Curriculum Topic :
Time-Harmonic Transmission Lines

THT4 : Arbitrary Loads on Time-Harmonic Lines

Module Outline:	
Prerequisite Skills	Competencies
Supplemental Reading and Resources	Assessments
Power Point Slides and Notes	

Prerequisite Skills

Prerequisites / Requirements:
THT3 Open- and Short-Circuit Terminations on Transmission Lines

Competencies

Competency THT.4: **Analyze the time-harmonic voltage and current on transmission lines terminated with arbitrary complex loads**

Competency Builders:

THT.4.1	Calculate voltage standing wave ratio for a line with an arbitrary load
THT.4.2	Calculate the Thevenin equivalent impedance for any time-harmonic line
THT.4.3	Solve a circuit containing one or more time-harmonic transmission lines

Supplemental Reading and Resources

Supplemental Reading Materials:

Prof. Peterson's online lecture notes 15

Assessments

The following questions and exercises may serve as either pre-assessment or post-assessment tests to evaluate student knowledge.

Question: THT.4.1 *Competency:* THT.4.1

What is the VSWR of a 50 Ω transmission line terminated with a 50 + j 50 Ω impedance?

Answer:

2.6 (linear) or 8.3 dB

Question: THT.4.2 *Competency:* THT.4.2

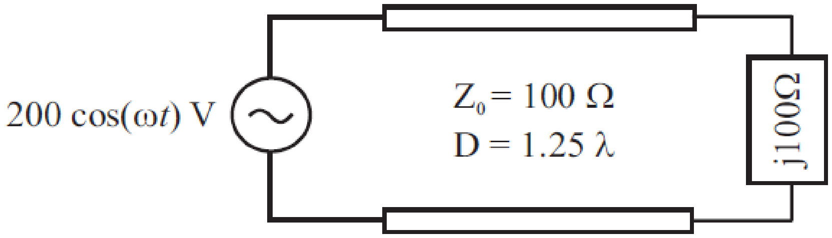
A 120 Ω transmission line is 0.372λ long and terminated with a 20 Ω resistor. What is the Thevenin equivalent impedance at the other end of the line?

Answer:

40 – j 118 Ω

Question: THT.4.3 *Competency:* THT.4.3

Sinusoidal Transmission Lines: Below is an ideal 200 volt sinusoidal source connected to a pure inductor via a 1.25λ transmission line. Write the phasor-form solutions for $\tilde{v}(z)$ and $\tilde{i}(z)$. What is the VSWR for this line? (**30 points**)



Answer:

The $j100\,\Omega$ load-transforms to the source side as a $-j100\,\Omega$ load, which is the Thevenin equivalent circuit of the transmission line. Hooked directly to the sinusoidal source, we can calculate that the input voltage is going to be $200\angle 0^\circ$ V and the input current is going to be $j2$ A. Now write down the general phasor solution to the transmission line equations:

$$\begin{aligned} \tilde{v}(z) &= V^+\exp(-j\beta z) + V^-\exp(+j\beta z) & \tilde{v}(0) &= V^+ + V^- &= 200\text{ V} \\ \tilde{i}(z) &= \frac{V^+}{Z_0}\exp(-j\beta z) - \frac{V^-}{Z_0}\exp(+j\beta z) & \tilde{i}(0) &= \frac{V^+}{Z_0} - \frac{V^-}{Z_0} &= j2\text{ A} \end{aligned}$$

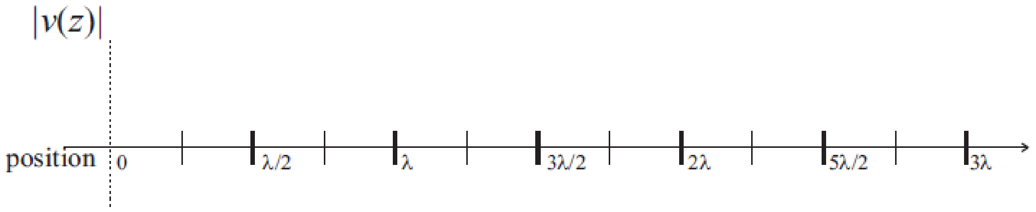
As shown above, we evaluate for current and voltage at the front of the line and equate this to our input voltage and current. This gives us two equations and two unknowns. Solving for V^+ and V^-, we get the following solution:

