Name: _____

GTID: _____

ECE 3025: Electromagnetics

TEST 2 (Spring 2008)

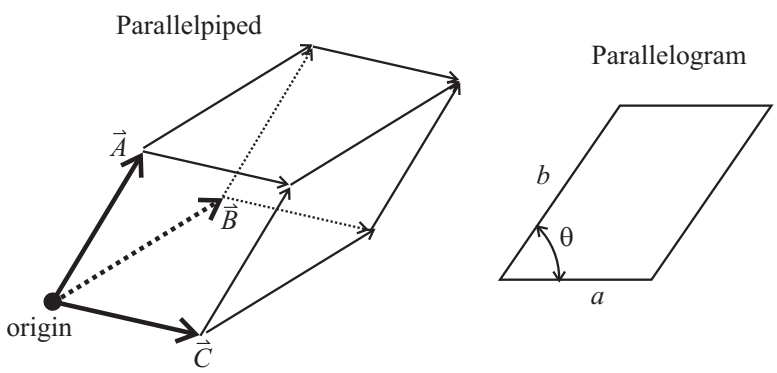
- Please read all instructions before continuing with the test.
- This is a **closed** notes, **closed** book, **closed** calculator, **closed** friend, **open** mind test. You should only have writing instruments on your desk when you take this test. If I find anything on your desk (excluding the test itself, writing instruments, and life-or-death medication) I will turn you in for an honor code violation. I am serious.
- Show all work. (It helps me give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.
- Work intelligently – read through the exam and do the easiest problems first. Save the hard ones for last.
- All necessary mathematical formulas are included either in the problem statements or the last few pages of this test.
- You have 80 minutes to complete this examination. When I announce a "last call" for examination papers, I will leave the room in 5 minutes. The fact that I do not have your examination in my possession will not stop me.
- I will not grade your examination if you fail to 1) put your name and GTID number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.
- Have a nice day!

Pledge Signature: _____

I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.

(1) **Power Storage:** Technologists have speculated that sinusoidal electric power could be generated at night (when public usage is low) and placed in storage for time T; after that period, it could be dumped back into the power grid during peak hours when the generation capacity is strained. Batteries are too expensive and inefficient to store such large amounts of power. Instead, engineers have discussed the possibility of using large circulating transmission lines buried in the earth (think of these as giant coaxial cables that loop around and connect back on themselves). The rings would have to be made out of special super-conductive material and cooled to eliminate the conductive losses ($\alpha_c = 0$). If the velocity of propagation was v_p (m/s), how low would any residual dielectric shunt losses have to be in order for a waveform to loose no more than half its power (-3 dB) over a time period T (s). Your answer should be a value for α_d measured in dB/km, simplified but obviously not evaluated. (**10 points**)

(2) **Vector Math:** Three edges of a *parallelpiped* are defined in three dimensions by the position vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ as shown in the diagram below. Using only vector math expressions, calculate the volume of the parallelpiped. Hint 1: the volume of a parallelpiped can be calculated by the area of one of its parallelogram faces multiplied by the normal distance between the face and its parallel counterpart. Hint 2: the area of a parallelogram is $ab\sin\theta$. (**11 points**)

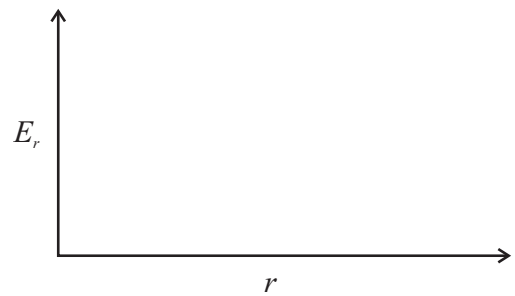


(3) **Electrostatic Charge Distributions:** All of the field distributions in this problem are free-space and may be written in the following form:

$$\vec{E}(r, \phi, \theta) = E_r(r)\hat{r}$$

Make a rough sketch in the graph provided of $E_r(r)$ for the following charge distributions. (**24 points**)

