ECE 6390 Homework 1: Orbital Mechanics

Solutions

1. (a) – (c) The MATLAB codes for solving these are practically the same with changes made only to the radial velocity, V_r, as specified in the problem set. One single code (part (a)) is at the end of this solution set. The calculations for obtaining the eccentricity, e, and period, T, are:

$$e = \sqrt{\left(1 - \frac{b^2}{a^2}\right)}$$

where b and a are the lengths of the minor and major axes resp.

$$T = \sqrt{\left(\frac{4\pi^2}{GM_p}a^3\right)} \approx simulation\_length \times \Delta t.$$

1. (d) Halley's Comet is the first comet observed and correctly predicted by the astronomer Edmund Comet. This part requires more precision to accurately obtain the eccentricity and period of the orbit. Little descrepancy in figures used for the radius could result in halving the period! However, full credit is given if calculations are done right and the orbit is elliptical as expected.

The table below summarizes the values for the period and eccentricity of each orbit. Reference: http://neo.jpl.nasa.gov/cgi-bin/neo_ele?type=NEC

Table 1: Summary of Period, T and eccentricity, e of each orbit

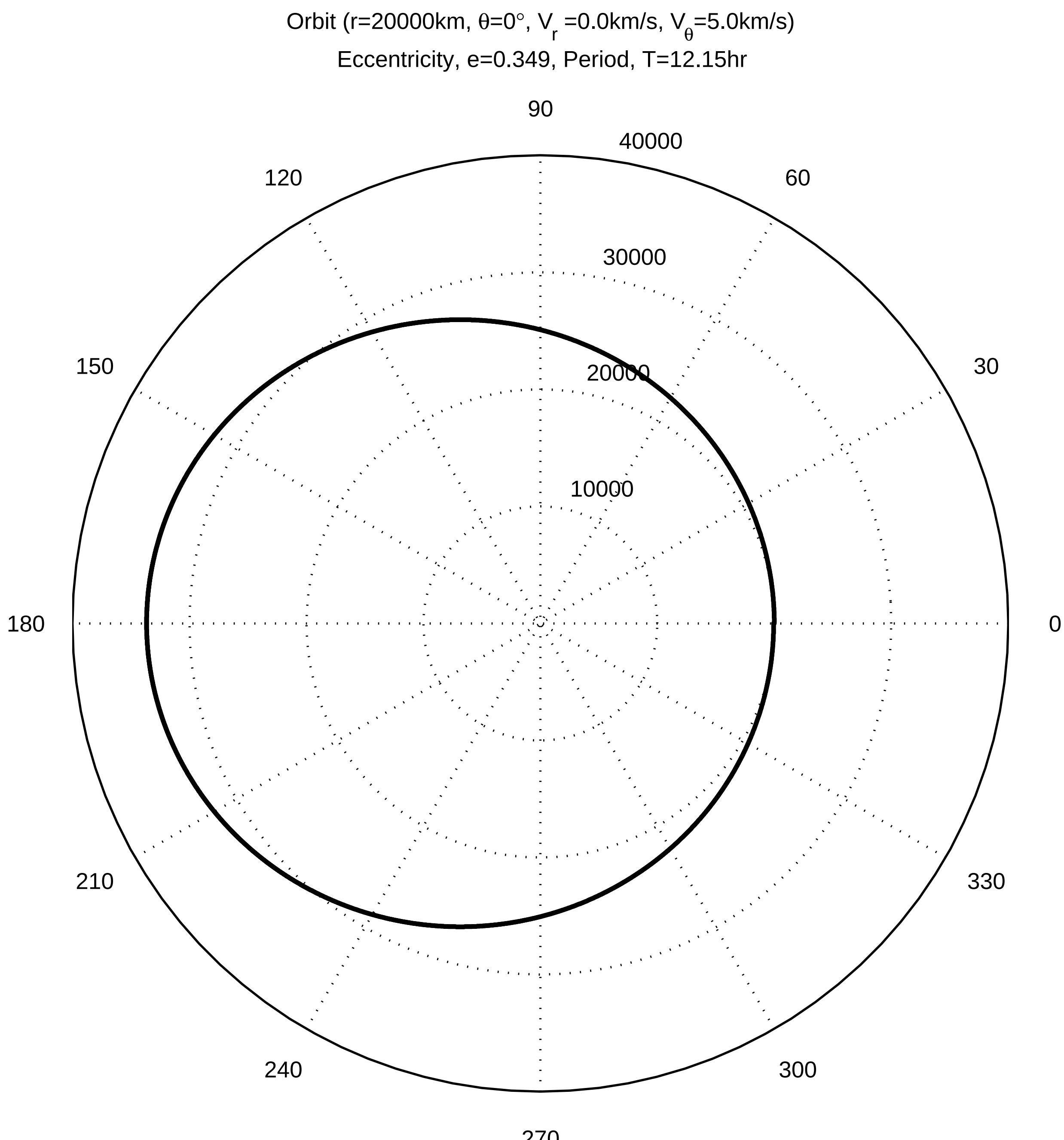
Orbit	**Period, T**	**Eccentricity, e**
1(a) Earth-orbit: $V_r = 0.000 km/s$	12.15 hr	0.349
1(b) Earth-orbit: $V_r = -3.000 km/s$	49.08 hr	0.874
1(c) Earth-orbit: $V_r = -6.000 km/s$	undefined	undefined
1(d) Halley's comet $r = 0.586 AU$	74.92yrs	0.984

```
clear all
close all
% Physical Constants
G = 6.672e-11;          % Gravitational Constant (Newton meters^2 kilograms^-2)
Mp = 5.974e24;          % Mass of the Planet Earth (kilograms)
