, the total period (time) it takes to travel one period around the planet in circular orbit depends on R^3/2; this is why distant satellites (such as the moon) take quite a long time to complete a single orbit (28 days).

Note: in some formulations of orbital equations, you will encounter "Kepler's constant" mu, which is equal to G times the mass of the planet. This was how Kepler formulated the original 3 laws for motion, assigning each body (such as the sun, the earth, or a planet) a Keplerian constant. It wasn't until later that the connection was made between a planet's Keplerian constant and its more-intrinsic property of total mass.

## Geosynchronous Earth Orbit

Example: Geosynchronous Orbit 3/8

Rotation Period: 23 hr. 56 min 4.09s *

$$T = 4.09\,s + 56\,min\,(60\,s/min) + 23\,hr\,(60\,min/hr)(60\,s/m) = 86{,}164.09\,s$$

Mass of Earth: $5.974 \times 10^{24}$ kg

$G = 6.672 \times 10^{-11}\ Nm^2/kg^2$

$$T = \frac{2\pi R^{3/2}}{\sqrt{G M_p}} \Rightarrow R = \left(\frac{G M_p T^2}{4\pi^2}\right)^{1/3}$$

$$R = 4.216 \times 10^{7}\,m = \underline{\underline{42{,}164.17\ km}}$$

The Most Important Answer in the Class!

*sidereal day: $\frac{24\,hrs \times 60\,min}{365.25} = 3.94\,min$ less time than a regular day.



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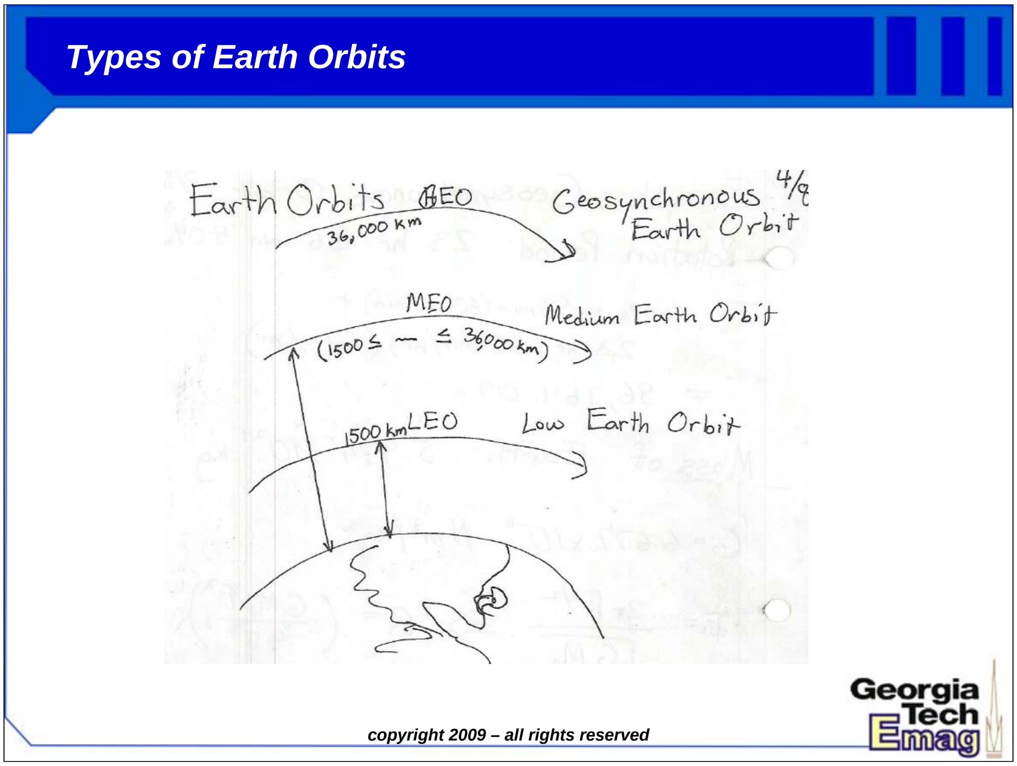
Now we have enough knowledge to calculate one of the most important results in the entire class: how far away do you have to launch a satellite to put it in a circular, geosynchronous earth orbit? A satellite in such an equatorial orbit would appear to hang in the sky at the same point, moving at the same radial speed as the rotation of the earth. Such satellites are extraordinarily useful, because we can point simple, high-gain dish antennas towards them and not deal with tracking issues. We can also cover the same, constant area of the globe depending on where we place the satellite, eliminating the need for coordinating coverage with a fast-moving constellation of lower-earth orbit satellites.

