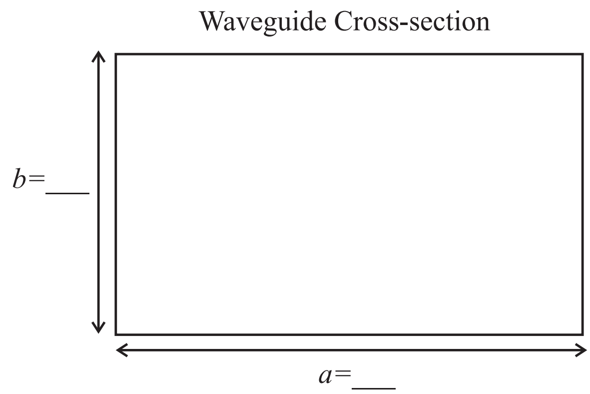
ECE 3065 Homework 7: More Waveguides

1. **Mystery Waveguide:** You are told that a rectangular, metallic, air-filled waveguide has cut-off frequencies of 3 GHz, 6 GHz, and 7.5 GHz for its lowest modes. Answer the following questions based on this scenario. Calculate the dimensions of this waveguide. Label the dimensions in meters below. In your diagram sketch the E-field distribution for this dominant mode. (10 points)



2. **Circular vs. Rectangular Waveguide:** You have been hired by small engineering company that specializes in making super-cheap microwave components. The first product that you are asked to design is a new ultra-cheap, air-filled metallic waveguide. Sections of this waveguide are manufactured by taking long rectangular sheets of tin and bending them around a cross-section of your choosing. You must decide between a square cross section or a circular cross section to produce a waveguide with a given low-frequency cut-off. (10 points)

   (a) Manufacturing cost of the waveguide is proportional to the perimeter of the cross-section. Calculate which cross section is cheaper to build.

   (b) Calculate which of these cross sections has more single-mode bandwidth (bandwidth between dominant and second-highest mode).

3. Bonus (+10 points): Complete the problem on the following page for a bonus.

ECE 3065

Dielectric Channel Waveguide

Problem: The dielectric slab waveguide discussed in the course confines light in only one transverse direction, say x-direction. Optical channel waveguides can provide confinement in both transverse directions. These waveguides are used in a variety of active and passive integrated optical devices such as lasers, modulators, switches and directional couplers. In this problem, we discuss a simple method to find the modes of such waveguides. Figure 1 shows the cross-section of a channel waveguide, composed of four dielectrics, substrate, lateral layer, cap layer, and a film material, with the refractive indices n_s,n_l, n_c and n_f, respectively. Light is essentially confined within the film material.

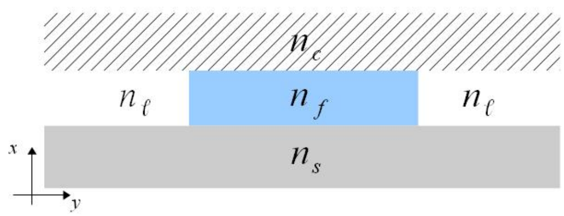


Figure 1: Cross-section of a dielectric channel waveguide.

In order to find the modes of this structure, we should solve the two dimensional vector wave equation:

$$\nabla_t^2 E_t + (n^2k^2 - \beta^2)E_t = 0, \tag{1}$$

in which, $\nabla_t = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y}$, E_t is the transverse component of the electric field, β is the propagation constant, i.e $\beta = \omega/v_p$ (v_p is the phase velocity), k is the wave vector, and $n = n(x,y)$ is the refractive index. This two-dimensional wave equation does not have a simple analytical solution for our structure of interest.

Part a: Show that if we can write the refractive index as sum of squares:

$$n^2(x,y) = n_0^2 + n_x^2(x) + n_y^2(y),$$

using $E_t(x,y) = X(x)Y(y)$, the two-dimensional wave equation can be separated into two one-dimensional wave equations as follows.

$$\frac{\partial^2 X}{\partial x^2} + (n_x^2k^2 - \beta_x^2)X = 0$$

$$\frac{\partial^2 Y}{\partial y^2} + (n_y^2k^2 - \beta_y^2)Y = 0$$

From these solutions, β is given by $\beta^2 = k^2 n_0^2 + \beta_x^2 + \beta_y^2$.

Part b: The condition of part (a) does not apply for the channel waveguide, in general. In the field-shadow method, illustrated in Figure 2.c, the field and the refractive indices in the shaded regions are ignored. Show that using this approximation, the channel guide can be split into two slab waveguides with separable index profiles. It is assumed that EM fields in the shaded regions of Figure 2.c, are weak and can be ignored (this is not true in near cut-off frequencies).

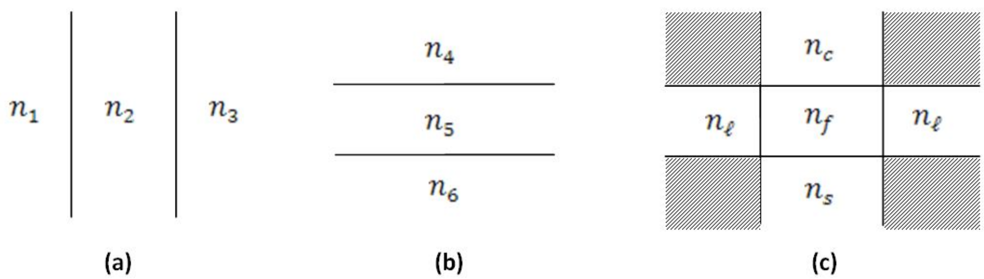


Figure 2: Method of field shadows.

Hint: *Assume two slab waveguides, Figure 2.a and 2.b. Choose the the refractive indices,* n_1 *to* n_6 *, such that the sum of squares gives exactly the refractive indices of the originial channel waveguide in the non-shaded regions of Figure 2.c. Ignore the refractive index in the shaded regions.*

Part c: From Part c, $E_t(x, y)$ can be written as the product of $X(x)$, the field of the x-slab waveguide, and $Y(y)$, the field of the y-slab waveguide (separation of variables). Using the field solutions of the planar slab waveguide, write the expressions for E_y of the mode TE_{00}, assuming a channel waveguide of the height a and the the width b.

Hint: *For the mode* TE_{00}*, the E-field is in y-direction, which corresponds to the mode* TE_0 *of the horizental slab waveguide and the mode* TM_0 *of the vertical slab waveguide.*

Part d: Dispersion curve is an important characteristic of an EM mode in a waveguide, showing the evolution of the propagation constant, β as a function of the frequency, ω. Write down the dispersion relations for the two horizental and vertical slab waveguides (β_x and β_y as a function of ω) for the mode TE_{00} found in part (c). Using the equation found of part (a) for the propagation constants, find the dispersion relation of the channel waveguide. Assuming that the cap and the lateral layer are filled with air ($n_c = n_\ell = 1$), the substrate is made from SiO_2 ($n_s = 1.45$) and the film layer from Si ($n_f = 3.475$). The width of the channel is 2 μm and its height is 1 μm. Draw this dispersion curve.

Part e: Using either the analytical solution of part (c), or a simulation tool (like Comsol), find the largest dimensions of the channel, for which the structure is single mode. Use the refractive indices of part d and the wavelength $\lambda = 1550\ nm$.

In practice, approximative methods like the method of field-shadows are used to obtain initial values for the dimensions of the waveguide. Then, the design is optimized using numerical methods, such as Finite Elements Method (FEM) or Finite Difference Time Domain (FDTD).