MAX_ITERATION = 3e4;    % Largest number of iterations

% Initial Conditions
dt = 10;                % increment (seconds)
r =  20e6;              % initial radius (meters)
theta = 0;              % initial angle (radians)
V_r = 0.00e3;           % r-component of velocity (meters/second)
V_theta = 5.00e3;       % theta-component of velocity (meters/second)

% Convert to Discrete Conditions
dr = V_r*dt;            % meters
dtheta = V_theta/r*dt;  % radians

% Initialize values for looped calculation
r_0 = r;
theta_0 = theta;
all_theta = theta;
all_r = r;

while abs(theta-theta_0) <= 2*pi && length(all_theta) < MAX_ITERATION,
% compute new state information
    r_new = r + dr;
    theta_new = theta + dtheta;
    dr_new = dr + ((r+0.5*dr)*dtheta^2 - G*Mp/(r^2)*(dt^2));
    dtheta_new = dtheta - (2*dr*dtheta/(r+0.5*dr));

% save and print new state information
    all_theta = [all_theta; theta_new ];
    all_r = [all_r; r_new ];

% reset variables for new calculation
    r = r_new; dr = dr_new;
    theta = theta_new;  dtheta = dtheta_new;
end;

% Convert polar coordinates to Cartesian (X-Y) coordinates (helps with
% eccentricity calculation)
[X,Y] = pol2cart(all_theta,all_r);


% eccentricity and period calculations
N = length(all_r);
opp_lengths = zeros(1,floor(N/2));
for k=1:floor(N/2)
    opp_lengths(k) = sqrt((X(k)-X(floor(N/2)+k))^2 + (Y(k)-Y(floor(N/2)+k))^2);
end
ecc = sqrt(1-(min(opp_lengths)/max(opp_lengths))^2);

% NOTE: using min(all_r) and max(all_r) may not yield accurate value for
% eccentricity as the major/minor axis does not necessarily cross the
% origin
```

