ECE 3065: Electromagnetic Applications
Solutions to the Final Exam (Spring 2004)

(1) **Short Answer Section** (**40 points**)

(a) higher

(b) 49

(c) zero (1) unity (2)

(d) homogeneous (1) source-free (2) linear (3) isotropic (4)

(e) source-free

(f) b)oundar(y c)ondition(s

(g) gain pattern

(h) reference

(i) false

(j) false

(k) Helical

(l) decrease

(m) horizontal

(n) 90 degrees

(o) diffraction

(2) **Descriptive Answer Section**

(a) **Refraction Through a Composite Medium:** We apply Snell's law of refraction at the first interface (medium 1 – medium 2) to obtain:

$$\sqrt{\epsilon_1}\sin\theta_1 = \sqrt{\epsilon_2}\sin\theta_2$$

We apply Snell's law of refraction at the second interface (medium 2 – medium 3) to obtain:

$$\sqrt{\epsilon_2}\sin\theta_2 = \sqrt{\epsilon_3}\sin\theta_3$$

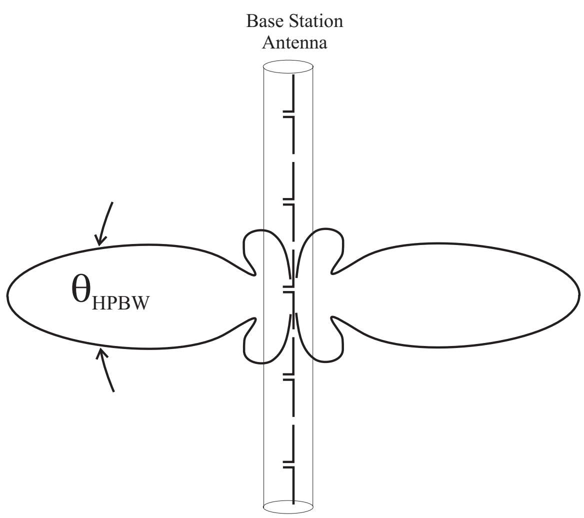
We continue down the layers in this manner to the last interface (medium $N-1$ – medium N) and apply transitivity to show that

$$\sqrt{\epsilon_1}\sin\theta_1 = \sqrt{\epsilon_2}\sin\theta_2 = \ldots\sqrt{\epsilon_N}\sin\theta_N$$

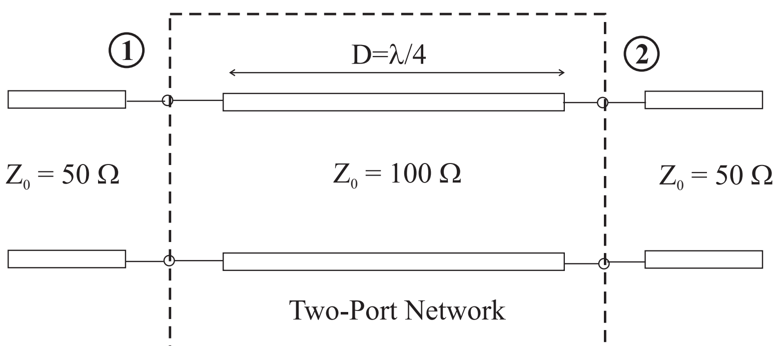
(b) **Three-port to Two-port Conversion:** If the network satisfies the zero and unity properties *even after a dummy load has been fixed to port 3*, then we know that no power is being delivered from ports 1 and 2 to port 3 ($s_{31} = s_{32} = 0$). After all, only a lossless network satisfies zero and unity properties, so a resistive load on port 3 cannot possibly be receiving any voltage across it to maintain this condition. Furthermore, when we enforce the zero property column-by-column, we find that $s_{13} = s_{23} = 0$ as well. Thus, we can write the final S-matrix as:

$$S = \begin{bmatrix} s_{11} & s_{12} & 0 \\ s_{21} & s_{22} & 0 \\ 0 & 0 & s_{33} \end{bmatrix}$$

(c) **Base Station Antenna:** This antenna, taken in its entirety, is much larger in the z-direction than a single half-wave dipole. Thus, we would expect the pattern to remain omnidirectional in azimuth, φ, but have a much smaller half-power beamwidth in elevation, θ. See the sketch below for an estimate of the far-field antenna gain pattern.



