Name: ______________________

GTID: ______________________

ECE 3065: Electromagnetic Applications

Final Exam (Spring 2004)

- Please read all instructions before continuing with the test.
- This is a **closed** notes, **closed** book, **closed** calculator, **closed** friend, **open** mind test. You should only have writing instruments on your desk when you take this test.
- Show all of your work. Work out the problems in the space underneath the corresponding problem or on the back of the page. Remember: it is easier for me to give partial credit if I can follow your work.
- Work intelligently – read through the exam and do the easiest problems first. Save the hard ones for last.
- All necessary mathematical formulas are included either in the problem statements or the last few pages of this test, which include formula charts of Fourier transform properties and Fourier transform pairs.
- You have 3 hours to complete this examination. When I announce a "last call" for examination papers, I will leave the room in 5 minutes. The fact that I do not have your examination in my possession will not stop me.
- I will not grade your examination if you fail to 1) put your name and GTID number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.
- Good luck!

Pledge Signature: ____

I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.

(1) **Short Answer Section** (**40 points**)

(a) ________________

For efficient antennas, a larger physical size usually indicates $\boxed{\textit{higher/lower}}$ peak gain.

(b) ________________

How many complex parameters does it take to characterize a 7-port linear device?

(c) ________________ (1) ________________ (2)

The S-matrix of a lossless N-port network will satisfy the $\boxed{\textit{Answer 1}}$ and $\boxed{\textit{Answer 2}}$ properties.

(d) ____________ (1) ____________ (2) ____________ (3) ____________ (4)

A simple medium has the following four properties: $\boxed{\textit{Answer 1}}$, $\boxed{\textit{Answer 2}}$, $\boxed{\textit{Answer 3}}$, and $\boxed{\textit{Answer 4}}$.

(e) ________________

The Helmoltz wave equation cannot be used to analyze antenna radiation because antennas violate the $\boxed{\textit{Answer}}$ assumption of a simple medium.

(f) ________________ (1) ________________ (2)

All of electromagnetic theory is essentially solving Maxwell's Equations for the appropriate b$\boxed{\textit{Answer 1}}$y c$\boxed{\textit{Answer 2}}$s.

(g) ________________

Of the following antenna characterizations – field pattern, power pattern, directivity, and gain pattern – which one allows you to calculate radio link budgets?

(h) ________________

An S-matrix measurement will depend on the $\boxed{\textit{Answer}}$ plane of the measurement.

(i) ________________

True or False: Only lossless networks have reciprocal S-matrices.

(j) ________________

True or False: A half-wave dipole has more peak gain than a quarter-wave monopole.

(k) ________________

A(n) $\boxed{\textit{Answer}}$ antenna is capable of producing circular polarization.

(l) ________________

Super-cooling a resonator circuit so that its metal begins to superconduct would *increase/decrease* the bandwidth.

(m) ________________

If a TV station wants to contain its radiation tightly within its FCC-mandated footprint, it should use a linear antenna with *Answer* polarization.

(n) ________________

Grazing incidence occurs when θ_i approaches *Answer*.

(o) ________________

Answer occurs whenever a wave bends around an obstructing object.

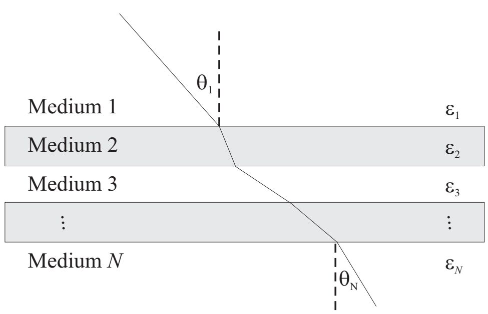
(2) **Descriptive Answer Section**

Write a **concise** answer to each question in the spaces provided beneath each problem statement. **Note:** Correct answers that are extremely verbose will be penalized.