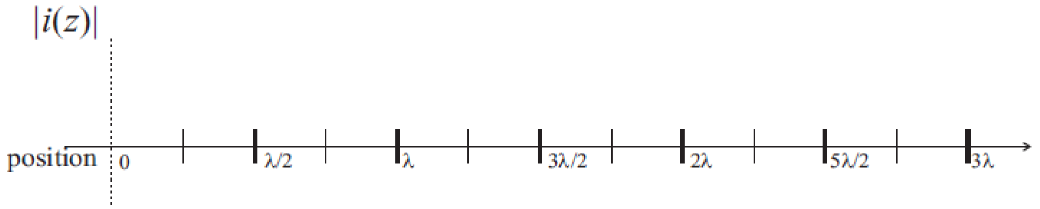
$$\begin{aligned} \tilde{v}(z) &= (100 + j100)\exp(-j\beta z) &+& \ (100 - j100)\exp(+j\beta z) \\ \tilde{i}(z) &= (1 + j)\exp(-j\beta z) &-& \ (1 - j)\exp(+j\beta z) \end{aligned}$$

Question: THT.4.4 *Competency:* THT.4.1,2,3

Sinusoidal T-lines: Answer the following questions based on time-harmonic estimation of a transmission line connected to a load R_L. (**35 points**)

(a) Sketch the total voltage and total current across a 3λ-length transmission line if the peak voltage is 10V and the VSWR is 2. The load is purely resistive and $R_L > Z_0$. Label voltage amplitudes (peaks and nulls) on the graph. (**15 points**)





(b) On your graph above, draw arrows pointing to the locations along z that would result in the *highest* possible Thevenin equivalent input impedance for the remaining section of transmission line. Place a circle around all of the locations along z that would likewise result in the lowest possible Thevenin equivalent input impedance. (**5 points**)

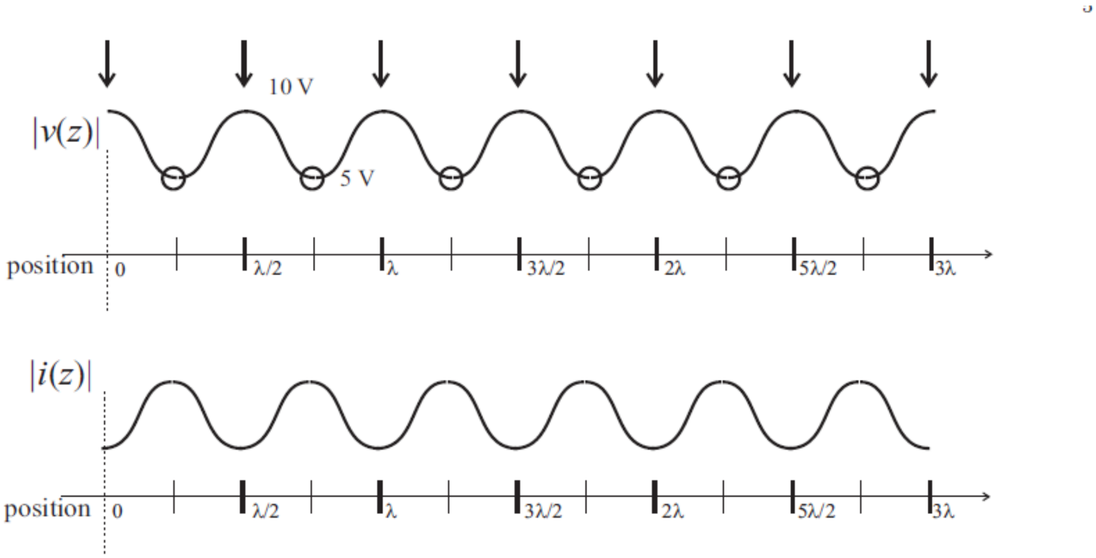
(c) Based on the information of part (a), solve for R_L in terms of Z_0. (**10 points**)

(d) Would the VSWR increase or decrease if the load was replaced with an inductor? Explain why. (**5 points**)

Answer:

Sinusoidal T-lines: Answer the following questions based on time-harmonic estimation of a transmission line connected to a load R_L.

(a) For VSWR of 2, the peak total voltage will be 10 V and the minimum voltage will be 5 V, with peaks and nulls alternating every $lambda/4$:



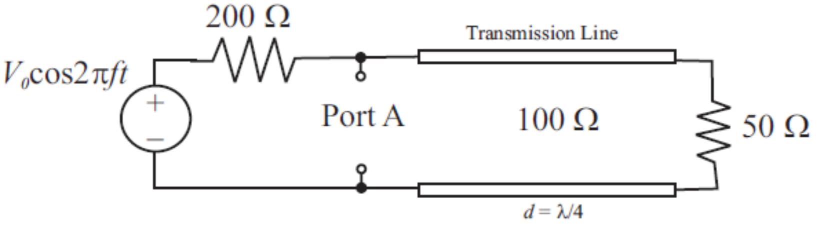
(b) Thevenin equivalent impedance is defined as $v(z)/i(z)$ at a given point; see above for labeling.