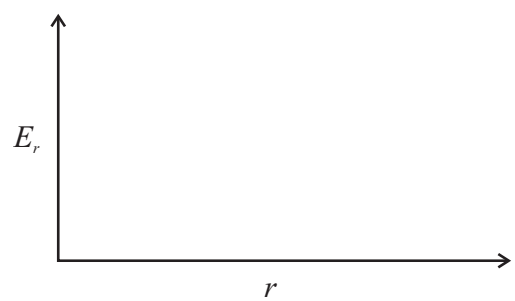
(a) At the origin is a +1 C point charge. Surrounding this charge is a spherical shell of uniform charge density centered at the origin at a radius of R and a total charge of -2 C. (**8 points**)



(b) There is a charge distribution in space of the form:

$$\rho_v(r, \phi, \theta) = \frac{\rho_o}{r}$$

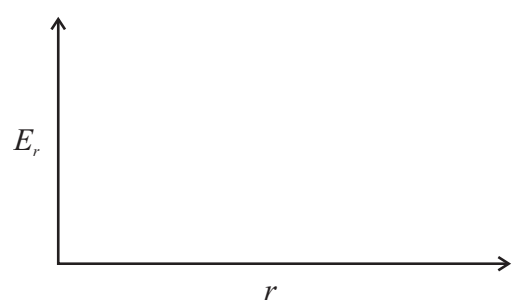
(**8 points**)



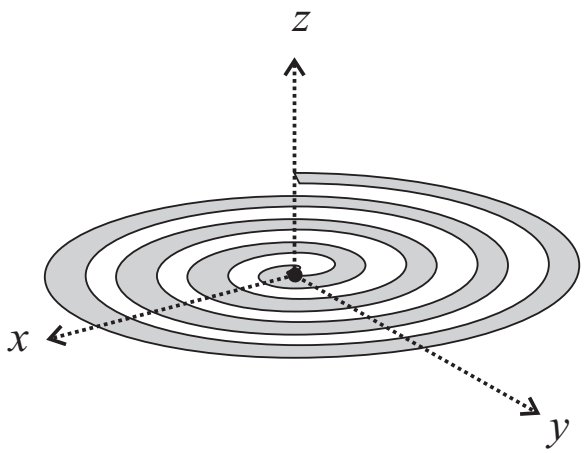
(c) There is a voltage distribution in space of the form:

$$V(r, \phi, \theta) = V_o \cos(2\pi r/\lambda)$$

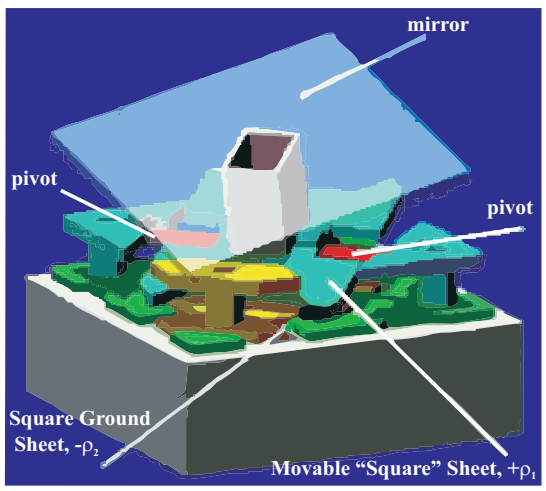
(**8 points**)

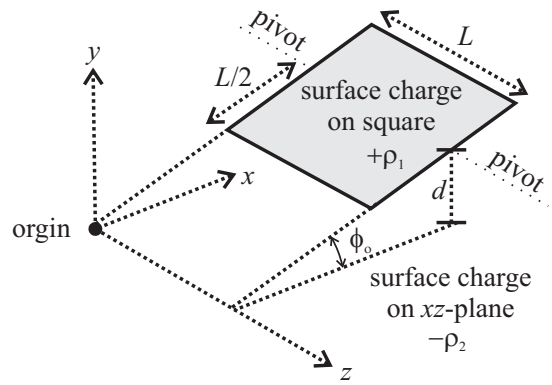


(4) **Voltage of a Spiral Charge:** Below is a flat, infinite Archimedean spiral of surface charge density. The surface charge density is given by $\rho_s(\rho, \phi) = \rho_o \exp(-\rho)$ (C/m^2) such that the charge density on the spiral arm exponentially decays away from the origin. The region of charge is on the xy-plane, bound by the region $\rho \leq \phi \leq \rho + \phi_o$, where ϕ_o is a constant related to the width of the spiral arm ($\phi_o = 0$ is infinitely thin, $\phi_o = 2\pi$ becomes a solid disk). Calculate the voltage at the origin relative to a zero-reference at infinity. Note: this problem can and should produce a tractable analytic solution (**25 points**).



