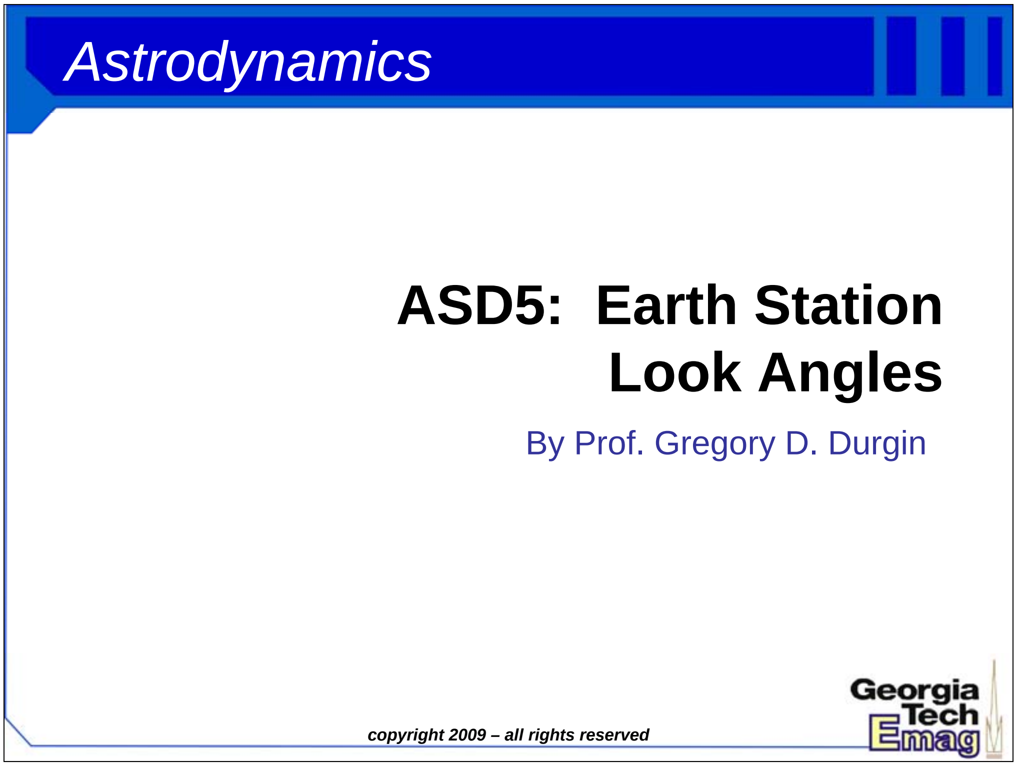
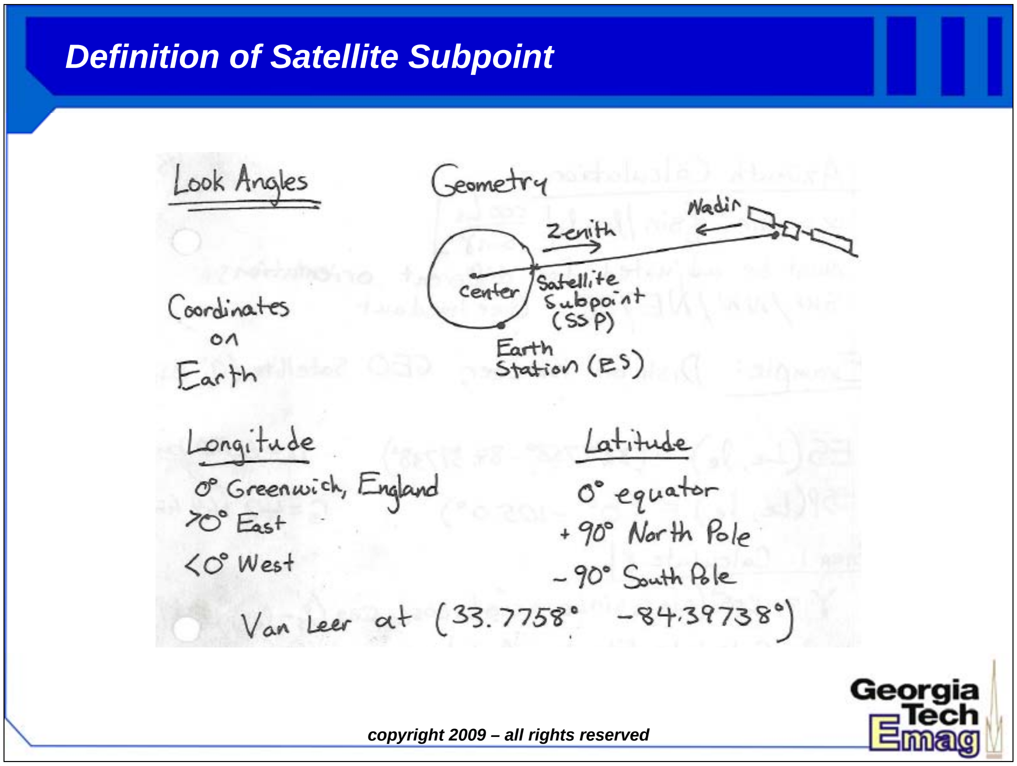


