ECPE 3614: Review of Small-Carrier Amplitude Modulation

This handout is a graphical review of small-carrier amplitude modulation techniques that we studied in class.

1 A Note on Complex Signal Spectra

All of the illustrations on this handout will use the following 4 time-domain signals and their equivalent analytical Fourier transforms:

\[
\begin{align*}
x_1(t) &= \sin(t) & &\quad \leftrightarrow \quad X_1(f) = u\left(\frac{1}{2} - |f|\right) \\
x_2(t) &= \frac{1}{2}\sin^2 \left(\frac{t}{2}\right) & &\quad \leftrightarrow \quad X_2(f) = (1 - 2|f|)u\left(\frac{1}{2} - |f|\right) \\
x_3(t) &= \frac{1}{2} \left[\sin(t + \frac{1}{2}) + \sin(t - \frac{1}{2})\right] & &\quad \leftrightarrow \quad X_3(f) = \cos(\pi f)u\left(\frac{1}{2} - |f|\right) \\
x_4(t) &= \frac{2}{\pi} J_0(\pi t) & &\quad \leftrightarrow \quad X_4(f) = \frac{u(\frac{1}{2} - |f|)}{\sqrt{1 - f^2}}
\end{align*}
\]

These pairs were computed from the transform table at the end of the handout, applying duality whenever necessary. It should be noted that \(x_1(t), x_2(t), x_3(t),\) and \(x_4(t)\) all produce \textit{real-valued} spectra. Of course, realistic band-limited signals that carry useful information will be complex-valued. The difference between an ideal, real-valued spectrum and a realistic, complex-valued spectrum is not always apparent from a two-dimensional plot. Rather, it is best to show the difference by plotting the signal spectrum on a skewed real and imaginary graph as shown below:

![Box Spectrum and Complex Spectrum](image)

The idealized signal spectra are useful for illustrating concepts in AM modulation. All of these concepts work equally well with complex-valued spectra – but the operations become much more difficult to visualize. So take everything you see in this handout with a grain of salt!

2 Double Sideband (DSB)

Below are the Fourier transforms of cosine and sine waves, which produce spectral lines in the frequency domain:

![Cosine Spectrum and Sine Spectrum](image)
The Fourier transform of the cosine produces even delta-s. The Fourier transform of the sine produces odd delta-s that are also imaginary.

AM modulation with a sine or cosine shifts a band-limited signal up to some carrier frequency, \( f_c \). Consider the case of cosine modulation. Follow the diagram below to see how a band-limited signal is modulated and demodulated using a cosine carrier wave. The steps of the diagram correspond to the following operations:

1. Start out with a baseband signal spectrum. This example uses \( X_1(f) \).

2. The signal \( x_1(t) \) is multiplied by \( \cos(2\pi f_c t) \) in the time domain. This has the effect of shifting copies of the baseband spectrum \( X_1(f) \) up and down the frequency axis. The resulting signal \( x_1(t)\cos(2\pi f_c t) \) is a Double Sideband, Suppressed Carrier (DSB-SC) amplitude modulated signal.

3. To demodulate this signal, the modulated signal is multiplied by \( \cos(2\pi f_c t) \) again in the time domain. The resulting signal, \( x_1(t)\cos^2(2\pi f_c t) \), still needs to be passed through an LPF to get rid of the high-frequency signal components that exist in the spectrum at \( f_c \).

4. This is the original recovered spectrum after sending the signal in Step 3 through the LPF.

**Cosine Modulation**

1. \( X_1(f) \)

2. Cosine Modulation

3. Second Cosine Multiply

AM modulation may be done with a sine wave carrier as well. Follow the diagram below to see how a band-limited signal is modulated and demodulated using a sine wave. The steps of the diagram correspond to the following operations:

1. Start out with a baseband signal spectrum. This example uses \( X_2(f) \).

2. The signal \( x_2(t) \) is multiplied by \( \sin(2\pi f_c t) \) in the time domain. This has the effect of shifting copies of the baseband spectrum \( X_2(f) \) up and down the frequency axis. Because the Fourier transform of a sine wave is odd, imaginary impulses, the frequency-shifted replicas of our triangle spectrum are also odd, imaginary copies. The resulting signal \( x_2(t)\sin(2\pi f_c t) \) is a DSB-SC AM modulated signal.
3. To demodulate this signal, the modulated signal is multiplied by $\sin(2\pi f_c t)$ again in the time domain. Notice how the imaginary copies of the triangle spectrum in Step 2 have flipped back to the real axis. The resulting signal, $x_2(t)\sin^2(2\pi f_c t)$, still needs to be passed through an LPF to get rid of the high-frequency signal components that exist in the spectrum at this point.

4. This is the original recovered spectrum after sending the signal in Step 3 through the LPF.

**Sine Modulation**

1. $X_1(f)$

2. Sine Modulation

3. Second Sine Multiply

4. Output of ILPF

**3 Single Sideband (SSB)**

In truth, we do not require a double sideband modulated signal to recover the original signal. Actually, we can use *Single Sideband* (SSB) modulation and still recover the exact original signal. This is illustrated by the figure below, which corresponds to the following steps:

1. Start out with a baseband signal spectrum. This example uses $X_3(f)$.

2. The signal $x_3(t)$ is multiplied by $\cos(2\pi f_c t)$ in the time domain. Once the copies of $X_3(f)$ have been shifted up and down the frequency axis, one of the sidebands is removed using an near-ideal filter. This particular illustration uses an IHPF to remove the lower sideband.

