Name:

GTID:

## ECE 3065: Electromagnetics

## TEST 1 (Spring 2004)

- Please read all instructions before continuing with the test.
- This is a closed notes, closed book, closed friend, open mind test. On your desk you should only have writing instruments, a calculator, and a compass+ruler for working Smith chart problems.
- Show all work. (It helps me give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.
- Work intelligently - read through the exam and do the easiest problems first. Save the hard ones for last.
- All necessary mathematical formulas are included either in the problem statements or the last few pages of this test.
- You have 75 minutes to complete this examination. When I announce a "last call" for examination papers, I will leave the room in 5 minutes. The fact that I do not have your examination in my possession will not stop me.
- I will not grade your examination if you fail to 1) put your name and GTID number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.
- Have a nice day!


## Pledge Signature:

I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.

## 1. Short Answer Section (24 points)

(a)

If the intrinsic impedance of a medium is complex, then we know that medium is Answer.
(b)

If you are trying to match a transmission line with input impedance of $1000 \Omega$ to a load of $10 \Omega$, you need a quarter-wavelength termination of impedance Answer.
(c)

A simple medium has the following four properties: $\overline{-}(2)(3)$, and Answer 4.
(d)

Snell's law and Fresnel reflection coefficients are derived by enforcing the Answer conditions on plane wave fields at the dielectric interface.
(e) $\qquad$ (1) (2)
A transmission line will have a VSWR of $+\infty$ if it terminates in either one of 4 types of loads (circuit elements): Answer 1, Answer 2, Answer 3, or Answer 4.
(f)

Answer is an example of an application that relies on the principle of total reflection.

Optional Survey Question: When did you take ECE 3025 and who was your instructor?
Year: $\qquad$ Semester: $\qquad$ Instructor: $\qquad$

## 2. Descriptive Answer Section (16 points)

Write a concise answer to each question in the spaces provided beneath each problem statement. Note: Correct answers that are extremely verbose will be penalized.
(a) Glare (8 points): Glare occurs when smooth surfaces such as water, glass, or metal reflect the light from the overhead sun directly into a person's eyes. Below are two sketches for $\perp$ and $\|$ reflection coefficients corresponding to different relative perimittivities in the transmitted medium and air in the incident medium. Based on these graphs, which polarization (vertical or horizontal with respect to an observer) should be dominant in the reflected wave? Assume that there are equal polarization components in the incident wave.
(b) Polarizing Sunglasses (8 points): To reduce glare, we use polarizing sunglasses, with lenses that contain conducting polymer chains aligned in the vertical direction. These polymer chains allow conductivity in the presence of $z$-polarized electric fields (along the length of the chains), but not for $x$ or $y$ polarizations. Thus, we say that the conductivity is anisotropic, and we can relate $\vec{E}$ and $\vec{J}$ through a matrix:

$$
\vec{J}=\left[\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & \sigma_{z}
\end{array}\right] \vec{E}
$$

Below is the diagram for polarizing sun glasses. Using this as a reference point, write a short explanation describing the operation of the polarizing sun glasses. What type of waves are propagating in the air and through the lenses and why? Which type of incident polarization do the glasses block?


## 3. Splice Match:

Below is a schematic for a high-frequency parallel-plate transmission line that connects two RF components. The parallel-plate transmission line consists of a 1 mm -thick dielectric layer sandwiched between two striplines of equal length (no ground plane like the microstrip). The initial width of the line is $w_{0}=0.53 \mathrm{~mm}$ to achieve a $50 \Omega$ impedance. You place a section of parallel-plate line with a different width $w_{m}$. Calculate the appropriate values for the matching trace width $w_{m}$ (in mm ), and the line lengths $D_{m}$ and $D_{s}$ (in wavelengths). Be sure to use the attached Smith Chart to show your work. (30 points)

| PCB with Parallel Plate |
| :--- | :--- | :--- |
| Transmission Lines |

4. Circular-Polarized Radio Wave: A right-hand circularly polarized wave is incident upon a dielectric surface at a grazing angle ( $\theta_{i}$ is close to $90^{\circ}$ ). The equation for its electric field is

$$
\tilde{\overrightarrow{\mathrm{E}}}_{i}(\overrightarrow{\mathrm{r}})=\frac{E_{i}}{\sqrt{2}}\left(\cos \theta_{i} \hat{\mathrm{x}}-\sin \theta_{i} \hat{\mathrm{z}}+j \hat{\mathrm{y}}\right) \exp \left(-j k\left[\sin \theta_{i} \hat{\mathrm{x}}+\cos \theta_{i} \hat{\mathrm{z}}\right] \cdot \overrightarrow{\mathrm{r}}\right)
$$

