THT1: Phasor Review

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Form of a Time-Harmonic Function

$$f(t) = A\cos(2\pi f t + \phi)$$

the three constants have the following meanings:

A: *amplitude* or *envelope* wave
f: harmonic *frequency*, with units of Hertz (Hz) or 1/s
ϕ: *phase* of the wave, with units of radians

The Phasor Transform

$$\text{Forward Transform:} \quad \tilde{X} = \mathcal{P}\{x(t)\}$$

$$\text{Inverse Transform:} \quad x(t) = \mathcal{P}^{-1}\{\tilde{X}\}$$

$$x(t) = A\cos(2\pi f t + \phi) \longrightarrow \tilde{X} = A\exp(j\phi)$$

back into the time domain, use the following formula:

$$x(t) = \text{Real}\{\tilde{X}\exp(j2\pi f t)\}$$

Example Phasor Calculation

Example 1.1: Basic Phasor Transform

Problem: Convert the function $7\sin(2\pi t)$ into the phasor domain and then back into the time domain.

Solution:

1. To go into the phasor domain, we first recognize that if $x(t) = 7\sin(2\pi t)$, we may also write this as

$$x(t) = 7\cos\left(2\pi t - \frac{\pi}{2}\right)$$

Following the formula in Equation (1.1.2), we can write this function in the phasor domain as

$$\tilde{X} = 7\exp\left(-j\frac{\pi}{2}\right)$$

2. To go back into the time domain is straightforward:

$$\begin{aligned} x(t) &= \text{Real}\{7\exp\left(-j\frac{\pi}{2}\right)\exp(j2\pi f t)\} \\ &= \text{Real}\left\{7\exp\left(j\left[2\pi f t - \frac{\pi}{2}\right]\right)\right\} \\ &= \text{Real}\left\{7\cos\left(2\pi f t - \frac{\pi}{2}\right) + j7\sin\left(2\pi f t - \frac{\pi}{2}\right)\right\} \\ &= 7\cos\left(2\pi f t - \frac{\pi}{2}\right) \\ &= 7\sin\left(2\pi f t\right) \end{aligned}$$

For the last steps, we applied the Euler formula for complex exponentials: $\exp(jx) = \cos x + j\sin x$.

Rectangular Form of Phasors

$$R\exp(j\phi) \quad = \quad = R\cos\phi + jR\sin\phi \quad = \quad X + jY \qquad ($$

Rectangular to Polar: $R = \sqrt{X^2 + Y^2} \quad \phi = \begin{cases} \tan^{-1}\left(\frac{Y}{X}\right) & X > 0 \\ \pi + \tan^{-1}\left(\frac{Y}{X}\right) & X < 0 \end{cases}$

Polar to Rectangular: $X = R\cos\phi \qquad Y = R\sin\phi$

Polar and Rectangular Form Geometry

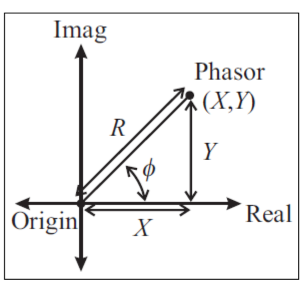
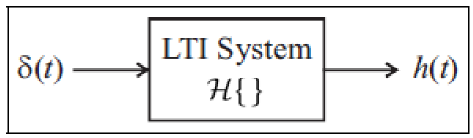


Figure 1.1. The rectangular form of a phasor marks a pair of Cartesian coordinates (X, Y) in the complex plane, with an alternate polar form representing magnitude R and phase ϕ.

Linear, Time-Invariant Systems

$$\mathcal{H}\{ax(t) + by(t)\} = a\mathcal{H}\{x(t)\} + b\mathcal{H}\{y(t)\}$$

$$h(t) = \mathcal{H}\{\delta(t)\}$$



Proof of Convolution for LTI Systems

$$x(t) = \int_{-\infty}^{\infty} x(\lambda)\delta(t-\lambda)\,dt$$

$$\begin{aligned} y(t) &= \mathcal{H}\{x(t)\} \\ &= \mathcal{H}\{\int_{-\infty}^{\infty} x(\lambda)\delta(t-\lambda)\,dt\} \\ &= \int_{-\infty}^{\infty} x(\lambda)\underbrace{\mathcal{H}\{\delta(t-\lambda)\}}_{h(t-\lambda)}\,dt \end{aligned}$$

$\longrightarrow$ *convolution integral.*

$$y(t) = x(t) \otimes h(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)\,dt$$

Sine Wave In, Sine Wave Out

$$A_1 \cos(2\pi f t + \phi_1) \otimes h(t) = A_2 \cos(2\pi f t + \phi_2)$$

$$\tilde{Y} = \tilde{H}\,\tilde{X} \quad \text{or} \quad A_y \exp(j\phi_y) = A_h\, A_x \exp(j[\phi_h + \phi_x])$$

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