(c) $R_L = 2Z_0$ results in a reflection coefficient of $+1/3$ and a VSWR of 2. Note that you could also achieve VSWR of 2 with a purely resistive $R_L = Z_0/2$, but this violates the condition of $R_L > Z_0$ given in the problem statement.

(d) At steady-state time-harmonic excitation, purely inductive loads result in VSWR of ∞, so clearly VSWR would increase.

Question: THT.4.5 *Competency:* THT.4.3

Sinusoidal Thevenin Equivalent: Below is a 100Ω steady-state, sinusoidal transmission line circuit that is a quarter-wavelength in length. Answer the following questions based on this scenario. (**26 points**)

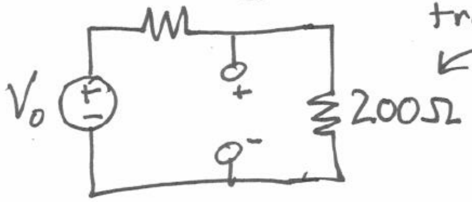


(a) What is the phasor-form Thevenin equivalent circuit as seen at port A for this system? (Reminder: Thevenin voltage is the open circuit voltage at port A; the Thevenin impedance is the open circuit voltage divided by the short circuit current.) (**16 points**)

(b) What is the $VSWR$ of this transmission line? (**10 points**)

Answer:

(a) What is the phasor-form Thevenin equivalent circuit as seen at port A for this system? (Reminder: Thevenin voltage is the open circuit voltage at port A; the Thevenin impedance is the open circuit voltage divided by the short circuit current.) (**16 points**)



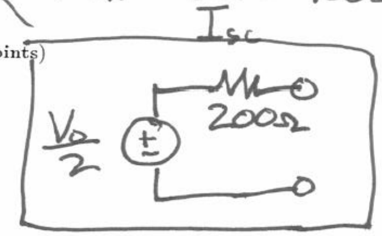
transform to $\left(\frac{Z_0^2}{Z_L}\right)$

$$V_{oc} = \frac{V_0}{2}$$

$$I_{sc} = \frac{V_0}{200\Omega}$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = 100\Omega$$

(b) What is the $VSWR$ of this transmission line? (**10 points**)



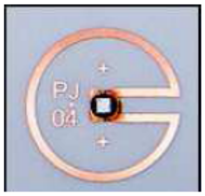
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50-100}{50+100}$$

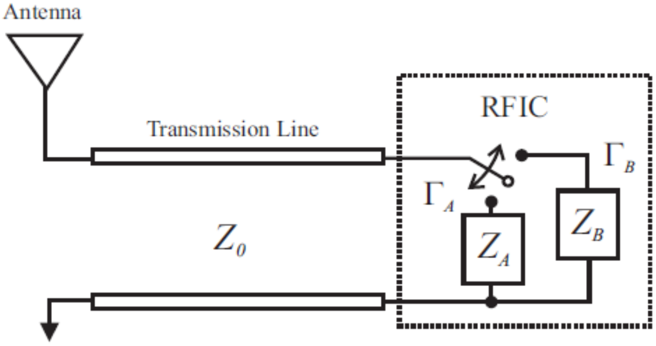
$$= -\frac{1}{3}$$

$$VSWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+|-\frac{1}{3}|}{1-|-\frac{1}{3}|} = \underline{\underline{2}}$$

