ECE 6390: Satellite Communications and Navigation Systems
Solutions to TEST 1 (Fall 2004)

1. **Short Answer Section (30 points)**

   (a) (1-4): linear, homogeneous, isotropic, source-free

   (b) equatorial (1), 0 (2), period (3)

   (c) bent-pipe

   (d) black body

   (e) 20

   (f) low-earth

   (g) spinner (1), momentum wheels (2)

   (h) moon

   (i) false (an object can pass a larger object without orbiting it if the relative velocity is large enough)

2. **Descriptive Answer Section (20 points)**

   (a) **Dish Antennas (10 points):** A thick, reflective layer of snow will move or aberrate the focal point of the dish, causing losses and in illumination efficiency. Unlike the smooth metallic reflector, the snow layer may also be electromagnetically rough at 12 GHz, introducing additional losses.

   (b) **Orbital Mechanics (10 points):**

$$
\begin{aligned}
\text{KE at Perigee } + \text{PE at Perigee} &= \text{KE at Apogee } + \text{PE at Apogee} \\
\frac{1}{2}mV_p^2 - \frac{m\mu}{r_p} &= \frac{1}{2}mV_a^2 - \frac{m\mu}{r_a} \\
V_p^2 - 2\frac{\mu}{r_p} &= V_a^2 - 2\frac{\mu}{r_a} \\
V_p^2 - 2\frac{\mu}{(1-e)a} &= V_a^2 - 2\frac{\mu}{(1+e)a} \\
V_p^2 - V_a^2 &= 2\frac{\mu}{(1-e)a} - 2\frac{\mu}{(1+e)a} \\
V_p^2 - V_a^2 &= \frac{4\mu e}{a(1-e^2)}
\end{aligned}
$$

3. **Link Budget for LEO Mobile Communications:**

First, compute the link budget for the satellite downlink:

$$\begin{aligned} P_R &= P_T + G_T + G_R - 20\log_{10}\left(\frac{4\pi}{\lambda}\right) - 20\log_{10}(r) \\ &= 33.5 \text{ dBW } + 31 \text{ dB } + 36 \text{ dB } - 53.6 \text{ dB } - 151.1 \text{ dB} \\ &= -104.2 \text{ dBW} \end{aligned}$$

The noise power is the result of thermal noise and device noise (mostly) from the LNA:

$$P_N = 10\log_{10}(k[T_p + T_d]B) = -130.8 \text{ dBW}$$

From this, the carrier-to-noise ratio in dB is

$$\frac{C}{N} = P_N - P_R = 26.5 \text{ dB}$$

4. **Solar Power from Space:**

Our satellite dish must project its power onto a 100m dish on the surface of the earth from 36,000 km away – no small feat! First, we must calculate the half-power beamwidth of this antenna, which will be extremely slender:

$$\theta_{\text{HPBW}} = 2\tan^{-1}\frac{50\text{m}}{36,000,000\text{m}} = 1.59 \times 10^{-4\circ}$$

We can then approximate the gain of the antennas using this simple rule of thumb for aperture antennas, noting that a dish antenna should have the same HPBW in azimuth and elevation:

$$G = \frac{30,000}{\theta^2_{\text{HPBW}}}$$

We know that the area of the antenna to achieve this gain is

$$\begin{aligned} A &= G \\ \frac{\pi}{4}D^2 &= \frac{30,000}{\theta^2_{\text{HPBW}}}\frac{\lambda^2}{4\pi} \end{aligned}$$

Solving for D, we obtain

$$D = \frac{\lambda\sqrt{30,000}}{\pi\theta_{\text{HPBW}}} = 2.7 \text{ km}$$

That's clearly too large to be practical. In fact, the astute engineer might notice that the far field for an antenna that large would be 10 million kilometers away!