(5) **DLP Chip:** Below is a 3D drawing and schematic of a dynamic light projection (DLP) mirror unit. The mirror is mounted on a pivoting metallic *square* (for modeling purposes) of side length L with uniform surface charge density $+\rho_1$. This square may tilt at an arbitrary angle ϕ_o with respect to an infinite (for modeling purposes) ground plane with uniform surface charge density $-\rho_2$. A value of $\phi_o = 0$ corresponds to the square and mirror perfectly parallel to the ground plane. The pivots bisect the square sheet, with the pivot points resting above the infinite ground plane by a distance d. Set-up any combination of integrals or expressions that solve for *total* electric field at an arbitrary point (x, y, z) in free space around the device. Simplify as much as possible, but do not evaluate non-trivial integral(s). (**30 points**)





Formula Sheet

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \qquad Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta D}{Z_0 + jZ_L \tan \beta D}$$

$$\alpha = \alpha_c + \alpha_d \,(\text{Nepers/m}) \qquad (\text{dB/m}){:}8.7\alpha$$

$$\vec{F} = Q\vec{E} \qquad \text{Point Charge at the Origin: } \vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon r^2}\hat{r}$$

$$\text{Charge Distributions: } \vec{E}(\vec{r}) = \int_L \frac{\rho_L(\vec{r}')(\vec{r} - \vec{r}')dL}{4\pi\epsilon||\vec{r} - \vec{r}'||^3} = \iint_S \frac{\rho_S(\vec{r}')(\vec{r} - \vec{r}')dS}{4\pi\epsilon||\vec{r} - \vec{r}'||^3} = \iiint_V \frac{\rho_V(\vec{r}')(\vec{r} - \vec{r}')dV}{4\pi\epsilon||\vec{r} - \vec{r}'||^3}$$

$$V(\vec{r}) = \frac{Q}{4\pi\epsilon r} = \int_L \frac{\rho_L(\vec{r}')dL}{4\pi\epsilon||\vec{r} - \vec{r}'||} = \iint_S \frac{\rho_S(\vec{r}')dS}{4\pi\epsilon||\vec{r} - \vec{r}'||} = \iiint_V \frac{\rho_V(\vec{r}')dV}{4\pi\epsilon||\vec{r} - \vec{r}'||}$$

$$\text{Variable of integration: } \vec{r}' \qquad \text{Point of observation: } \vec{r}$$

$$\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \qquad \nabla \times \vec{E} = \det \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\vec{D} = \epsilon\vec{E} \qquad \vec{E} = -\nabla V \qquad \nabla \cdot \vec{D} = \rho_v \qquad V_B - V_A = -\int_A^B \vec{E} \cdot d\hat{l}$$

$$\text{Cross Product: } \vec{a} \times \vec{b} = \det \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \|\vec{a}\|\|\vec{b}\| \sin \theta_{ab} \hat{n}$$

$$\text{Dot Product: } \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = \|\vec{a}\|\|\vec{b}\| \cos \theta_{ab}$$

$$\text{Sphere Volume: } \frac{4\pi}{3} r^3 \qquad \text{Sphere Surface Area: } 4\pi r^2$$

$$\text{Infinite Charge Sheet on } xy\text{-plane: } \vec{E} = \frac{\rho_o}{2\epsilon}\hat{z}\,(\text{flip sign for } z < 0) = \frac{\rho_o}{2\epsilon}\text{sgn}(z)\hat{z}$$

DIVERGENCE

CARTESIAN $\nabla \cdot \mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$

CYLINDRICAL $\nabla \cdot \mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$

SPHERICAL $\nabla \cdot \mathbf{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(D_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial D_\phi}{\partial \phi}$

GRADIENT

CARTESIAN $\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$

CYLINDRICAL $\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$

SPHERICAL $\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r \sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$