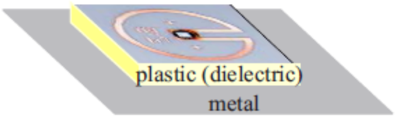
Question: THT.4.6 *Competency:* THT.4.3

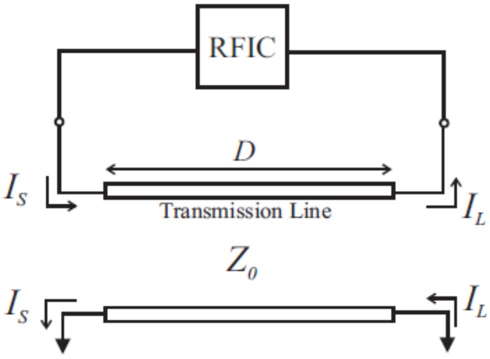
RFID on Metallic Surfaces: Below is a picture of an Impinj$^{\text{TM}}$ 915 MHz inductive RFID tag (left), consisting of an radio-frequency integrated circuit (RFIC) in the center connected to a small loop antenna. This is usually fabricated atop a piece of insulating plastic. One way to model this system is shown below (right) where the antenna sends a forward-propagating voltage down a coplanar strip transmission line terminated by the RFIC. The RFIC can modulate information on the reflected sinusoidal wave by switching between different loads Z_A and Z_B.





When the RFID tag is placed on a metal surface, the topology of the circuit changes dramatically. One way to model this new topology is with a *microstrip* transmission line; the antenna and feed traces now beat against the metal surface as if it were a ground plane underneath the plastic/dielectric, just like a printed circuit board!





Answer the following questions based on this scenario. (**36 points**)

(a) For the free-space RFID tag, if the chip is excited by the antenna with a forward propagating wave of 1V, what is the amplitude and phase of the reflected signals if Z_A is an open circuit and Z_B is a perfect short? (**5 points**)

(b) For the free-space RFID tag, what is the VSWR (in dB) for either transmission state in (a)? (**5 points**)

(c) When the RFID tag is brought closer and closer to metal, explain how Z_0 changes in the second circuit model. (**10 points**)

(d) What is the mathematical relationship between V^+ and V^- (in terms of β, D, and Z_0) on the transmission line in the near-metal circuit model? (Hint: What does Kirchoff's Current Law state about the relationship between I_L and I_S?) (**16 points**)

(e) (**Bonus +5 Points**): What is the Thevenin equivalent of the antenna in this circuit as seen from the terminals of the RFIC in terms of β, D, and Z_0?

Answer:

RFID on Metal Surfaces:

(a) The reflected voltage will be +1 V for the open circuit load, -1 V for the short circuit load.

(b) Both open and short circuits result in a VSWR of ∞ dB.

(c) As the RFID tag is brought closer to a metal surface, this is equivalent to shrinking b in the microstrip transmission line model. Thus, the line becomes more capacitive and less inductive. Therefore, Z_0 drops.

(d) Start with the phasor solution for current and voltage on a transmission line:

$$\tilde{v}(z) = V^+ \exp(-j\beta z) + V^- \exp(+j\beta z) \qquad \tilde{i}(z) = \frac{V^+}{Z_0}\exp(-j\beta z) - \frac{V^-}{Z_0}\exp(+j\beta z)$$

Note that $I_S = \tilde{i}(z=0)$ and $I_L = \tilde{i}(z=D)$ and that, according to KCL, $I_S = I_L$ in the current loop that includes the RFIC. Thus, we may write

$$\frac{V^+}{Z_0}\exp(-j\beta 0) - \frac{V^-}{Z_0}\exp(+j\beta 0) = \frac{V^+}{Z_0}\exp(-j\beta D) - \frac{V^-}{Z_0}\exp(+j\beta D)$$

which simplifies to

$$\frac{V^-}{V^+} = \frac{1-\exp(-j\beta D)}{1-\exp(j\beta D)} = -\exp(-j\beta D)$$

(e) If you got the answer in part (d), the jump to a Thevenin equivalent impedance is not too difficult. The voltage across the terminals of the RFIC, if we do a KVL, is going to be $V_{th} = \tilde{v}(z=0) - \tilde{v}(z=D)$:

$$\tilde{v}(z) = V^+ \exp(-j\beta z) + (V^+ \cdot -\exp(-j\beta D))\exp(+j\beta z)$$

$$V_{th} = V^+(1-\exp(-j\beta D)) - V^+(\exp(-j\beta D) - 1) = 2V^+(1-\exp(-j\beta D))$$

The corresponding current into the transmission line is

$$I_{th} = \frac{V^+}{Z_0}(1+\exp(-j\beta D))$$

Finishing our computation,

$$Z_{th} = \frac{V_{th}}{I_{th}} = \frac{2Z_0(1-\exp(-j\beta D))}{1+\exp(-j\beta D)} = j2Z_0\tan(\beta D/2)$$

Does this make physical sense? If the transmission line has length zero, then its Thevenin equivalent impedance is 0Ω, which makes sense. As length is added, the lossless transmission line in this topology appears to be an inductor, storing reactive power without permanently expending it.