In this lecture, we learn how to translate orbital trajectory into a look angle from a vantage point on the earth. In previous lectures, we learned how a satellite travels through space, describing the basic 2-body orbital problem as well as some of the things that go wrong with it. But one of the most important questions to ask in satellite communications is: where do we point our dish antenna in order to receive a certain satellite. So this lecture is about how to translate a position in space into a direction to point our satellite dish antenna.

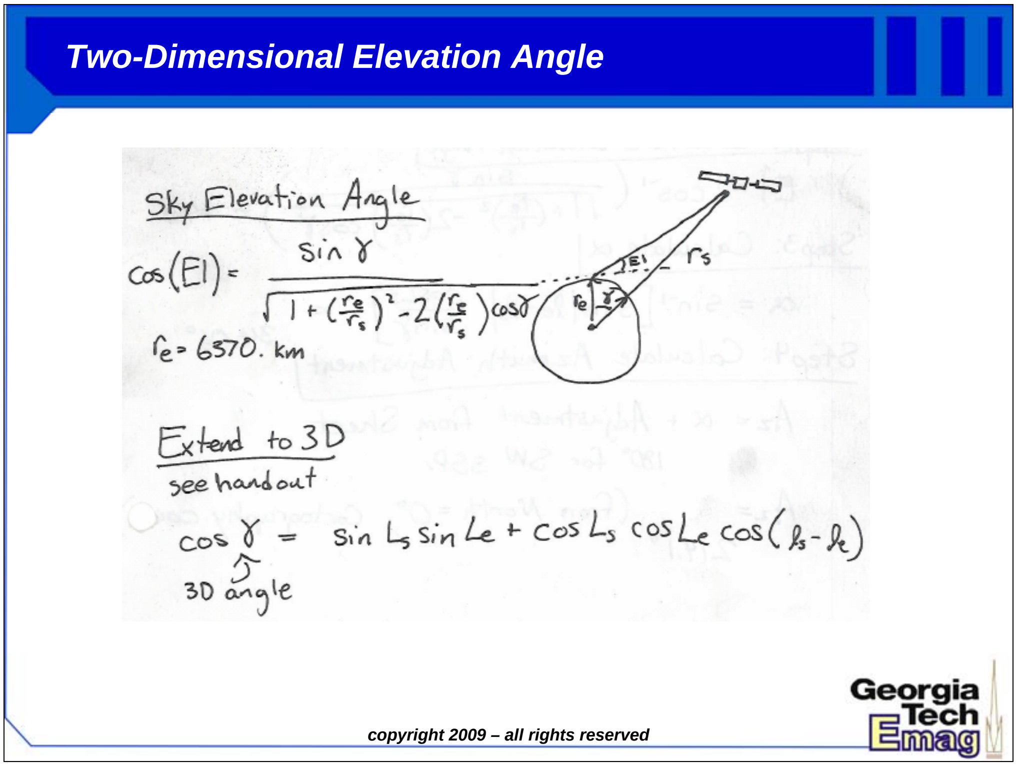
After all, many satellite systems require a large dish antenna to complete the radio link. Sometimes this is because we need a great deal of gain to get enough carrier power into our receiver; other times, it has more to do with getting a dish antenna with beamwidth narrow enough to spatially reject co-channel interfering satellites, to hone in on a single satellite and nothing else in an otherwise chatty and noisy spectrum. In either case, we need to be careful and precise to point our dish antenna since, as we will explore in greater detail later, dish antennas can achieve incredibly high gains, but only in a single direction; if we increase the efficiency in which an antenna collects power in one direction, we must necessarily decrease average efficiency in all the other directions – antennas are not amplifiers after all. So we need to be careful how we point them, or else we will either miss that narrow lobe of highest gain or, even worse, we will move it straight on top of an undesired satellite operating in the same band.