CURL

CARTESIAN $\nabla \times \mathbf{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\mathbf{a}_z$

CYLINDRICAL $\nabla \times \mathbf{H} = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right)\mathbf{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)\mathbf{a}_\phi + \frac{1}{\rho}\left[\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi}\right]\mathbf{a}_z$

SPHERICAL $\nabla \times \mathbf{H} = \frac{1}{r \sin\theta}\left[\frac{\partial(H_\phi \sin\theta)}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi}\right]\mathbf{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial \phi} - \frac{\partial(rH_\phi)}{\partial r}\right]\mathbf{a}_\theta + \frac{1}{r}\left[\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta}\right]\mathbf{a}_\phi$

LAPLACIAN

CARTESIAN $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

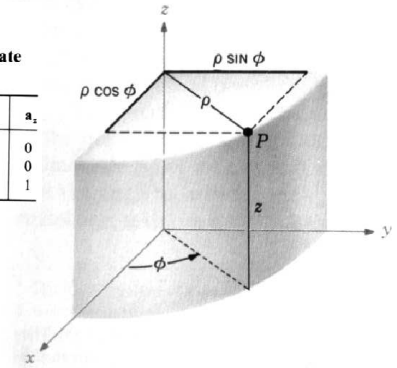
CYLINDRICAL $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$

SPHERICAL $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$

Cylindrical Coordinates

Dot products of unit vectors in cylindrical and cartesian coordinate systems

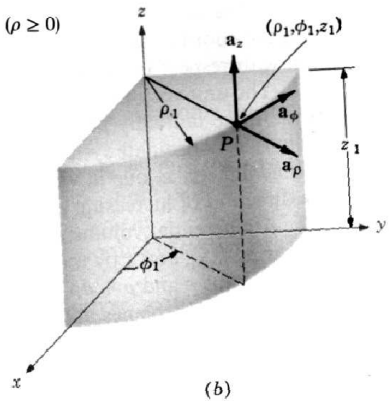
	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	1



$\rho = \sqrt{x^2 + y^2}$ $(\rho \geq 0)$

$\phi = \tan^{-1}\frac{y}{x}$

$z = z$

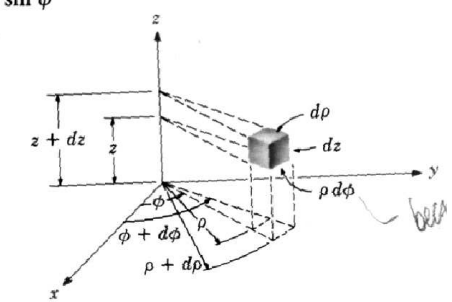


(b)

$x = \rho \cos\phi$

$y = \rho \sin\phi$

$z = z$

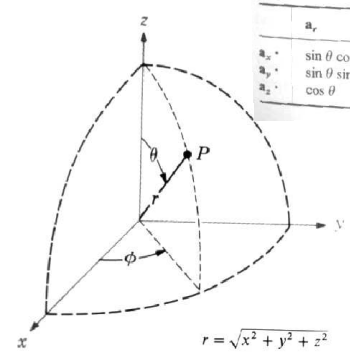


(c)

Spherical Coordinates

Dot products of unit vectors in spherical and cartesian coordinate systems

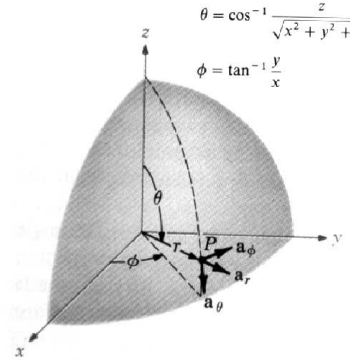
	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0



$r = \sqrt{x^2 + y^2 + z^2}$ $(r \geq 0)$

$\theta = \cos^{-1}\frac{z}{\sqrt{x^2 + y^2 + z^2}}$ $(0° \leq \theta \leq 180°)$

$\phi = \tan^{-1}\frac{y}{x}$



$x = r \sin\theta \cos\phi$

$y = r \sin\theta \sin\phi$

$z = r \cos\theta$