Practical Geometrical Behavior of Knife-Edge Diffraction

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Introduction

The scalar Knife-Edge Diffraction (KED) solution is a workhorse for RF and optics engineers who regularly deal with practical diffraction phenomena. Yet the approximate, polarization-independent KED result is formulated in a way that does not provide direct physical insight. In this paper, we demonstrate how the KED formula contains similar underlying physics to other geometrically-based half-screen diffraction solutions.

The analysis proves that the diffracted wave emanating from a knife edge is an ideal middle ground between the Sommerfeld solutions for perpendicular (soft) and parallel (hard) polarizations. This justifies the widespread use of KED for radio wave propagation whenever polarization or composition of the diffracting screen is unknown. The GTD coefficient derived in this paper also allows engineers to tackle difficult multiple diffraction problems with geometrical techniques, rather than more cumbersome procedures such as Millington's technique [1].

Breaking Down Edge Diffraction Formulas

The KED problem is formulated for a semi-infinite half-plane in between a point source and an observation point, as shown in Figure 1. Thus, a line drawn from the source to the observation point will clear the diffracting knife edge by a distance d. A negative value of d implies obstruction. The famous KED solution for this case is

$$E_{\diamond} = \left(\frac{1+j}{2}\right)\frac{E_o \exp(-jk[r_o+r])}{k(r+r_o)}\sqrt{\frac{2}{\pi}}\mathrm{F}\left(-d\sin\phi_i\sqrt{\frac{k}{2}\left(\frac{1}{r_o}+\frac{1}{r}\right)}\right) \qquad (1)$$

where k is wavenumber ($k=\frac{2\pi}{\lambda}$) and r, r_o, d, and θ are geometrical terms summarized in Figure 1 [2]. $\mathrm{F}(\cdot)$ is the Fresnel integral function:

$$\mathrm{F}(a) = \int_a^{\infty} \exp\left(-j\tau^2\right)\, d\tau \qquad (2)$$

The KED result presumes an isotropic radiator of the form

$$E_{\diamond i} = \frac{E_o \exp(-jk[r_o+r])}{k(r_o+r)} \qquad (3)$$

on the source-side of the screen. Equation (1) is an approximate result, independent of incident field polarization (the $\diamond$-subscript is used to denote a scalar result). However it has proven resiliently useful in optics and radio wave propagation.

There is a powerful geometrical interpretation of Equation (1) which allows it to be recast in terms of distances between the line-of-sight path, $r+r_o$, and the total

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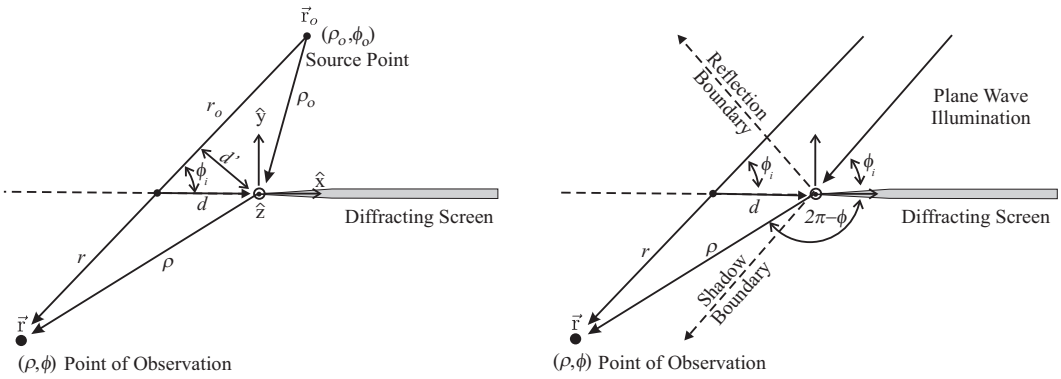


Figure 1: Geometry used for diffraction by a thin semi-infinite plane (i.e. a "knife edge").

length of the two line segments formed from source-to-edge-to-observation. This difference may be expressed mathematically as Δr:

$$\Delta r = \overbrace{\sqrt{r_o^2 + d'^2}}^{\text{source-to-edge}} + \overbrace{\sqrt{r^2 + d'^2}}^{\text{edge-to-observation}} - r - r_o \approx \frac{d'^2}{2}\left(\frac{1}{r_o} + \frac{1}{r}\right) \tag{4}$$

Interestingly, the square-root of this term appears in the argument of the Fresnel integral function that describe the principle amplitude changes in Equation (1).