One key caveat: we must match the radial velocity of the satellite with 1 sidereal day (23 hours, 56 minutes, 4.09s), which is not the same as a solar day (24 hours). This is because our perception of a solar day is the time it takes for the sun to travel back to the same point in the sky as the day before; this time not only includes the absolute radial velocity of earth, but also a small amount of angular dislocation due to the revolution around the earth. Not surprisingly, if we took the difference between a sidereal day and a solar day (about 4 minutes), multiplied it by 365 (and a quarter), we'd get 24 hours! Why? Because as the earth winds around the sun, it effectively "erases" one solar day of sunrises and sunsets during the course of a year. So there are actually 366 (and a quarter) sidereal days in a year. The phenomenon is similar to the one in Jules Verne's "Around the World in 80 Days", when his protagonist completes his race, despite thinking he was finishing the last leg of his journey to London on the 81st day. By traveling westward, he had effectively experienced 1 more sunrise and sunset during the course of his 80-day travel than his London spectators.

The magical radius for geosynchronous circular orbits is about 42,000 km as measured from the center of the earth – or 36,000 km in terms of altitude.



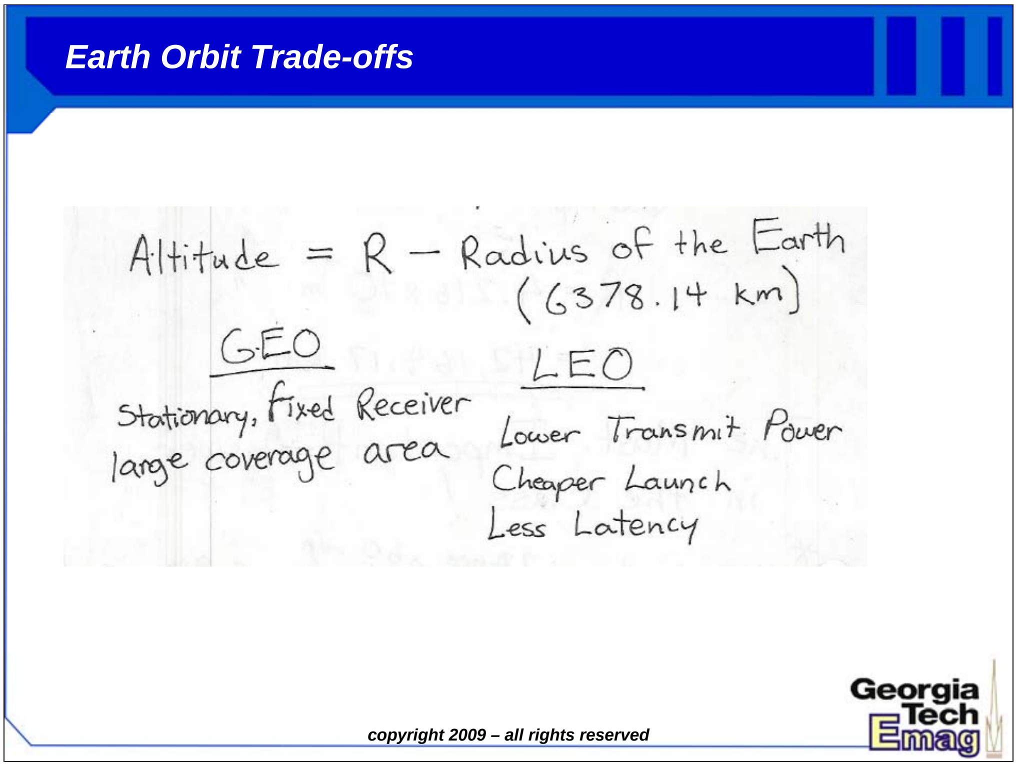

This number gives us a nice way to classify satellites in earth orbit. As satellites get higher and higher, they move slower, cover more of the earth, and get much more expensive to put there. In general, we speak of 4 types of earth orbital altitudes

• Low Earth Orbit (LEO) is any orbit with an altitude less than 1500 km. These are cheapest to launch, but zip across the sky very quickly. Due to low launch and replacement costs, many communications satellites use networks of LEO satellites to trunk data and mobile telephony from one point on the earth to another. This orbit is also used for surveying and weather satellites.

• Medium Earth Orbit (MEO) is any satellite orbit above 1500 km and less than 36,000 km. These orbits are less populated because they are more expensive to launch into. Earth coverage is better here, however, so some satellite systems that require multiple simultaneous satellites for operation (GPS) operate here.

• Geosynchronous Earth Orbit (GEO) is the designation for any satellite that is 36,000 km above the surface of the earth. This orbit is again highly populated because of the unique functionality a stationary satellite provides for broadcasting information to fixed dishes on earth.