3. To demodulate this signal, the modulated signal is multiplied by $\cos(2\pi f_c t)$ again in the time domain. The resulting signal still needs to be passed through an LPF to get rid of the high-frequency signal components.

4. The original spectrum is recovered after sending the signal in Step 3 through the LPF.

SSB AM cuts the amount of transmitted signal bandwidth in half compared to DSB AM. This additional space in the frequency domain may be used for *frequency division multiplexing* (FDM) of other signals.
*Vestigial Sideband* (VSB) modulation is closely related to SSB modulation. For VSB, either the lower or upper sideband of the DSB AM signal is removed using a non-ideal filter which leaves a small *vestige* of the spectrum behind. VSB AM has less bandwidth than DSB, but more bandwidth than SSB.

**Single Sideband (SSB) Modulation**

1. $X_s(f)$

2. SSB Modulation

3. Second Cosine Multiply

4. Output of ILPF

4. Quadrature Amplitude Modulation (QAM)

Like SSB, QAM is another way of "doubling" the spectral efficiency of signal transmission. Instead of slicing off sidebands, QAM takes advantage of the *orthogonality* of the in-phase and quadrature channels to transmit two different signals simultaneously through the same frequency band. The in-phase channel refers to a signal modulated with a cosine. The quadrature channel refers to a signal modulated with a sine. When the in-phase and quadrature channels are demodulated with cosines and sines, respectively, each channel appears invisible to the other.

The picture below illustrates two signals ($x_1(t)$ and $x_2(t)$) modulated using QAM. Signal $x_1(t)$ is modulated on the in-phase channel and signal $x_2(t)$ is modulated on the quadrature channel. Notice how the cross-channel spectra vanish at baseband in the demodulation steps.

5. Can We Use Both QAM and SSB to Multiplex Signals?

If using QAM or SSB doubles the spectral efficiency of DSB-SC AM, we may be tempted to use both techniques together to quadruple the spectral efficiency. Unfortunately, using one type of modulation scheme precludes the use of the other. To give an example, consider the figure at the end of the handout. In this example, we have modulated the single sidebands of 4 signals, $x_1(t)$, $x_2(t)$, $x_3(t)$, and $x_4(t)$, around a carrier frequency of $f_c$. Two signals ($x_1(t)$ and $x_3(t)$) are modulated using the in-phase channel, and two signals ($x_2(t)$ and $x_4(t)$) are modulated using the quadrature channel. The 4 signals take up the same bandwidth as a single DSB-SC AM signal.

Let us try to extract signal $x_3(t)$ from the multiplexed signal. Referring to the figure, we filter out the lower sideband first, since this just contains $x_1(t)$ and $x_2(t)$. Then, we down-
Quadrature Amplitude Modulation

Output of In-Phase ILPF

Output of Quadrature ILPF

Cosine Modulation

Sine Modulation

Cosine Multiply

Sine Multiply

Add In-phase and Quadrature Channels
convert the spectrum by multiplying by $\cos(2\pi f_0 t)$. Our hope is that, in doing this, we will also get rid of the quadrature sideband that corresponds to $x_4(t)$. But that is not what happens. After down-shifting the spectrum and filtering out the high-frequency components, the sideband corresponding to $x_4(t)$ still exists in the baseband signal. There is a basic principle here to be learned: removing the sideband from a DSB-SC AM signal destroys the ability to distinguish between the in-phase and quadrature channels.

At this point, we may be tempted to say “Look at the figure. I can see the difference between the $S_3(f)$ on the real plane and $S_4(f)$ on the imaginary plane. Shouldn’t there be a way to separate them?” Here is the problem: this academic example using real-valued spectrum makes the problem of separation appear simpler than it really is. Imagine if $S_3(f)$ and $S_4(f)$ were more realistic, complex-valued spectra of the type illustrated on page 1. The two spectra would be tangled together in such a way that it would be impossible for you (or the receiver) to determine which spectral components had been modulated onto which channel.

6 Appendix

<table>
<thead>
<tr>
<th>Useful Fourier Transform Pairs</th>
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<tbody>
<tr>
<td><strong>TIME DOMAIN</strong></td>
</tr>
<tr>
<td>Cosine: $\cos(2\pi f_0 t)$</td>
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<tr>
<td>Sine: $\sin(2\pi f_0 t)$</td>
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<tr>
<td>Box: $u\left(\frac{T}{2} -</td>
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<td>Triangle: $(1 -</td>
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<tr>
<td>Cosine Pulse: $\cos\left(\frac{\pi t}{T}\right)u\left(\frac{T}{2} -</td>
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<td>U-Shape: $\frac{u\left(\frac{T}{2} -</td>
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6
Why QAM + SSB Does Not Work

QAM + SSB Modulation of 4 Different Signals

Output of ILPF

Multiply By Cosine

Pass Through IHPF Before Downconversion