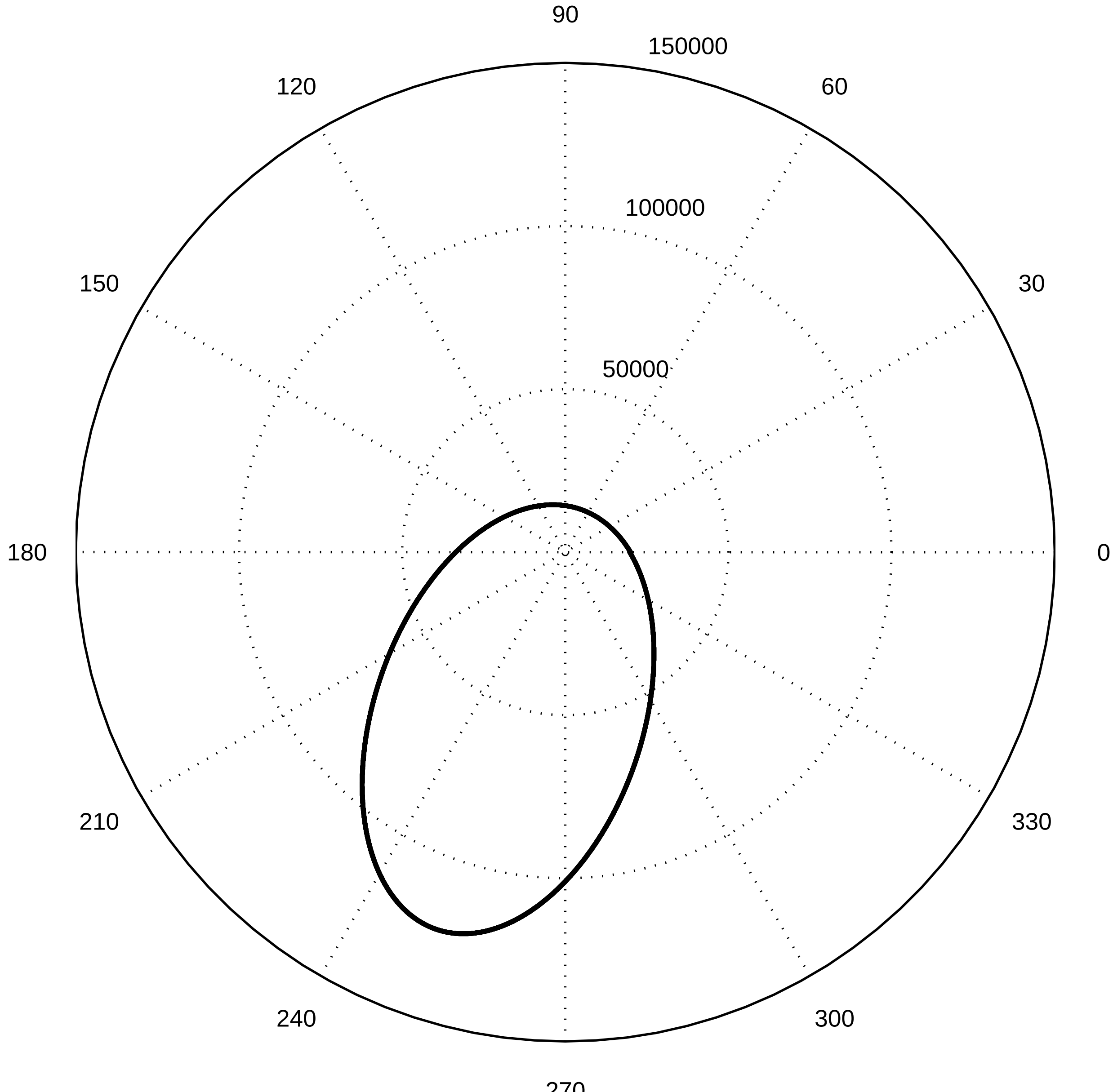


```
if(length(all_theta) < MAX_ITERATION)
    Period = dt*length(all_theta); % unit in seconds (same as unit of dt)
else
    error(’dt value is possibly too coarse, try a different value’);
end

figure(1), polar( all_theta, all_r/1000, ’r.’ );
title( sprintf(’Orbit (r=%1.0fkm, \\theta=%1.0f\\circ, V_r =%1.1fkm/s, V_{\\theta}=%1.1fkm/s)
```