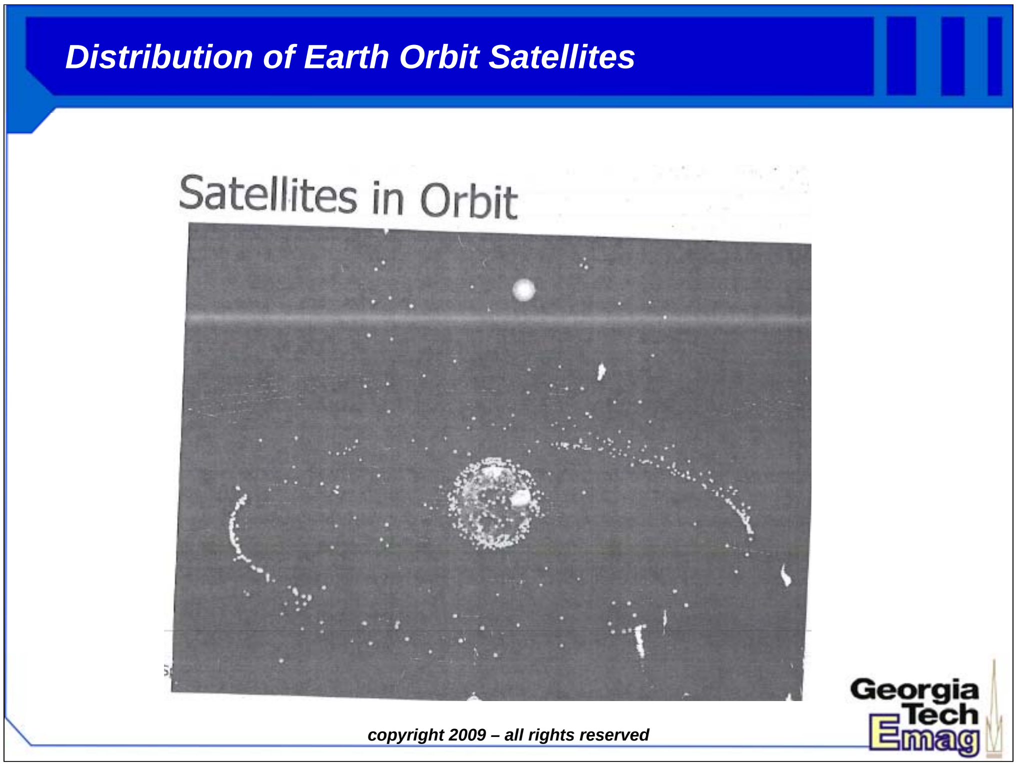
• High Earth Orbit (HEO) is any orbit above the GEO 36,000 km altitude. Extremely rare for anything other than scientific spacecraft or de-orbited GEO satellites.




Don't forget the difference between altitude and radius of orbital calculations. The problem wordings in this class will always make it very clear what information you are given for calculations. The exact sea-level radius of the earth is 6378.14 km, so it is trivial to convert from radius to altitude … you just need to pay close attention to what values you are using.

The difference in altitude makes it very clear what the advantages in LEO vs. GEO satellites. In addition to cheaper launch costs, LEOS require much less transmit power for radio signals to overcome the orders-of-magnitude less distances to earth and the much higher attendant path losses. This was actually the most significant factor in choosing low LEO satellites for global mobile telephony – transmit power at a mobile phone required to get a signal to a satellite was too high above LEOs; since the mobile phone was placed right next to a human head, the transmit power required to get to a GEO satellite would have been comparable in magnitude to sticking your head in a microwave oven.

Additionally, there is less temporal latency in an earth-satellite-earth LEO link than a GEO link, which may be a consideration in some links. It takes almost a quarter of a second for a signal to make a round trip relay through a GEO satellite – one of the few examples of when the transit time of a radio wave is significant enough to pay attention to.




Here's a diagram of where satellites are with respect to the earth. Notice the dense cloud of satellites swarming in LEO around the earth. Note also how uniformly they are spaced about the planet, not limited to any one path.

Conversely, the MEO region is relatively sparse, with just a few surveying and positioning satellites sprinkled in this region.

Then we arrive at the dense ring of GEO satellites, with many companies and governments desiring that broadcasting ability to fixed-dish antenna earth stations. Although it's not readily apparent from the low-quality picture, you can probably discern exactly where North and South America and Asia are when this particular orbital plot was captured. The very thick band of satellites on the left hover above North and South America; the broader but slightly less dense band of satellites on the right serve Europe and Asia. The huge expanses of the Pacific and Atlantic do not warrant many GEO launches. Satellite TV companies discriminate against fish!