ECE 6390 Solutions to Homework 7: GPS

1. (see next page)

Problem 1:

2. (a) The given parameters for the GPS signal are:
RF bandwidth, $B = 2\ MHz$, bit rate $R_b = 50\ bit/sec$, and chip rate $R_c = 1.023\ Mcps$.
This means that the processing gain,

$$M = \frac{R_c}{R_b} = \frac{1.023 \times 10^6}{50} = 20460$$

With the noise temperature, $T_{sys} = 270K$, the noise power is $N_p = kTB$, where Boltzmann's constant, $k = 1.39 \times 10^{-23} J/K$.
$\Rightarrow N_p = 7.506 \times 10^{-15} W$
From table 12.2 in the text (Pratt et al), we choose the received power, $P_R = 10^{-16} W$. With eight interferers ($Q = 8$) of equal power, the SNR for the C/A code after despreading can be computed as:

$$SNR = \frac{MP_R}{N_p + (Q-1)P_R} = \frac{20460 \times 10^{-16}}{7.506 \times 10^{-15} + 7 \times 10^{-16}} \approx 249.33$$

GPS uses binary phase shift keying (BPSK), so $BER = \frac{1}{2} erfc\left(\sqrt{SNR}\right) \approx 9.303 \times 10^{-111}$. In 10 seconds, there are $10 \times R_b = 500\ bits$, which means that the average error in 10 seconds is $500 \times 9.303 \times 10^{-111} \approx 4.65 \times 10^{-108}$.

(b) This problem is similar to part (a), but requires solving in reverse. We obtain the SNR, using BER formula for BPSK as before, that gives a BER = 0.1. The obtained value, using trial and error, is $SNR = 0.8210$.
With jamming power, P_J, included, the SNR formula becomes:

$$\begin{aligned} SNR &= \frac{M \cdot P_R}{N_p + (Q-1)P_R + P_J} \\ \Rightarrow P_J &= \frac{M \cdot P_R}{SNR} - N_p - (Q-1)P_R \approx 2.484 \times 10^{-12} W \end{aligned}$$

Assuming a jamming frequency of $f_c = 1.575\ MHz$, then the link budget formula could be used to obtain the required jamming power,

$$P_T = \frac{P_J (4\pi R)^2}{G_R G_T \lambda^2} = \frac{2.484 \times 10^{-12} \cdot (4\pi 5 \times 10^3)^2}{1 \cdot 10^{0.7} \times \left(\frac{3 \times 10^8}{1.575 \times 10^9}\right)^2} \approx 53.9\ mW$$

Very Interesting how jamming could be easily implemented here!

2. **GPS Problem:**

Below are the ranging equations for the four satellite points:

$$
\begin{aligned}
(PR_1 + c\tau)^2 &= (X_1 - U_x)^2 + (Y_1 - U_y)^2 + (Z_1 - U_z)^2 \\
(PR_2 + c\tau)^2 &= (X_2 - U_x)^2 + (Y_2 - U_y)^2 + (Z_2 - U_z)^2 \\
(PR_3 + c\tau)^2 &= (X_3 - U_x)^2 + (Y_3 - U_y)^2 + (Z_3 - U_z)^2 \\
(PR_4 + c\tau)^2 &= (X_4 - U_x)^2 + (Y_4 - U_y)^2 + (Z_4 - U_z)^2
\end{aligned}
$$

We are allowed to assume perfect synchronization of the GPS receiver with the GPS clock ($\tau = 0$), so that we can linearize the above system by taking difference of pairs of ranging equations:

$$
\begin{aligned}
PR_1^2 - PR_2^2 &= X_1^2 - X_2^2 + 2(X_2 - X_1)U_x + Y_1^2 - Y_2^2 + 2(Y_2 - Y_1)U_y + Z_1^2 - Z_2^2 + 2(Z_2 - Z_1)U_z \\
PR_2^2 - PR_3^2 &= X_2^2 - X_3^2 + 2(X_3 - X_2)U_x + Y_2^2 - Y_3^2 + 2(Y_3 - Y_2)U_y + Z_2^2 - Z_3^2 + 2(Z_3 - Z_2)U_z \\
PR_3^2 - PR_4^2 &= X_3^2 - X_4^2 + 2(X_4 - X_3)U_x + Y_3^2 - Y_4^2 + 2(Y_4 - Y_3)U_y + Z_3^2 - Z_4^2 + 2(Z_4 - Z_3)U_z
\end{aligned}
$$

This provides 3 equations with 3 unknowns – user position (U_x, U_y, U_z). Now let us calculate the Cartesian coordinates of the satellites

	Sat 1	Sat 2	Sat 3	Sat 4
X (km)	12229.2420	22.2652	-1239.1036	-1963.3663
Y (km)	-23318.5826	-18760.3115	-24988.4822	-20166.3794
Z (km)	3631.2185	18829.4083	8974.2288	17203.4513

When these values and the pseudo-ranges are inserted into the linear system of equations, we have enough information to solve for the Cartesian coordinates of the user: (518.5371, -5278.3008, 3546.7052) km.

Converting this coordinate into latitude, longitude, and elevation produces $33.7715°$, $-84.3893°$.