

# THT2: Sinusoids on Transmission Lines

By Prof. Gregory D. Durgin

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## Time-Harmonic Systems and Phasors

Phasors

$V(t) = A \cos(2\pi ft + \phi)$

three pieces of information

For a linear time invariant system:

$A \cos(2\pi ft + \phi_1) \rightarrow \text{LTI system} \rightarrow B \cos(2\pi ft + \phi_2)$

We only need to track 2 pieces of info

take this for granted

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## From Phasor-Domain to Time Domain

Phasors

$V(t) = A \cos(2\pi ft + \phi)$   
 three pieces of information

time invariant

For a linear system:

$A \cos(2\pi ft + \phi_1) \rightarrow \boxed{\text{LTI system}} \rightarrow B \cos(2\pi ft + \phi_2)$   
 take this for granted

We only need to track 2 pieces of info

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## Example of Sinusoidal Resonance

$V_L(t)$   
 $\Gamma_G = -1$      $\Gamma_L = 1$

General Form

$$V_L(t) = (1 + \Gamma_L) \sum_{n=0}^{\infty} (\Gamma_L \Gamma_G)^n V_S(t - [2n+1]T)$$

$$= 2 \sum_{n=0}^{\infty} (-1)^n A \cos(2\pi f [t - [2n+1]T])$$

Phasor Transform:

$$\tilde{V}_L = 2 \sum_{n=0}^{\infty} (-1)^n A \exp(-j 2\pi f [2n+1]t)$$

$$= 2A \exp(-j 2\pi f T) \sum_{n=0}^{\infty} (-1)^n \exp(-j 4\pi f n T)$$

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## Recast Solution in terms of $D$ and $\beta$

$$\lambda = \frac{2\pi}{\beta} \quad \beta = \frac{2\pi}{\lambda} \quad T = \frac{D}{v_p} = \frac{D}{\lambda f} = \frac{\beta D}{2\pi f}$$

wavenumber  
radians/meter

$$\tilde{V}_L = 2A \exp(-j \frac{2\pi f \beta D}{2\pi f}) \sum_{n=0}^{\infty} (-1)^n \exp(-j \frac{4\pi n f D}{2\pi f})$$

$$\tilde{V}_L = 2A \exp(-j \beta D) \sum_{n=0}^{\infty} (-1)^n \exp(-j 2n \beta D)$$

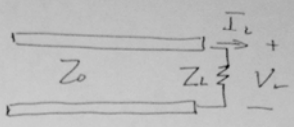
$$\tilde{V}_L = \frac{2A \exp(-j \beta D)}{1 + \exp(-j 2\beta D)} \times \frac{\exp(j \beta D)}{\exp(j \beta D)}$$

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## What is Impedance Transformation?

at load:

$$I_L = \frac{V_L}{Z_L}$$


$$V(D) = V_o^+ \exp(-j \beta D) + V_o^- \exp(j \beta D) = V_L$$

$$I(D) = I_L = \frac{V_L}{Z_L} = \frac{V_o^+}{Z_0} \exp(-j \beta D) - \frac{V_o^-}{Z_0} \exp(j \beta D)$$

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## Solving for Forward and Backward Waves

Solved for  $V_o^+$ ,  $V_o^-$

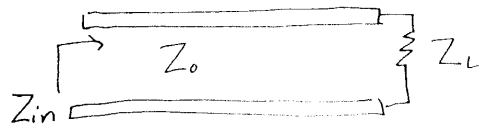
$$V_o^+ = \frac{1}{2} \exp(j\beta D) \left(1 + \frac{Z_o}{Z_L}\right) V_L$$

$$V_o^- = \frac{1}{2} \exp(j\beta D) \left(1 - \frac{Z_o}{Z_L}\right) V_L$$

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## Load Transformation



$$Z_{in} = Z_o \left[ \frac{Z_L + j Z_o \tan \beta D}{Z_o + j Z_L \tan \beta D} \right]$$

Special cases:

Matched:  $Z_L = Z_o$

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