(3) **S-Parameters of Transmission Line Circuits:** We must apply circuit analysis to our device:



First start by calculating the reflection coefficient at port 1. Transform the 50Ω load at port 2 across the quarter-wavelength section of 100Ω line to get the following equivalent load as seen at the first terminal:

$$Z_{\text{eq},\lambda/4} = \frac{Z_0^2}{Z_L} = 200\Omega$$

Use this equivalent load to calculate the reflection coefficient, which is equal to s_{11} and, by the symmetry of the problem, s_{22}:

$$s_{11} = s_{22} = \Gamma = \frac{Z_{\text{eq},\lambda/4} - Z_0}{Z_{\text{eq},\lambda/4} + Z_0} = \frac{3}{5}$$

Now let us turn our attention to s_{21}. The full wave voltage solution on the transmission line as a function of position, z, is given by:

$$\tilde{v}(z) = K \left[\exp\left(-j \underbrace{\frac{2\pi}{\lambda}}_{\beta} z \right) \underbrace{-\frac{1}{3}}_{\Gamma} \exp\left(j \frac{2\pi}{\lambda} z \right) \right]$$

If we define $z = -\frac{\lambda}{4}$ to be the left side of the transmission line, then

$$\tilde{v}\ (z)|_{z=-\frac{\lambda}{4}} = K \left[j - \frac{1}{3}(-j) \right] = j\frac{4}{3}K$$

This must also be equal to the total voltage across port 1:

$$\text{Port 1 Voltage} = V_1^+ + V_1^- = V_1^+(1 + s_{11}) = \frac{8}{5}V_1^+$$

Equating this result with the previous result gives us the following relationship: $K = -j\frac{6}{5}V_1^+$. Now evaluate this result for the right side of the transmission line ($z = 0$).

$$\tilde{v}\ (z)|_{z=0} = K \left[1 - \frac{1}{3} \right] = \frac{2}{3}\left(-j\frac{6}{5}V_1^+ \right) = -j\frac{4}{5}V_1^+$$

Since s-parameters are measured with 50Ω lines that have matched loads, this voltage much be equal to V_2^-. Therefore,

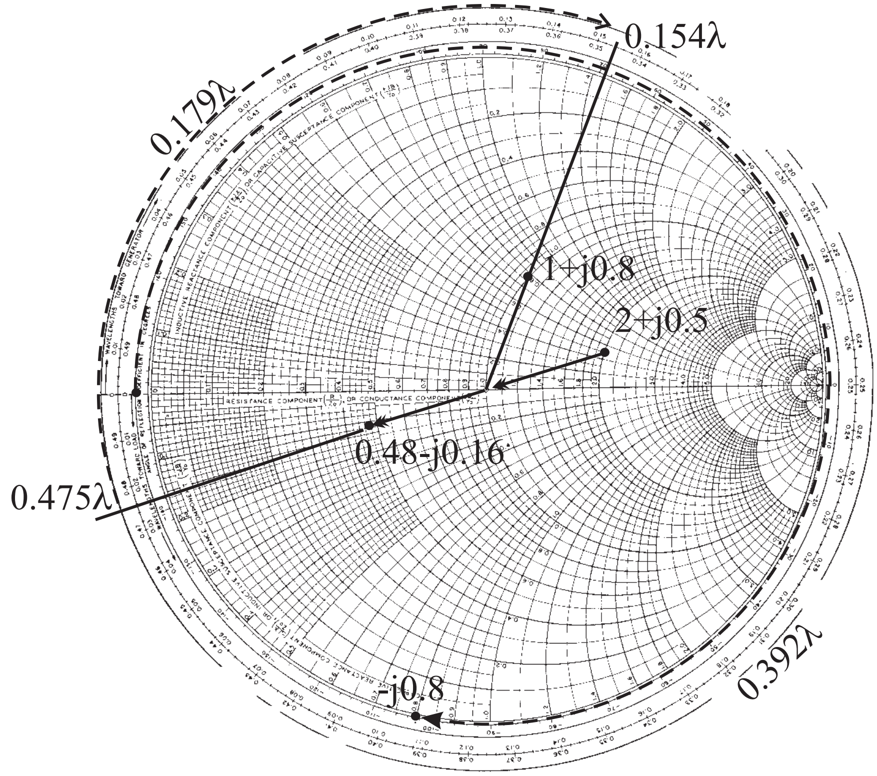
$$s_{21} = \frac{V_2^-}{V_1^+} = -j\frac{4}{5}$$

