THT5: Lossy Transmission Lines

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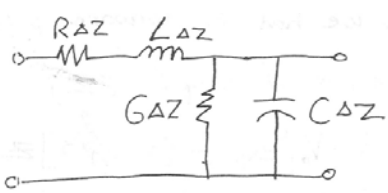
Why Are Lines Lossy?

Teaser Question: When was the first undersea telegraph installed?



For Lossy lines, either R or G is non-zero.

Notes:

1) We usually study lossy line effects for sinusoidal excitation, since loss is not as important for DC switching and logic pulses. These traces are usually too short to notice loss. This is changing, however, as frequencies continue to climb.

Mathematical Solution for Lossy Line

2) For sinusoidal excitation, solution is

$$V(z) = V_0^+ \exp(-\gamma z) + V_0^- \exp(+\gamma z)$$

$$i(z) = \frac{V_0^+}{Z_0} \exp(-\gamma z) - \frac{V_0^-}{Z_0} \exp(+\gamma z)$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \leftarrow \text{now imaginary}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

3) What causes loss?
- Ohmic resistance from metal, (
- Conduction in separating medium, (
- Radiation effects

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Time-Domain Solution

Look Carefully:

Let us say we had a forward prop. wave

$$V(z) = V_0^+ \exp(-\gamma z)$$

$$= V_0^+ \exp(-[\alpha + j\beta] z)$$

$$= V_0^+ \exp(-\alpha z) \exp(-j\beta z)$$

$$V(t, z) = \mathrm{Re}\{V(z) \exp(j\omega t)\}$$

$$= V_0^+ \exp(-\alpha z) \cos(\omega t - \beta z)$$

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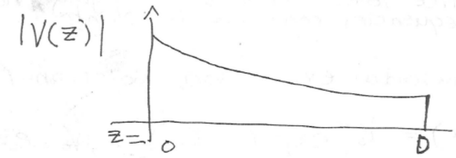
Magnitude of Voltage over Distance

Envelope



α has units of Napiers per meter

More useful value is dB/m

$$P(z) = \frac{|V_0^+|^2}{2Z_0} \cdot \left| \exp(-\alpha z) \exp(-j\beta z) \right|^2$$

$$= \frac{|V_0^+|^2}{2Z_0} \exp(-2\alpha z)$$

Attenuation Coefficients

$$P(z + 1m) = \frac{|V_0^+|^2}{2Z_0} \exp(-2\alpha z) \exp(-2\alpha \cdot 1m)$$

$$\text{dB Loss/m} = 10 \log_{10} \frac{P(z)}{P(z+1m)}$$

$$= 10 \log_{10} \exp(+2\alpha) = 10 \log_{10} 10^{+2\alpha \log_{10} e}$$

$$= +20\alpha \log_{10} e$$

$$\text{Loss} = 8.7\alpha \text{ dB/m}$$

Example: Converting a Spec

Example	Loss @ 1.0 GHz	Loss @ 2.0 GHz
Coaxial Cable	1.6 dB/m	3.0 dB/m
	$\alpha = 0.184$ Nep/m	$\alpha = 0.345$ Nep/m

Nepers/m

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Low Loss Transmission Line Formulas

Low-Loss Lines

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= j\omega\sqrt{LC}\sqrt{\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)}$$

$$\cong j\omega\sqrt{LC}\left[1 + \frac{R}{j\omega 2L}\right]\left[1 + \frac{G}{j\omega 2C}\right]$$

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Key Low-Loss Results

further approximation:

$$\gamma \cong j\omega\sqrt{LC}\left(1 + \frac{1}{j\omega 2}\left[\frac{R}{L} + \frac{G}{C}\right]\right)$$

then

$$\beta \cong \omega\sqrt{LC} \qquad \alpha \cong \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}$$

$$Z_o \cong \sqrt{\frac{L}{C}}$$

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Lossy Line Load Transformation

$$Z_{in} = Z_o\left[\frac{Z_L + Z_o \tanh(\gamma L)}{Z_o + Z_L \tanh(\gamma L)}\right]$$

hyperbolic vs. regular tangent

$$\tan x = \frac{\exp(jx) - \exp(-jx)}{j\left[\exp(jx) + \exp(-jx)\right]}$$

$$\tanh x = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)}$$

$$\tanh j\beta D = j \tan \beta D$$

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Frequency-Dependent Attenuation

$$\alpha = \underbrace{\frac{R}{2}\sqrt{\frac{C}{L}}}_{\substack{\alpha_c \\ \text{conductor} \\ \text{losses}}} + \underbrace{\frac{G}{2}\sqrt{\frac{L}{C}}}_{\substack{\alpha_d \\ \text{dielectric} \\ \text{losses}}}$$

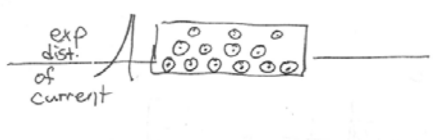
$$\alpha_c = \frac{R}{2Z_0} \qquad \alpha_d = \frac{GZ_0}{2}$$

Starts out small for low frequencies and creeps upward with increasing frequency. Eventually, this term dominates.

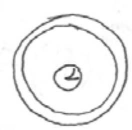
Loss on a Coaxial Cable

Skin Effect for High Frequencies

Microstrip Cross Section



Coaxial X-Section



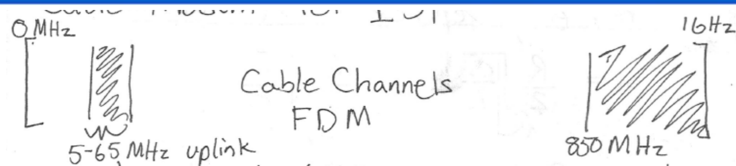
for coaxial Line

Coax

$$\alpha_c = \frac{\sqrt{f}}{4Z_0}\sqrt{\frac{\mu}{\pi\sigma}}\left[\frac{1}{a} + \frac{1}{b}\right]$$

$\sigma \rightarrow$ conductivity

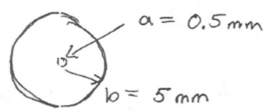
Example: Cable Modem ISP



broken into 6 MHz analog TV channels

TV cable Specs

Cu: $5.96 \times 10^{7}\,\Omega^{-1}m^{-1}$ conductivity



$Z_o = 75\Omega$

(not-so-lossy line)

assume ① low-loss line ② skin effect dominates loss term.

Solution

What is the loss/m for cables carrying:

ⓐ the lowest uplink frequency? (5 MHz)

ⓑ the highest down link frequency? (1GHz)

$$\alpha(f) \cong \alpha_c$$

$$= \frac{\sqrt{f}}{4Z_o}\sqrt{\frac{\mu}{\sigma\pi}}\left[\frac{1}{a} + \frac{1}{b}\right]$$

$$= \frac{\sqrt{f}}{4(75\Omega)}\underbrace{\sqrt{\frac{4\pi\times10^{-7}}{\pi\, 5.96\times10^{7}\,\Omega^{-1}m^{-1}}}}_{8.19\times10^{-8}}\underbrace{\left[\frac{1}{.0005} + \frac{1}{0.005}\right]}_{2200}$$

$$= 6\times10^{-7}\sqrt{f}$$

$\alpha(5MHz) = 1.3\times10^{-3}$ Nepers/m $= 0.012$ dB/m

$\alpha(1GHz) = 0.019$ Nepers/m $= 0.165$ dB/m