First, recognize that we use latitude and longitude to specify position on the earth. In our look angle problem, we will specify the earth station (ES) location in terms of longitude and latitude in addition to the satellite subpoint (SSP). The SSP is the point on the earth where a straight line drawn from the earth's center to a satellite position would pass through the surface of the earth. In this geometry, a person standing on the SSP would observe the satellite to be directly overhead (a direction called zenith).

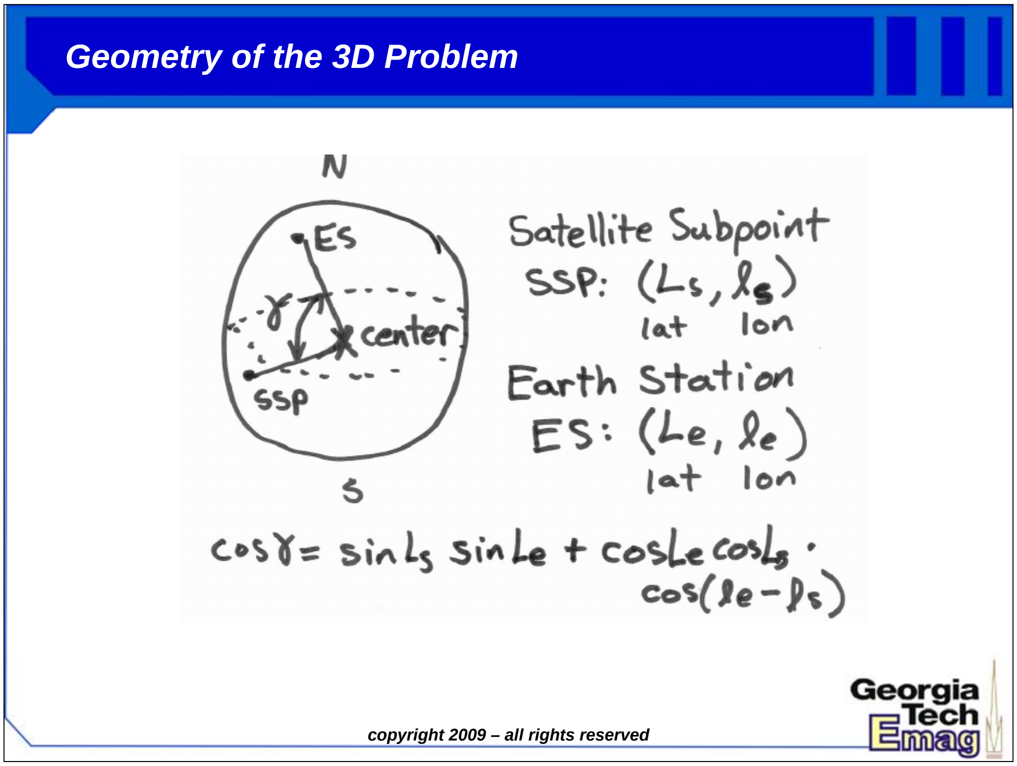
Keep in mind that latitude represents the degree to which we approach the north or south poles on the earth. Under this convention, zero degrees corresponds to the equator: +90 degrees and -90 degrees correspond to geographic north and south poles, respectively. Longitude represents the east/west bearing of a location on the earth's surface. Zero degrees passes through Greenwich, England; negative values count off in an easterly direction while positive values count off in a westerly direction.