Note that s_{21} is also, by symmetry, s_{12}. The final S-matrix is

$$S = \begin{bmatrix} \frac{3}{5} & -j\frac{4}{5} \\ -j\frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

Keep in mind that this device is just an ideal, lossless transmission line. Since there is nothing inside to absorb power, this S-matrix should obey the zero and unity properties – a powerful way to check our mathematics!

(4) **Stub Line Matching Network:** The normalized impedance for this load is $2 + \frac{1}{2}j$, which is plotted on the following Smith chart. The load must be mirrored to the other side of the chart so that it is a normalized admittance, $0.48 - 0.16j$. We then rotate this 0.179λ until the real part is 1; this gives us a value of $1 + 0.8j$. To remove the $+0.8j$, we attach a parallel, open-circuit stub of length 0.392λ (remember that, in admittance, an open circuit load starts at real 0 on the Smith chart).



(5) **Waveguide Propagation:**

(a) Below is the cut-off equation with the tunnel parameters substituted in:

$$(f_c)_{x0} = \frac{1}{2\sqrt{\mu_0\epsilon_0}}\sqrt{\left(\frac{x}{a}\right)^2 + \left(\frac{0}{b}\right)^2} = x\frac{1.5\times10^8 \text{ m/s}}{10 \text{ m}} = 850 \text{ MHz}$$

The value of x that satisfies this relationship exactly is 56.67. Since x represents a modal number, it must be an integer. Thus, the maximum value is 56 (57 would be cut-off).

(b) First, let us calculate the cut-off frequencies for the TE_{10} and TM_{77} modes:

$$(f_c)_{10} = \frac{1}{2\sqrt{\mu_0\epsilon_0}}\sqrt{\left(\frac{1}{10 \text{ m}}\right)^2 + \left(\frac{0}{6 \text{ m}}\right)^2} = 15 \text{ MHz}$$

$$(f_c)_{77} = \frac{1}{2\sqrt{\mu_0\epsilon_0}}\sqrt{\left(\frac{7}{10 \text{ m}}\right)^2 + \left(\frac{7}{6 \text{ m}}\right)^2} = 204 \text{ MHz}$$

We know that the group velocity for these modes will be given by:

$$v_g = \frac{1}{\sqrt{\mu_0\epsilon_0}}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

which produces a group velocity of 3.00×10^8 m/s for the TE_{10} mode and 2.91×10^8 the TM_{77} mode.

After travelling the maximum distance of 1 kilometer (from the base station in the center of the tunnel to a user at the end), this will result in a transit time of 3.33 μs for the TE_{10} mode and a transit time of 3.43 μs for the TM_{77} mode. The difference, 10 ns, is the dispersion between the two modes. This is usually tolerable in a digital cellular link since the pulses are still relatively long. In the GSM standard, for example, digital pulses are separated in time by 10 μs and can tolerate dispersion of about 1 μs. The 10 ns you calculated in this example may *not* be tolerable in a wireless data link, however, where the bit rate is much higher and the digital bits separated by much less time.

(6) **Link Budget:** We plug all of our given information into the logarithmic link budget equation:

$$\underbrace{P_R}_{-82\,\text{dBm}} = P_T + \underbrace{G_T}_{8\,\text{dBi}} + \overbrace{G_R}^{0\,\text{dBi}} - \underbrace{20\log_{10}\left(\frac{4\pi}{\lambda}\right)}_{47.8\,\text{dB}} - \overbrace{20\log_{10}(r)}^{54.0\,\text{dB}} - \underbrace{\text{Excess Loss}}_{16.0\,\text{dB}}$$

A transmit power of 27.8 dBm is required for this transmission. In the linear scale, this is 600 mW of power.