<u>Curriculum Topic</u> :
Time-Harmonic Transmission Lines

<u>THT1</u> : Phasor Review

Module Outline:	
Prerequisite Skills	Competencies
Supplemental Reading and Resources	Assessments
Power Point Slides and Notes	

Prerequisite Skills

Prerequisites / Requirements:
MATH 4 semesters of calculus
PHYS 2 semesters of undergraduate physics

Competencies

Competency THT.1: Understand and apply the phasor transform

Competency Builders:
THT.1.1 Convert a time-harmonic, time-domain function into a phasor.
THT.1.2 Convert a phasor into a time-domain function.
THT.1.3 Understand why phasors are useful for linear, time-invariant systems.

Supplemental Reading and Resources

Supplemental Reading Materials:

Prof. Durgin's online phasor notes.

Assessments

The following questions and exercises may serve as either pre-assessment or post-assessment tests to evaluate student knowledge.

Question: THT.1.1 *Competency:* THT.1.1

What is the phasor-domain expression for the following equation: $5\cos(10\pi t + 7x^2)$?

Answer:

$5\exp(j7x^2)$

Question: THT.1.2 *Competency:* THT.1.2

What is the time-domain function x(t) of the phasor $\boldsymbol{X} = 25\exp(-j5z)$ if the frequency is 2 kHz?

Answer:

$x(t) = 25\cos(4000\pi t - 5z)$

Question: THT.1.3 *Competency:* THT.1.3

Mathematically show that when the input of an LTI system is a sinusoid, the output is also a sinusoid. (5 points)

Answer:

To show why this is true, start with the basic convolution integral and substitute a generic sinusoidal input $x(t) = A\cos(2\pi f t + \phi)$:

$$
\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)\,d\lambda \\
&= \int_{-\infty}^{\infty} x(t-\lambda)h(\lambda)\,d\lambda \\
&= \int_{-\infty}^{\infty} A\cos(2\pi f[t-\lambda]+\phi)h(\lambda)\,d\lambda \\
&= A\left[\cos(2\pi f t)\underbrace{\int_{-\infty}^{\infty}\cos(\phi-2\pi f\lambda)h(\lambda)\,d\lambda}_{H_x} - \sin(2\pi f t)\underbrace{\int_{-\infty}^{\infty}\sin(\phi-2\pi f\lambda)h(\lambda)\,d\lambda}_{H_y}\right] \\
&= A\left[H_x\cos(2\pi f t) - H_y\sin(2\pi f t)\right] \\
&= A\sqrt{H_x^2+H_y^2}\cos\left(2\pi f t - \tan^{-1}\frac{H_y}{H_x}\right)
\end{aligned}
$$

After a series of manipulations, we see that the final answer is a sinusoid whose overall amplitude and phase depend on the system characteristics. Since H_x and H_y evaluate to constants, the result is *always* true for any LTI system; the frequency f does not change. You have just proven the most important applied theorem in electrical engineering.