Of prime importance to this class, the Van Leer building in Atlanta is (33.7758 lat, -84.39738 lon).



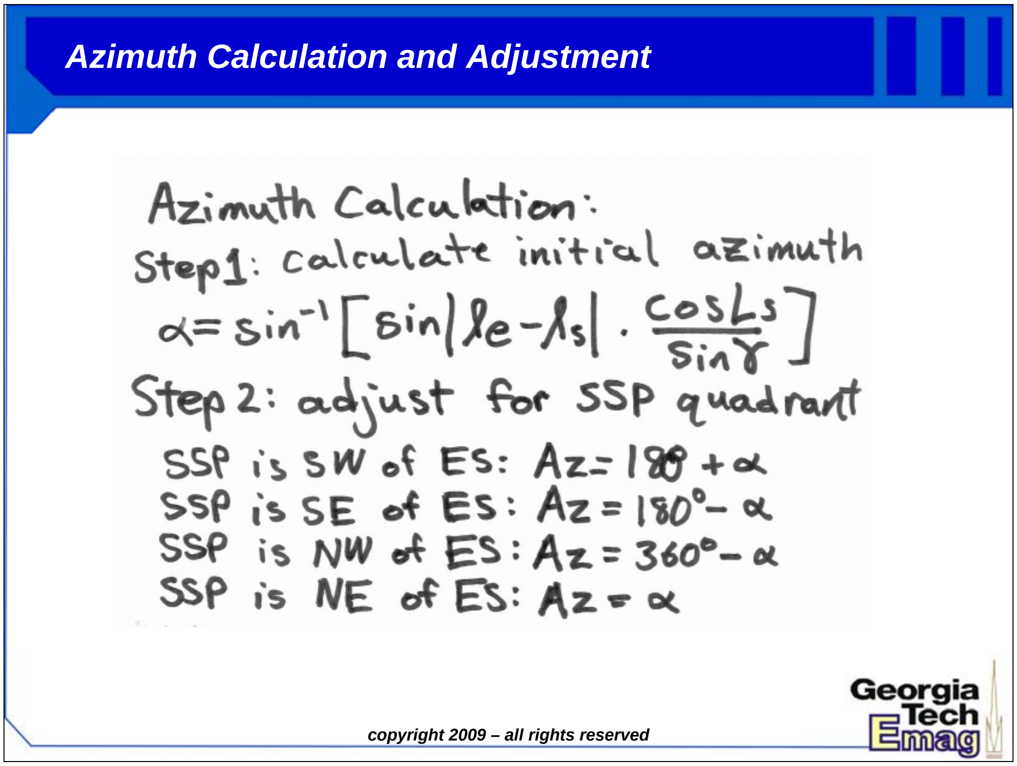
In two dimensions, a look angle calculation is straightforward enough. If we are given the radius of the earth and the radius of the satellite at a given instant in time, as well as the angle gamma that subtends these two directions at the earth's center, then we may determine exactly the rest of the sides and angles of the triangle shown above (law of sines and cosines).

The elevation angle that we wish to calculate in this case is the obtuse angle shown above minus 90 degrees. This is because a line drawn tangent to the circle at the ES point is perpendicular to the earth radius at this point – and this line represents the effective horizon that we would experience at this point. In this 2D world, we would have to elevate our dish antenna an angle (El) above the horizon in order to track the satellite's position.