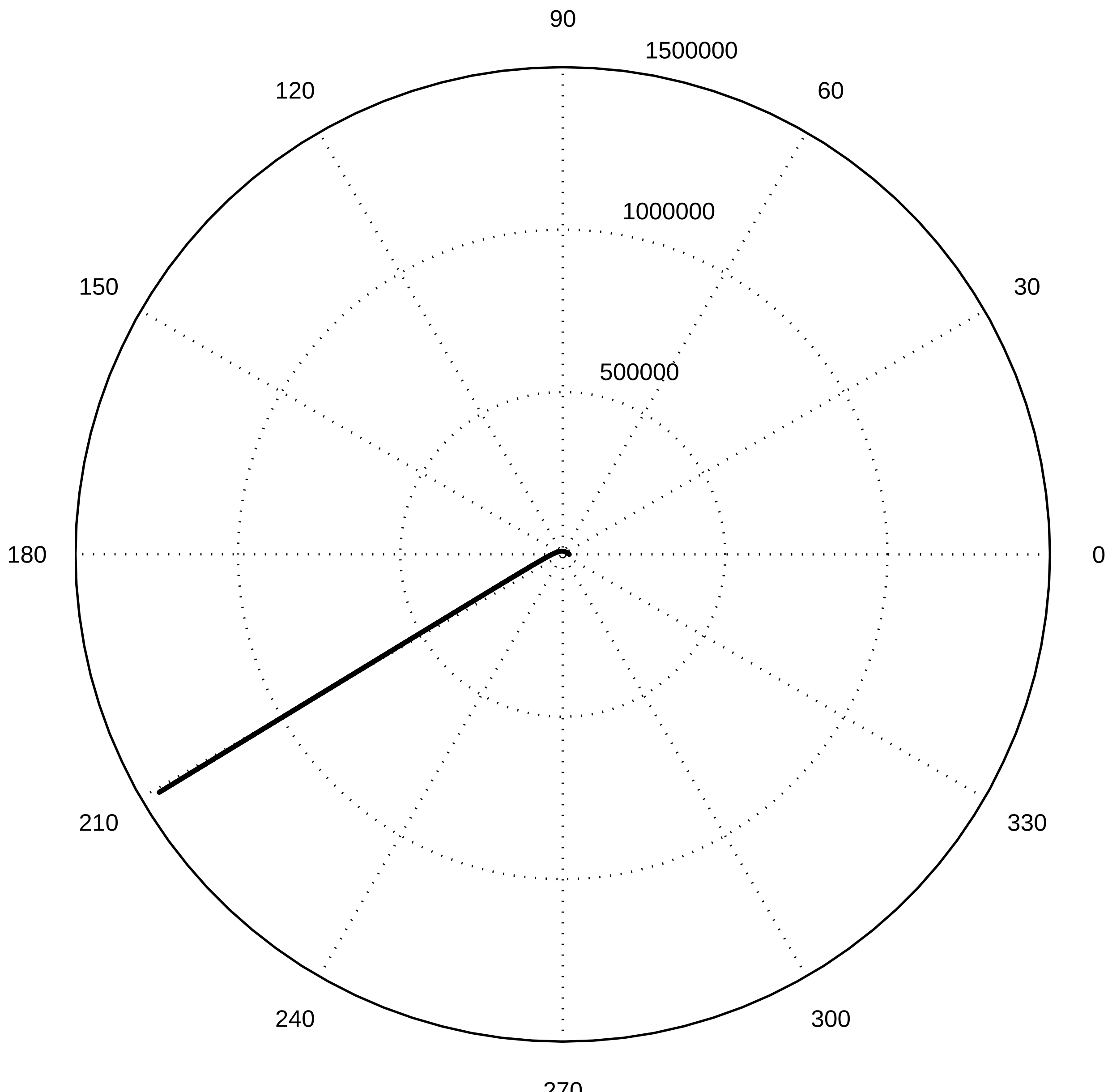
```
% SOLUTION TO ECE6390 Homework #1
% Halley’s Comet (part (d))
clear all
close all
% Physical Constants
G = 6.672e-11;          % Gravitational Constant (Newton meters^2 kilograms^-2)
Mp = 1.98e30;           % Mass of the Sun (kilograms)
MAX_ITERATION = 3e4;    % Largest number of iterations

AU = 150e9;             % astronomical unit (meters)
% Initial Conditions
dt = 100000;            % increment (seconds)
r = 0.586*AU;           % initial radius (meters)
theta = 0;              % initial angle (radians)
V_r = 0;                % r-component of velocity (meters/second)
V_theta = 54.4e3;       % theta-component of velocity (meters/second)

% Convert to Discrete Conditions
dr = V_r*dt;             % meters
dtheta = V_theta/r*dt;   % radians

% Initialize values for looped calculation
r_0 = r;
theta_0 = theta;
all_theta = theta;
all_r = r;

while abs(theta-theta_0) <= 2*pi && length(all_theta) < MAX_ITERATION,
% compute new state information
    r_new = r + dr;
    theta_new = theta + dtheta;
    dr_new = dr + ((r+0.5*dr)*dtheta^2 - G*Mp/(r^2)*(dt^2));
    dtheta_new = dtheta - (2*dr*dtheta/(r+0.5*dr));

% save and print new state information
    all_theta = [all_theta; theta_new ];
    all_r = [all_r; r_new ];

% reset variables for new calculation
    r = r_new; dr = dr_new;
    theta = theta_new;  dtheta = dtheta_new;
end;

% Convert polar coordinates to Cartesian (X-Y) coordinates (helps with
% eccentricity calculation)
[X,Y] = pol2cart(all_theta,all_r);

% eccentricity and period calculations
N = length(all_r);
opp_lengths = zeros(1,floor(N/2));
for k=1:floor(N/2)
    opp_lengths(k) = sqrt((X(k)-X(floor(N/2)+k))^2 + (Y(k)-Y(floor(N/2)+k))^2);
end
ecc = sqrt(1-(min(opp_lengths)/max(opp_lengths))^2);

% NOTE: using min(all_r) and max(all_r) may not yield accurate value for
% eccentricity as the major/minor axis does not necessarily cross the
% origin

if(length(all_theta) < MAX_ITERATION)
    Period = dt*length(all_theta); % unit in seconds (same as unit of dt)
else
    error(’dt value is possibly too coarse, try a different value’);
end
```


Orbit (r=20000km, θ=0°, V_r =0.0km/s, V_θ=5.0km/s)
Eccentricity, e=0.349, Period, T=12.15hr
90
40000
120
60
30000
150
20000
30
10000
180
0
210
330
240
300
270

Orbit (r=20000km, θ=0°, V_r =−3.0km/s, V_θ=5.0km/s)
Eccentricity, e=0.874, Period, T=49.08hr



Orbit (r=20000km, θ=0°, V_r =−6.0km/s, V_θ=5.0km/s)
Eccentricity and Period are undefined



Halley's Comet Orbit (r=87900000km, θ=0°, V_r =0.0km/s, V_θ=54.4km/s)
Eccentricity, e=0.984, Period, T=75.92yrs