KED Asymptotic Expansion

We may subtract out the geometrical optics portion of the KED solution from Equation (1) over the region $\phi < \phi_i + \pi$ to achieve the diffracted field:

$$E_d = \mp\sqrt{\frac{2}{\pi}}\left(\frac{1+j}{2}\right)\frac{E_o \exp(-jk(r_o + r))}{k(r + r_o)}\mathrm{F}\left(\pm d \sin\phi_i \sqrt{\frac{k}{2}\left(\frac{1}{r_o} + \frac{1}{r}\right)}\right) \tag{5}$$

where the top signs are valid for the incidence region ($\phi < \phi_i + \pi$) and the bottom signs are valid for the shadowed region ($\phi > \phi_i + \pi$). We may recast this in terms of the path length difference Δr from Equation (4):

$$E_d(\rho,\phi) = \mp\sqrt{\frac{2}{\pi}}\left(\frac{1+j}{2}\right)\frac{E_o \exp(-jk(\rho_o + \rho - \Delta r))}{k(\rho_o + \rho - \Delta r)}\mathrm{F}\left(\pm\sqrt{k\Delta r}\right) \tag{6}$$

Half-plane diffraction problems are convenient in analysis, since asymptotic expansions can be achieved simply by integrating F() by parts repeatedly:

$$\mathrm{F}(a) = \underbrace{-\frac{j}{2a}\exp(-ja^2)}_{O(1/a)} + \underbrace{\frac{1}{4a^3}\exp(-ja^2)}_{O(1/a^3)} - \underbrace{\frac{1}{12}\int_a^\infty \frac{1}{x^4}\exp(-jx^2)\,dx}_{O(1/a^5)\ +\ \text{higher-order terms}} \tag{7}$$

This method does not get any closer to a closed-form result for the integral, but we have generated a series expansion with respect to $1/a$. Keeping only the first

term in Equation (7) and ignoring the higher-order behavior, the diffracted field of Equation (6) may be written as

$$E_d(\rho,\phi) \approx \underbrace{\frac{E_o}{k\rho_o}\exp(-jk\rho_o)}_{\text{incident field}}\underbrace{\frac{\exp(-jk\rho)}{\sqrt{k\rho}}\sqrt{\frac{\rho_o}{\rho_o+\rho}}}_{\text{diffracted wave}}\underbrace{\frac{-1}{2\sqrt{\pi}}\exp\left(-j\frac{\pi}{4}\right)\frac{\sqrt{\rho_o\rho(\rho_o+\rho)}}{(\rho_o+\rho-\Delta r)\sqrt{\Delta r}}}_{\text{diffraction coefficient}} \tag{8}$$

The diffracted wave in Equation (8) now takes the geometrical form of a localized wave peeling off the knife edge. The diffraction coefficient (all of the leftover terms) should be relatively insensitive to the source distance ρ_o from the edge, as GTD would predict. Thus, we may take the limit as $\rho_o \to \infty$, which would correspond to plane-wave incidence:

$$D_\diamond(\phi,\phi_i) = \frac{-1}{2\sqrt{\pi}}\exp\left(-j\frac{\pi}{4}\right)\sqrt{\frac{\rho}{\Delta r}} \tag{9}$$

which approximately resembles a GTD result obtained by Giovaneli in [3]. Based on the plane wave geometry, apply the law-of-sines in Figure 1 to simplify the diffracted wave solution:

$$\sqrt{\frac{\rho}{\Delta r}} = \sqrt{\frac{2r\rho}{d^2\sin^2\phi_i}} = \frac{1}{|\sin(\phi-\phi_i)|}\sqrt{\frac{2\sin\phi}{\sin\phi_i}} \tag{10}$$

After substitutions and simplifications, the diffraction coefficient may be expressed solely as a function of angular geometry. The end result is summarized in Table 1 and compared to other well-known, half-screen diffraction coefficients.

Table 1: Summary of common half-plane GTD diffraction coefficients [4].

Solution Type	Diffraction Coefficient
Sommerfeld-$\perp$	$D_\perp(\phi,\phi_i) = \frac{-\exp\left(-\frac{j\pi}{4}\right)}{2\sqrt{2\pi}}\left[\sec\left(\frac{\phi-\phi_i}{2}\right)-\sec\left(\frac{\phi+\phi_i}{2}\right)\right]$
Sommerfeld-$\parallel$	$D_\parallel(\phi,\phi_i) = \frac{-\exp\left(-\frac{j\pi}{4}\right)}{2\sqrt{2\pi}}\left[\sec\left(\frac{\phi-\phi_i}{2}\right)+\sec\left(\frac{\phi+\phi_i}{2}\right)\right]$
Felsen Absorber	$D_\diamond^{(F)}(\phi,\phi_i) = \frac{-1}{\sqrt{2\pi}}\left[\frac{1}{\pi-\lvert\phi-\phi_i\rvert}+\frac{1}{\pi+\lvert\phi-\phi_i\rvert}\right]$
KED Screen	$D_\diamond(\phi,\phi_i) = \frac{-\exp\left(-j\frac{\pi}{4}\right)}{\sqrt{2\pi}}\lvert\csc(\phi-\phi_i)\rvert\sqrt{-\frac{\sin\phi}{\sin\phi_i}}$