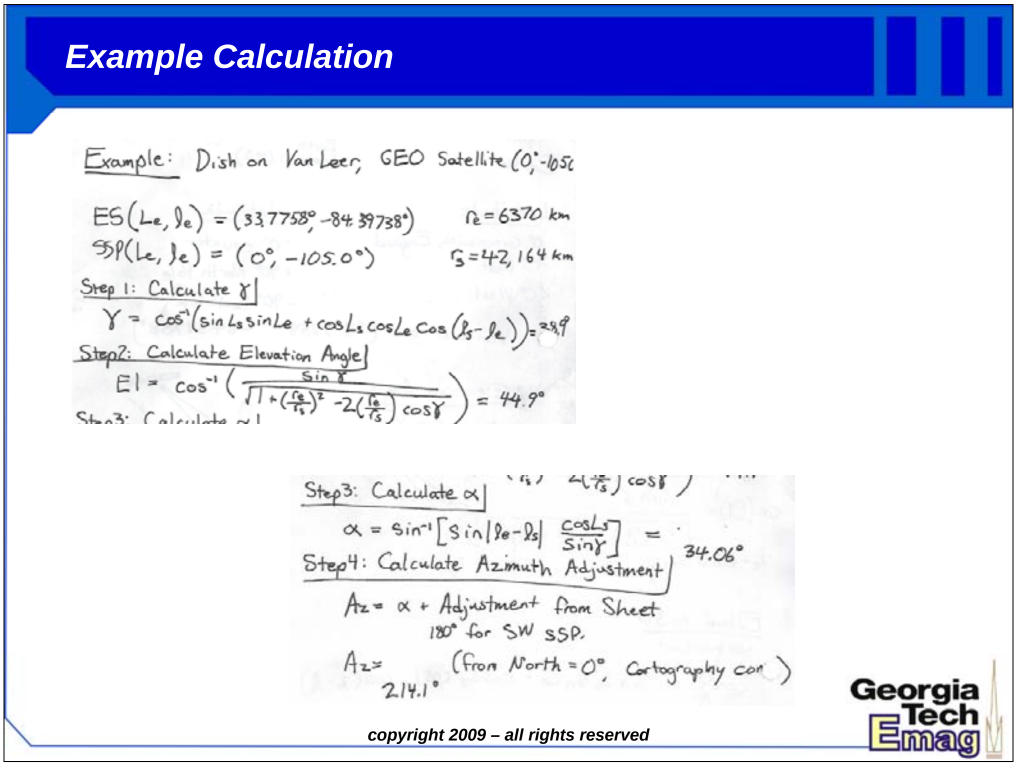
To translate this into 3D, we must first calculate this angular distance gamma from two pairs of longitude and latitude, one for the SSP and one for the ES. Given the geometry above, we need to define several helper variables (psi, beta, and little delta) based on the earth station position (Le, le) and the SSP (Ls,ls). Uppercase L refers to latitude and lowercase l refers to longitude in the system of equations above.

From the equation above, we may calculate the angle gamma from this pair of location specifications. After this calculation, we may use the formula on the previous page to calculate elevation angle. But in 3D, this is only a portion of what is necessary to point a dish. We also need the azimuthal bearing – which direction along the horizon to point the dish antenna.



The azimuth calculation is just a bit trickier (this is not included in the book). The formula above calculates a preliminary azimuth bearing angle, alpha, based on the longitude and latitude of both ES and SSP. Once calculated, one must use the table to make a piece-wise adjustment to this angle before arriving at the final azimuth angle (Az).

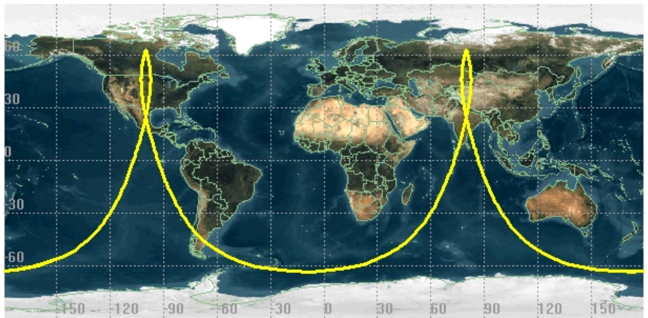
The final (Az) angle is measured with respect to North, using the cartographer's convention of representing increasing angles with clockwise motion on a standard Mercator projection map. Thus, North corresponds to 0 degrees, East corresponds to 90 degrees, South corresponds to 180 degrees, and West corresponds to 270 degrees. Note that this is contrary to the mathematical and engineering conventions of representing "East" (the x-axis) as 0 degrees, "North" (the y-axis) as 90 degrees, "West" as 180 degrees and "South" as 270 degrees.



