ECE 6390: Satellite Communications and Navigation Systems
Solutions to TEST 1 (Summer 2015)

1. **Circular Orbits:**

   (a) $T = \sqrt{\frac{4\pi^2 a^3}{M_e G}} = 2.4 \times 10^6$s or 27.5 days. Note that this is an approximate calculation of a lunar *sidereal month* that does not take into consideration that the earth-moon system revolves with slight eccentricity around a *barycenter* that lies approximately two-thirds of an earth radius from the center of the earth. A lunar sidereal month is not the same as a lunar *synodic* month (29.5 days), which is the time it takes for the moon to achieve the same *visual* phase as seen from earth.

   (b) If the three bodies are perfectly aligned in the order moon-satellite-earth, then the satellite is approximately 42,000 km from the center of earth and 343,000 km from the center of the moon. These forces will opposed one another, resulting in a net force of 22.553 N on the satellite (the minimum).

   If the three bodies are perfectly aligned in the order satellite-earth-moon, then the satellite is still 42,000 km from the center fo the earth and 427,000 km from the center of the moon. These forces will reinforce on another in this order, resulting in a net force of 22.587 N on the satellite (the maximum).

   The difference between these total forces is 0.034 N.

2. **Elliptical Orbits:** An earth station on the edge of coverage for this satellite would form a right triangle with the satellite and the center of the earth and inner angle of 10 degrees. Under this geometry, apogee would occur at 36.8km, leading to an eccentricity of 0.68.

3. **Link Budget:** 160 MHz

4. **Orbital Mechanics:** e = 1/3

5. **Plane Waves:**

   (a) Grouping terms, we have:

   $$\tilde{\vec{H}}(\vec{r}) = 72.1\,[0.83\hat{x} + 0.55\hat{y}]\exp\left(-j361.1\,[0.55x - 83y - 0.06z]\right)\ \mu\text{A/m}$$

   The corresponding electric field is:

   $$\tilde{\vec{E}}(\vec{r}) = 27.2\,[-0.04\hat{x} - 0.03\hat{y} + 0.998\hat{z}]\exp\left(-j361.1\,[0.55x - 83y - 0.06z]\right)\ \text{mV/m}$$

   (b) The wavenumber is 361.1 rad/m, so the wavelength is 0.0174 m, and the frequency is 17.2 GHz.

   (c) The radio wave is arriving from only 3.1 degrees above the horizon, indicating an earth-based transmitter.