Comparison of the Diffraction Coefficients

In Figure 2, the graph GTD diffracted power of both Sommerfeld-$\perp$ and -$\parallel$incidence PEC waves is shown, with each normalized against the KED GTD coefficient for the same observation angle; incidence angle is at $\phi_i = 90°$. Notice that for observations near the shadow boundary, all GTD coefficients are nearly identical, independent of incident angle. When the observation point drops deeper into the shadow region ($\phi \to 360°$), the $\parallel$-polarization PEC result leads to much more diffracted power than the $\perp$-polarization. The converse is true for observations made close to the open aperture in the problem ($\phi \to 180°$).

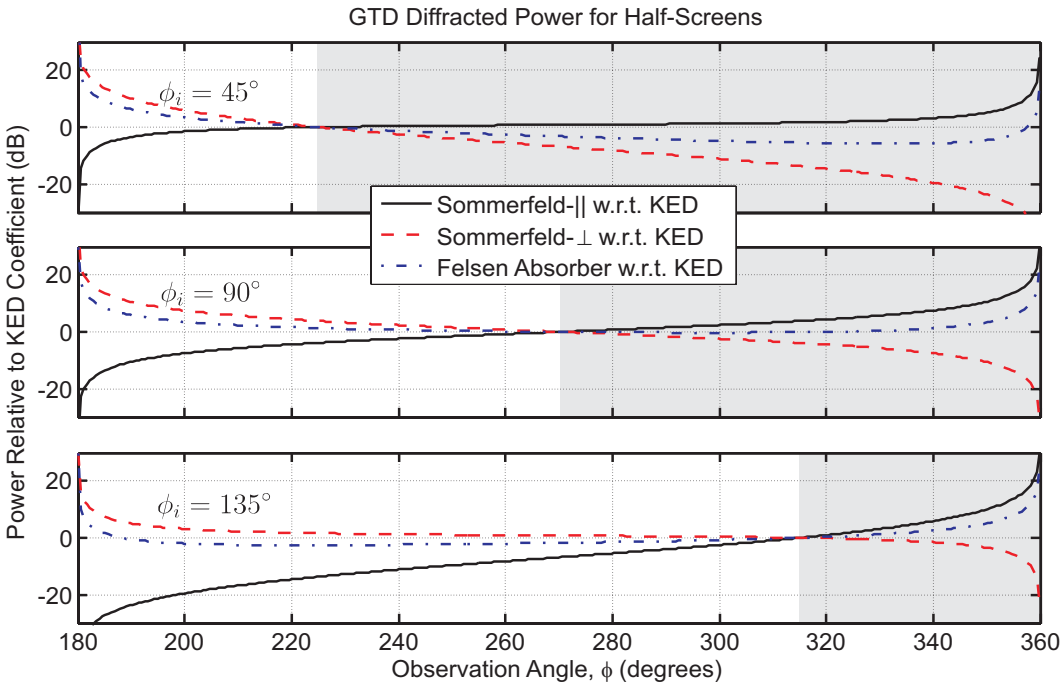


Figure 2: Comparison of GTD coefficients with respect to the KED-based coefficient across observation angles $180° \leq \phi \leq 360°$. Graphs are for $\phi_i = 45°$, $90°$ (normal), and $135°$ incidence. Gray area indicates the shadow region.

Perhaps most interestingly, the normal-incident KED diffraction coefficient is an *exact* dB-average of the two PEC half-planes across the entire observation area. In the linear scale, the KED diffraction coefficient is then the geometric mean of the two Sommerfeld-based coefficients:

$$|D_\diamond(\phi, \pi/2)|^2 = \Big| D_\perp(\phi, \pi/2) \Big| \Big| D_\parallel(\phi, \pi/2) \Big| \tag{11}$$

for all observation angles $180° \leq \phi \leq 360°$. Despite its approximations, the KED coefficient represents a profoundly useful middle-ground in diffraction problems where polarization or screen composition is unknown. Even more bizarre is that by forcing KED into a geometrical framework, some of the problematic phase behavior of the KED result near the plane of the screen has been fixed.

References

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