(a) **Refraction Through a Composite Medium:** Below is a sketch of N parallel layers of dielectric, nonmagnetic materials. Each layer has a different permittivity ϵ and thickness. Show that, no matter what the thickness or composition of layers 2 through $N-1$, the exit angle θ_N is related to the incident angle θ_1 by the following relationship:

$$\sqrt{\epsilon_1}\sin\theta_1 = \sqrt{\epsilon_N}\sin\theta_N$$

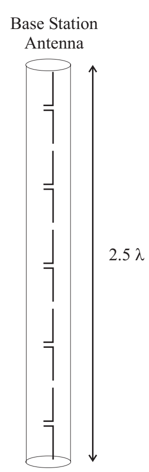
(**10 points**)



(b) **Three-port to Two-port Conversion:** You are given a three-port network with known S-matrix measured with 50Ω lines. You screw a 50Ω "dummy load" resistor to port 3. You measure the new two-port device and find that it satisfies the unity and zero properties. Based on this information, circle the s-parameters in the original three-port S-matrix below that we know for sure are zero-valued. (**10 points**)

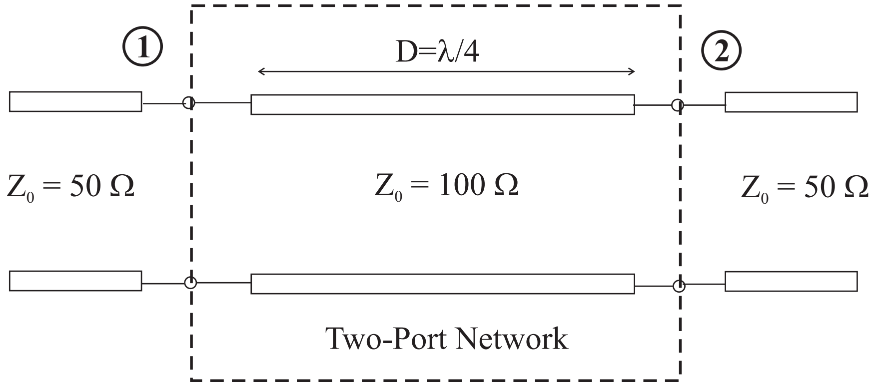
$$S = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix}$$

(c) **Base Station Antenna:** A common design for cellular base station antennas involves a vertical stack of half-wave dipoles (shown below) that are electrically in phase with one another. Without doing any direct calculation, what would the gain pattern for this antenna look like with respect to a half-wave dipole antenna? Why is this used instead of a half-wave dipole? (**10 points**)



(3) **S-Parameters of Transmission Line Circuits:**

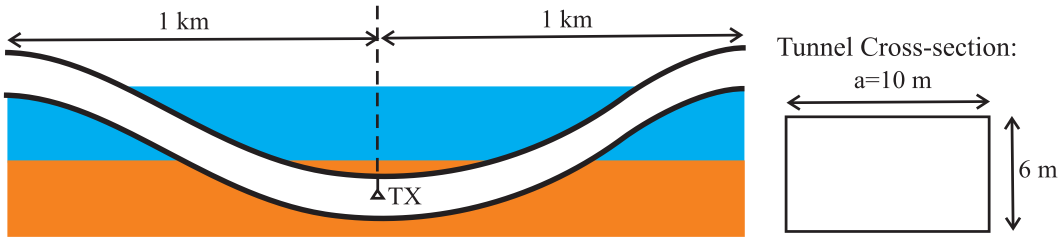
Find the S-matrix for the circuit below. (**20 points**)



(4) **Stub Line Matching Network:** Design a parallel, open-circuit stub-line match for a 100Ω transmission line that terminates in a $200+j50\Omega$ load. Calculate how far from the load should the stub be placed (in wavelengths) and the length of the open-circuit stub line (also in wavelengths). Show your work on the attached Smith Chart. (**20 points**)

(5) **Waveguide Propagation: (20 points)**

A section of the Chesapeake bay-bridge tunnel is 2 kilometers long with a width of 10 meters and a height of 6 meters. In the middle of this tunnel is a small 850 MHz cellular base station that provides coverage for mobile users in the tunnel. This station is low-powered and uses the tunnel like a rectangular metallic waveguide to communicate with motorists. Answer the following questions based on this scenario, which is illustrated below:



(a) Find the highest value for x such that the TE_{x0} mode still propagates through this tunnel. (**5 points**)

(b) For a cell phone user operating at the end the tunnel, what is the dispersion (in nanoseconds) between the power carried by the dominant TE_{10} mode and the TM_{77} mode. (**10 points**)

(c) What assumption are we making about the tunnel walls when we model the propagation this way? (**5 points**)

(6) **Link Budget:** (**20 points**)

