# Curriculum Topic :
# Time-Harmonic Transmission Lines

## THT2 : Sinusoids on Transmission Lines

| *Module Outline:* | |
|---|---|
| Prerequisite Skills | Competencies |
| Supplemental Reading and Resources | Assessments |
| Power Point Slides and Notes | |

### Prerequisite Skills

*Prerequisites / Requirements:*
**THT1** Phasor Review

### Competencies

**Competency THT.2:** **Describe the mathematical form of sinusoidal waves on a transmission line**

*Competency Builders:*
THT.2.1 Write the voltage and current solutions for a time-harmonic transmission line in the phasor domain
THT.2.2 Quantify the relationship between wavenumber, wavelength, and frequency

### Supplemental Reading and Resources

*Supplemental Reading Materials:*

Prof. Peterson's online lecture notes 13

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## Assessments

The following questions and exercises may serve as either pre-assessment or post-assessment tests to evaluate student knowledge.

*Question:* THT.2.1 *Competency:* THT.2.1

True or False: Under steady-state time-harmonic excitation, a useful equivalent circuit for a transmission line is two short circuit pathways.

*Answer:*

false

*Question:* THT.2.2 *Competency:* THT.2.2

A transmission line with velocity of propagation 1.7 x $10^8$ m/s is excited at 2.45 GHz. What is the wavenumber of this line?

*Answer:*

90.6 rad/m

*Question:* THT.2.3 *Competency:* THT.2.1

**Transmission Line with Sinusoidal Excitation:** (30 points) Below are phasor-domain voltage and current to a transmission line operating with steady-state sinusoids with frequency $f$:

$$\tilde{v}(z) = 100\exp(-j\beta[z-D]) + 50\exp(j\pi + j\beta[z-D])$$

$$\tilde{i}(z) = \exp(-j\beta[z-D]) - \frac{1}{2}\exp(j\pi + j\beta[z-D])$$

where $z=0$ is the source side and $z=D$ is the load side. Perform the following analysis:

(a) In the equations above, circle the portion of the solution representing the backward-propagating current waveform. (5 points)

(b) In the equations above, box the forward-propagating *amplitude* of the voltage waveform. (5 points)

(c) Write a simplified expression for time-varying voltage evaluated at the front of the line ($z=0$). Your answer should be in the time domain, $v(t)$. (5 points)

(d) What is the characteristic impedance, $Z_0$, of this line? (5 points)

(e) What is the VSWR of voltages on this line? (5 points)

(f) What is the load impedance, $Z_L$, for this line? (5 points)

*Answer:*

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(a) In the equations above, circle the portion of the solution representing the backward-propagating current waveform. *The term* $\frac{1}{2}\exp(j\pi + j\beta[z-D])$ *should be circled.*

(b) In the equations above, box the forward-propagating *amplitude* of the voltage waveform. *The amplitude 100 should be boxed.*

(c) This follows from the phasor inverse transform definition (see formula sheet)

$$\begin{aligned} v(t,0) &= \text{Real}\left\{\tilde{v}(0)\exp(j2\pi ft)\right\} \\ &= 100\cos(2\pi ft + \beta D) + 50\cos(2\pi ft - \beta D + \pi) \\ &= 100\cos(2\pi ft + \beta D) - 50\cos(2\pi ft - \beta D) \end{aligned}$$

Anyone who made it to line two got full credit.

(d) $Z_0 = 100\Omega$

(e) There are two ways of doing this. First, we immediately see that $|\Gamma| = 0.5$ because the reflected voltage/current wave is half the magnitude of the forward voltage/current wave. This can be plugged directly into one of the VSWR formulas to get VSWR=3. The same result can be obtained from the following reasoning: VSWR is the ratio of max voltage to min voltage on the line; the max voltage occurs when the forward wave adds in phase with the backwards wave (100+50 V); the min voltage occurs when the forward wave adds out-of-phase (destructively) with the backwards wave (100-50 V); thus, the VSWR will be 150/50 or 3.

(f) For this problem, I gave full credit to anyone who put down line 1 of the following answer, but did not finish the calculation. Very few people got full credit for this problem.

$$\begin{aligned} Z_L &= \frac{\tilde{v}(D)}{\tilde{i}(D)} \\ &= \frac{100 + 50\exp(j\pi)}{1 - \frac{1}{2}\exp(j\pi)} \\ &= \frac{100 - 50}{1 + \frac{1}{2}} \\ &= \frac{100}{3}\Omega \end{aligned}$$

Many people tried to solve this from the VSWR by using the property $|\Gamma| = \frac{1}{2}$ to backsolve $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$. This actually doesn't work because it only gives you the *magnitude* of the reflection coefficient. Many people worked the problem with $\Gamma = \frac{1}{2}$ and found $Z_L = 300\Omega$, when in fact the reflection coefficient for this particular example was $\Gamma = -\frac{1}{2}$.

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