Here is a sample calculation to demonstrate the technique in 4 easy steps. In it, we demonstrate the calculation of look angles for a geostationary satellite with SSP (0,-105.0) at our earth station located in Van Leer (Atlanta, GA).

The final calculation indicates that we need to point the antenna at an elevation of nearly 45 degrees above the horizon and turn the dish around to an azimuthal angle of 214 degrees (SSW).

Ground Track – Molniya Orbit



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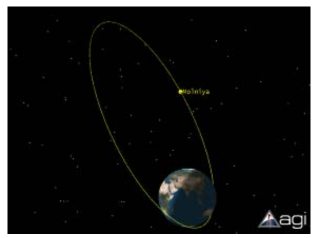


Although we discussed elliptical orbits, we didn't necessarily talk about applications that require elliptical orbits – most of our discussions of real satellites have centered on circular orbits. Here is one example of an eccentric orbit called a Molniya orbit.

Developed by the Soviets during the cold war, the Moltniya orbit has these attributes:

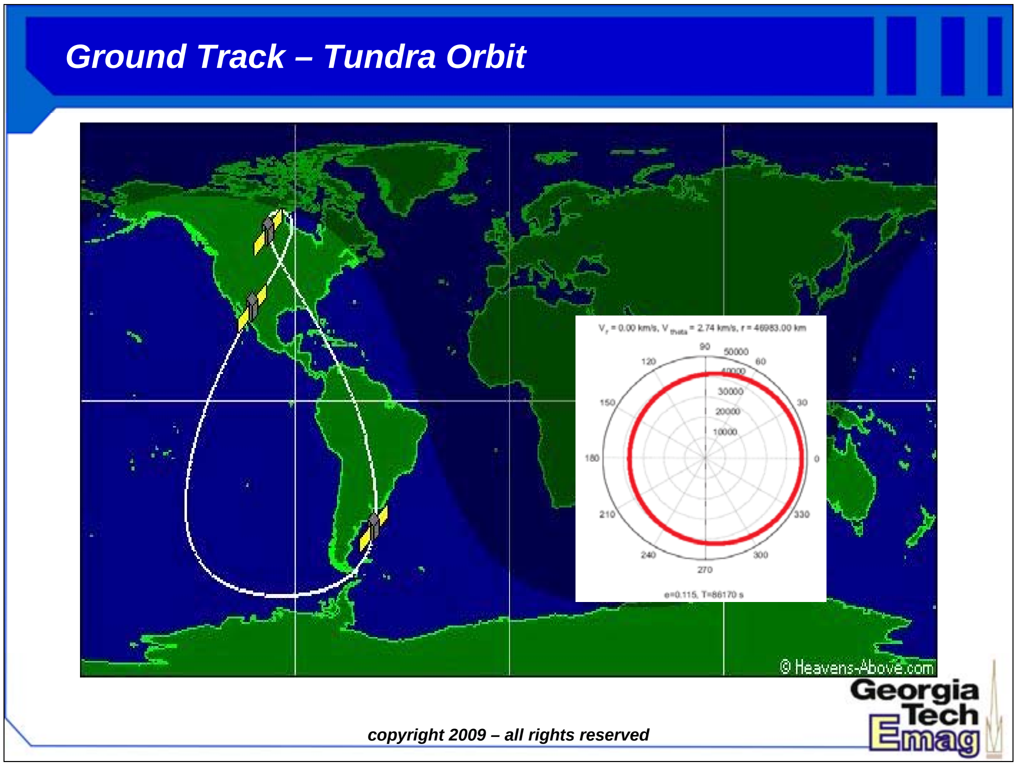
• Inclination 63 degrees – this inclination was found to minimize orbital perturbations and precession errors, extending the lifetime of the orbit.