You are designing IEEE 802.11a wireless internet links in a residential area. This link operates at 5.85 GHz (the U-NII – unlicensed national information infrastructure band) and delivers high-data rate internet from an outdoor wireless access point atop a neighborhood utility pole to indoor desktop and laptop computers with a wireless PC/PCMCIA transceiver card. Here are some useful facts about the link:

- The longest TR separation distance is about 500m. You may assume free space propagation between the base station and the house.
- Brick houses with foil-backed insulation lead to the highest penetration losses. The research literature states that these houses contribute 16.0 dB of loss in excess of free space losses.
- The base station antenna has a gain value of 8 dBi. The smaller, less-efficient computer antennas usually have a gain value of 0 dBi.
- Reliable transmission at the target data rate requires a minimum received power of -82 dBm.

(a) What is the minimum transmit power in dBm for establishing this link in the worst-case scenario (highest TR separation distance into a brick house)? (**10 points**)

(b) Convert your answer from (a) into Watts (linear scale). (**5 points**)

Cheat Sheet

Scattering Relationship (3-port): $$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \end{bmatrix}$$

N-port zero property: $$\sum_{i=1}^{N} s_{ij} s_{ik}^* = 0 \quad \text{or} \quad \sum_{i=1}^{N} s_{ji} s_{ki}^* = 0 \quad \text{for j} \neq \text{k}$$

N-port unity property: $$\sum_{i=1}^{N} |s_{ij}|^2 = 1 \quad \text{or} \quad \sum_{j=1}^{N} |s_{ij}|^2 = 1$$

$$V(z) = K\left[\exp(-j\beta z) + \Gamma_L \exp(j\beta z)\right] \text{ (for } z = 0 \text{ at load)} \qquad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta D}{Z_0 + jZ_L \tan \beta D}$$

Quarter Wavelength: $$Z_{in} = \frac{Z_0^2}{Z_L}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \qquad \mu_0 = 4\pi \times 10^{-7} \text{H/m} \qquad c = 3.0 \times 10^8 \text{ m/s}$$

$$\lambda f = v_p \qquad \omega = 2\pi f \qquad \beta = \frac{2\pi}{\lambda} \qquad D = T v_p$$

Planar Waveguide: $$(f_c)_m = \frac{m}{2a\sqrt{\mu\epsilon}}$$

Rectangular Waveguide: $$(f_c)_{mn} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Group Velocity: $$v_g = \frac{1}{\sqrt{\epsilon\mu}} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

Logarithmic Free Space Link Budget

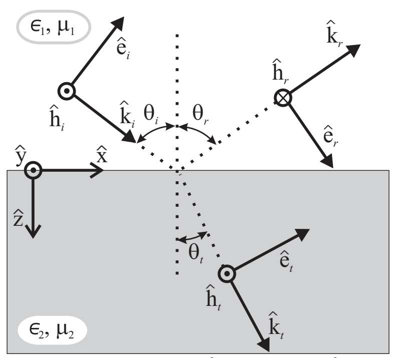
$$\text{P}_R = \text{P}_T + G_T + G_R - 20\log_{10}(r) - 20\log_{10}(f) + 20\log_{10}(c/4\pi) - \text{Extra Loss}$$

$$G(\theta, \phi) = \epsilon \frac{U(\theta, \phi)}{U_{av}}$$

$$U_{av} = \frac{1}{4\pi} \int_0^{\pi} \int_0^{2\pi} P(\theta, \phi) \sin(\theta)\, d\theta\, d\phi$$

Fresnel Reflection Coefficients for a Dielectric Interface

$\parallel$ Polarization



$$\Gamma_{\parallel} = -\frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

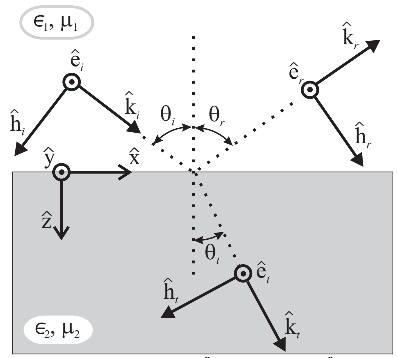
$$\tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} = \frac{\cos\theta_i}{\cos\theta_t}\left(1 + \Gamma_{\parallel}\right)$$

