THT2: Sinusoids on Transmission Lines

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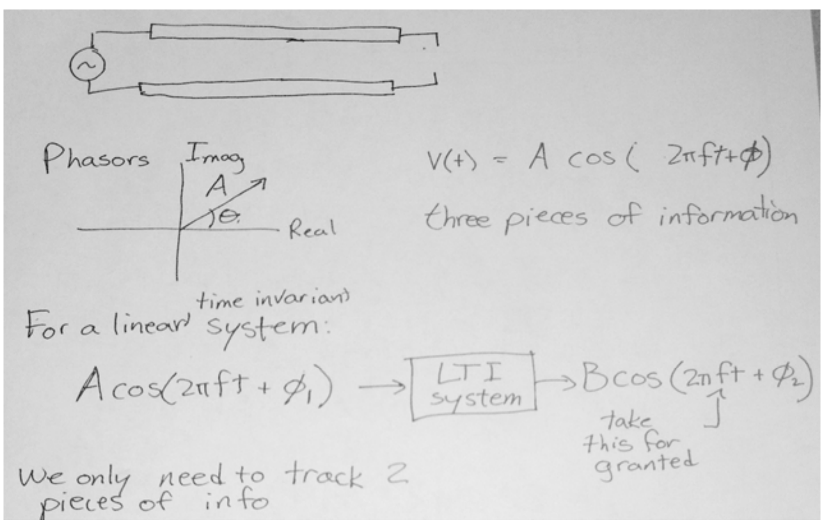
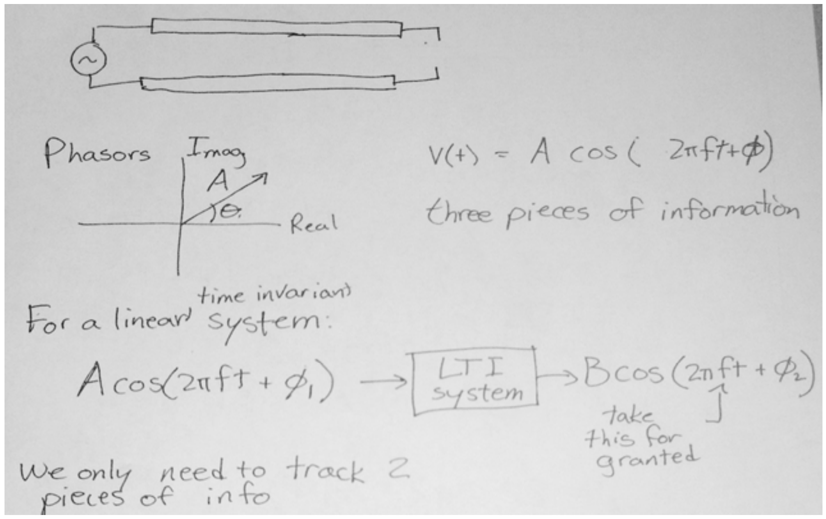
Time-Harmonic Systems and Phasors



From Phasor-Domain to Time Domain



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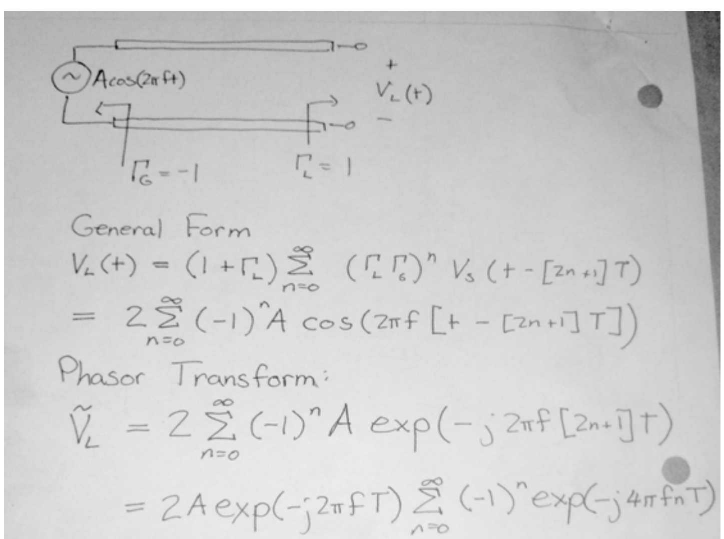
Example of Sinusoidal Resonance



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Recast Solution in terms of D and β

$\lambda = \frac{2\pi}{\beta}$ $\beta = \frac{2\pi}{\lambda}$ $T = \frac{D}{v_p} = \frac{D}{\lambda f} = \frac{\beta D}{2\pi f}$

wavenumber
radians/meter

$$\tilde{V}_L = 2A\exp\left(-j\,\frac{2\pi f\,\beta D}{2\pi f}\right)\sum_{n=0}^{\infty}(-1)^n \exp\left(-j\,\frac{4\pi n f\,\beta D}{2\pi f}\right)$$

$$\tilde{V}_L = 2A\exp(-j\beta D)\sum_{n=0}^{\infty}(-1)^n \exp(-j2n\beta D)$$

$$\tilde{V}_L = \frac{2A\exp(-j\beta D)}{1+\exp(-j2\beta D)} \times \frac{\exp(j\beta D)}{\exp(j\beta D)}$$

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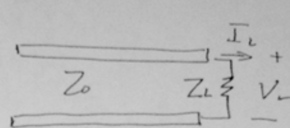
What is Impedance Transformation?

at load:

$$I_L = \frac{V_L}{Z_L}$$



$$V(D) = V_o^+\exp(-j\beta D) + V_o^-\exp(j\beta D) = V_L$$

$$I(D) = I_L = \frac{V_L}{Z_L} = \frac{V_o^+}{Z_o}\exp(-j\beta D) - \frac{V_o^-}{Z_o}\exp(j\beta D)$$

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Solving for Forward and Backward Waves

Solved for V_0^+, V_0^-

$$V_0^+ = \frac{1}{2}\exp(j\beta D)\left(1 + \frac{Z_0}{Z_L}\right)V_L$$

$$V_0^- = \frac{1}{2}\exp(-j\beta D)\left(1 - \frac{Z_0}{Z_L}\right)V_L$$

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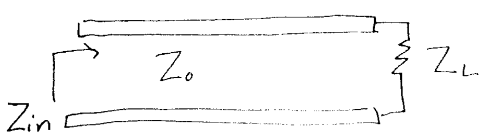
Load Transformation



$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan\beta D}{Z_0 + jZ_L\tan\beta D}\right]$$

Special cases:

Matched: $Z_L = Z_0$

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