• Period of 12 hours – this orbit would repeat twice a day

• Eccentricity – the orbit would have perigee in the southern hemisphere and be in effectively LEO, while reaching apogee in effective MEO on the Northern hemisphere

The map above shows the ground track (the trace of the satellite subpoint over time) of a Molniya orbit over the course of a 24 hour period. Note how the SSP spends time in both north and south hemispheres, but spends much more time in the north due to the highly eccentric orbit; the satellite whips around perigee in the southern hemisphere due to Kepler's second law and slowly hovers at apogee above the northern hemisphere. This winds up being a very good orbit for covering high-altitude locations – much better than geostationary orbits.

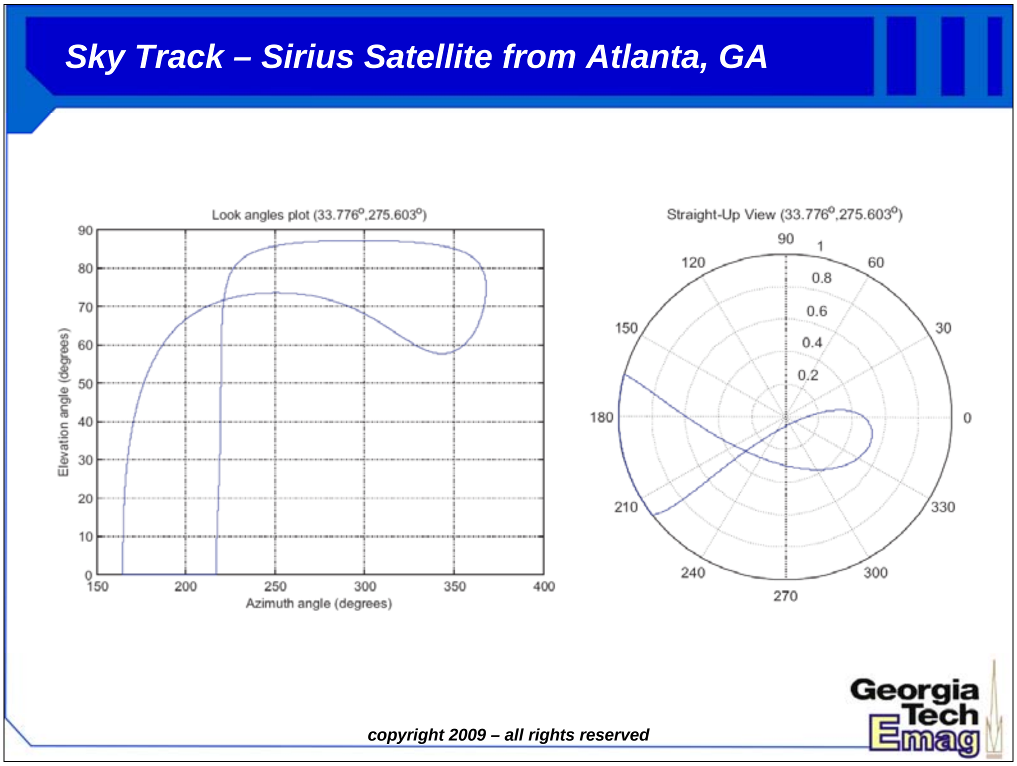
Can you guess the principle application for this orbit? Note that it is ideal for spying on North America! For the first half of the day, the satellite spends most of its time recording data over North America. For the second half of the day, the satellite can dump its data back over Russia.



A related orbit, the “Tundra” orbit, was also developed by the Soviet union. It is also inclined by 63 degrees, eccentric, but with a total period of 1 sidereal day, making it a geosynchronous (not geostationary) earth orbit. In the configuration, the satellite can be made to hover over North America most of the time, spending its faster-moving perigee whipping around the Southern hemisphere. Although a satellite in this orbit does not spend 100% of its time covering North America like a geostationary satellite, the SSP does spend most of its time at higher altitudes where the look angle is relatively high in the sky from a receiver antenna’s point of view in North America. This is particularly nice if you are trying to cover mountainous or forested areas; objects on the horizon can often degrade geostationary satellite links in the far north because their elevation look angles are so low to the horizon.