$$\begin{aligned} \hat{\mathrm{k}}_i &= \sin\theta_i \hat{\mathrm{x}} + \cos\theta_i \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_i &= \cos\theta_i \hat{\mathrm{x}} - \sin\theta_i \hat{\mathrm{z}} \\ \hat{\mathrm{h}}_i &= \hat{\mathrm{y}} \end{aligned}$$

$$\begin{aligned} \hat{\mathrm{k}}_r &= \sin\theta_r \hat{\mathrm{x}} - \cos\theta_r \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_r &= \cos\theta_r \hat{\mathrm{x}} + \sin\theta_r \hat{\mathrm{z}} \\ \hat{\mathrm{h}}_r &= -\hat{\mathrm{y}} \end{aligned}$$

$$\begin{aligned} \hat{\mathrm{k}}_t &= \sin\theta_t \hat{\mathrm{x}} + \cos\theta_t \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_t &= \cos\theta_t \hat{\mathrm{x}} - \sin\theta_t \hat{\mathrm{z}} \\ \hat{\mathrm{h}}_t &= \hat{\mathrm{y}} \end{aligned}$$

$\perp$ Polarization



$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\tau_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = 1 + \Gamma_{\perp}$$

$$\begin{aligned} \hat{\mathrm{k}}_i &= \sin\theta_i \hat{\mathrm{x}} + \cos\theta_i \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_i &= \hat{\mathrm{y}} \\ \hat{\mathrm{h}}_i &= -\cos\theta_i \hat{\mathrm{x}} + \sin\theta_i \hat{\mathrm{z}} \end{aligned}$$

$$\begin{aligned} \hat{\mathrm{k}}_r &= \sin\theta_r \hat{\mathrm{x}} - \cos\theta_r \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_r &= \hat{\mathrm{y}} \\ \hat{\mathrm{h}}_r &= \cos\theta_r \hat{\mathrm{x}} + \sin\theta_r \hat{\mathrm{z}} \end{aligned}$$

$$\begin{aligned} \hat{\mathrm{k}}_t &= \sin\theta_t \hat{\mathrm{x}} + \cos\theta_t \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_t &= \hat{\mathrm{y}} \\ \hat{\mathrm{h}}_t &= -\cos\theta_t \hat{\mathrm{x}} + \sin\theta_t \hat{\mathrm{z}} \end{aligned}$$

General Plane Wave Solution

$$\vec{\mathsf{E}}_\diamond(\vec{\mathrm{r}}) = E_\diamond \, \hat{\mathrm{e}}_\diamond \, \exp\left(j\left[\phi - k\hat{\mathrm{k}}_\diamond \cdot \vec{\mathrm{r}}\right]\right) \qquad \vec{\mathsf{H}}_\diamond(\vec{\mathrm{r}}) = \frac{E_\diamond}{\eta} \hat{\mathrm{h}}_\diamond \exp\left(j\left[\phi - k\hat{\mathrm{k}}_\diamond \cdot \vec{\mathrm{r}}\right]\right)$$

$\vec{\mathrm{r}} = x\hat{\mathrm{x}} + y\hat{\mathrm{y}} + z\hat{\mathrm{z}}$ $\quad \eta = \sqrt{\frac{\mu}{\epsilon}}$ $\quad k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_p}$ $\quad \diamond \to i$ (incident) **or** r (reflected) **or** t (transmitted)

Snell's Law of Reflection

$$\theta_i = \theta_r$$

Snell's Law of Refraction

$$\frac{\sin\theta_i}{v_{p1}} = \frac{\sin\theta_t}{v_{p2}} \quad \text{where } v_p = \frac{1}{\sqrt{\epsilon\mu}}$$

Physical Quantities

θ_i	angle of incidence	E	electric field amplitude (V/m)
θ_r	angle of reflection	$\Gamma_{\parallel,\perp}$	reflection coefficient ($\frac{E_r}{E_i}$)
θ_t	angle of transmission	$\tau_{\parallel,\perp}$	transmission coefficient ($\frac{E_t}{E_i}$)
$\hat{\mathrm{e}}$	electric field polarization	η	intrinsic impedance (Ω, Ohms)
$\hat{\mathrm{h}}$	magnetic field polarization	v_p	velocity of propagation (m/s)
$\hat{\mathrm{k}}$	direction of propagation	k	wavenumber (radians/m)
μ	magnetic permeability (H/m)	λ	wavelength (m)
ϵ	electric permittivity (F/m)	f	frequency (Hz)