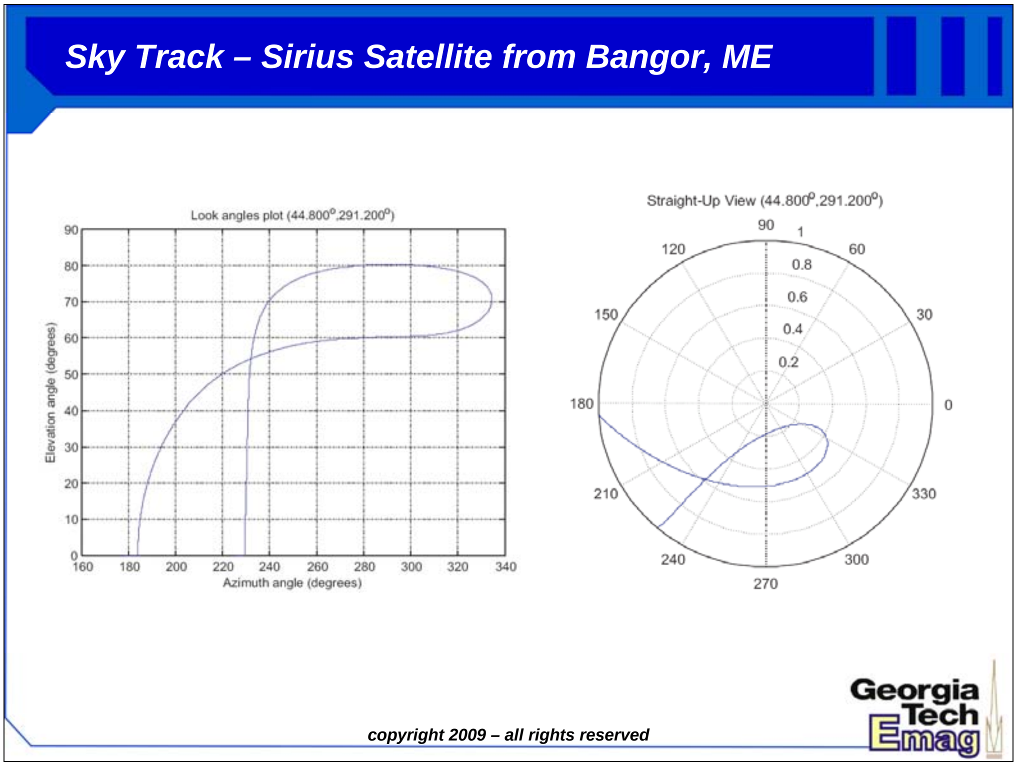
This orbit has a nice application today: it is used by Sirius satellite radio to provide quality radio coverage to mobile users. The original Sirius strategy was to place 3 satellites in Tundra orbits so that there would always be one directly overtop North America during the course of a day. Contrast this with the original XM radio strategy of using two geostationary satellites (named “Rock” and “Roll”), each providing complete coverage that sometimes struggled to give service at higher latitudes.

XM and Sirius merged in 2008.



These graphs demonstrate the sky tracks of a satellite in Tundra orbit over North America. This sky track is for Atlanta, GA. Remember in our previous calculation how a geostationary satellite required about 45 degrees of dish elevation for proper reception on the roof of Van Leer. This is certainly achievable, but notice how much better this sky track above is. The elevation angle is mostly above 45; the azimuth is all over the place, but satellite radio targets a large mobile audience who don't really have the luxury of pointing their antennas.

That's not to say that pointing antennas is out of the question for this application. If a single fixed receiver is used to pipe in satellite radio to a residential building or office or shopping mall, the coverage for the geostationary XM satellites would be preferred since we could use a larger, directional antenna aligned in the direction of the radiating satellite.



Here is a sky track for Bangor, Maine, which is obviously a lot farther north than Atlanta (44.8 degrees latitude). Notice how well the satellite in Tundra orbit still stays above the 45-degree elevation line in its sky track. By now, however, the geostationary satellites would have dropped fairly low to the horizon. Their signals would be easy to degrade with foliage or terrain obstruction. Stephen King probably subscribed to Sirius satellite radio instead of XM radio when it first came.