(a) Make the grazing assumption for reflection coefficients and write the equation describing the reflected electric field, $\tilde{\overrightarrow{\mathrm{E}}}_{r}(\overrightarrow{\mathrm{r}})$. ( $\mathbf{1 5}$ points)
(b) What is the total electric field (incident plus reflected) on the $z=0$ plane? ( $\mathbf{1 5}$ points)

## Cheat Sheet

Electrical Properties of Common Transmission Lines

|  | $L(\mathrm{H} / \mathrm{m})$ | $C(\mathrm{~F} / \mathrm{m})$ | Impedance, $Z_{0}$ | $v_{p}$ |
| :---: | :---: | :---: | :---: | :---: |
| Parallel Plate | $\frac{\mu d}{w}$ | $\frac{\epsilon w}{d}$ | $\frac{d}{w} \sqrt{\frac{\mu}{\epsilon}}$ | $\frac{1}{\sqrt{\mu \epsilon}}$ |
| Coaxial Cable | $\frac{\mu}{2 \pi} \ln \left(\frac{b}{a}\right)$ | $\frac{2 \pi \epsilon}{\ln \left(\frac{b}{a}\right)}$ | $\frac{1}{2 \pi} \ln \left(\frac{b}{a}\right) \sqrt{\frac{\mu}{\epsilon}}$ | $\frac{1}{\sqrt{\mu \epsilon}}$ |
| Stripline | $\frac{Z_{0}}{v_{p}}$ | $\frac{1}{v_{p} Z_{0}}$ | $\frac{1}{2 \pi} \sqrt{\frac{\mu}{\epsilon_{\text {eff }}}} \ln \left(\frac{8 b}{a}+\frac{a}{4 b}\right)$ | $a<b$ |
|  |  |  | $\sqrt{\frac{\mu}{\epsilon_{\text {eff }}}} \frac{1}{1.393+\frac{a}{b}+\frac{2}{3} \ln \left(\frac{a}{b}+\frac{13}{9}\right)}$ | $a>b$ |

$* \epsilon_{\text {eff }}=\epsilon_{0}\left[\frac{\epsilon_{r}+1}{2}+\frac{\epsilon_{r}-1}{2} \frac{1}{\sqrt{1+12 b / a}}\right] \quad \epsilon_{0}=8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m} \quad \mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}$

$$
\lambda f=v_{p} \quad \omega=2 \pi f \quad \beta=\frac{2 \pi}{\lambda} \quad D=T v_{p}
$$

Reflection: $\Gamma_{L, G}=\frac{Z_{L, G}-Z_{0}}{Z_{L, G}+Z_{0}} \quad$ Transmission: $1+\Gamma_{L, G}$

Phasor Transform: $A \cos (2 \pi f t+\phi) \longrightarrow A \exp (j \phi)$
Reverse Transform: $\tilde{x} \longrightarrow \operatorname{Real}\{\tilde{x} \exp (j 2 \pi f t)\}$

$$
\begin{gathered}
v(z)=V^{+} \exp (-\gamma z)+V^{-} \exp (+\gamma z) \\
i(z)=\frac{V^{+}}{Z_{0}} \exp (-\gamma z)-\frac{V^{-}}{Z_{0}} \exp (+\gamma z) \\
\gamma=\alpha+j \beta=\sqrt{(R+j \omega L)(G+j \omega C)} \\
Z_{\text {in }}=Z_{0} \frac{Z_{L}+j Z_{0} \tan \beta D}{Z_{0}+j Z_{L} \tan \beta D}
\end{gathered}
$$

Quarter Wave Match: $Z_{M}=\sqrt{Z_{0} Z_{L}}$

$$
V S W R=\frac{V_{\max }}{V_{\min }}=\frac{1+\left|\Gamma_{L}\right|}{1-\left|\Gamma_{L}\right|}
$$

Fresnel Reflection Coefficients for a Dielectric Interface

| \|| Polarization $\begin{gathered} \Gamma_{\\|}=\frac{\eta_{1} \cos \theta_{i}-\eta_{2} \cos \theta_{t}}{\eta_{1} \cos \theta_{i}+\eta_{2} \cos \theta_{t}} \\ \tau_{\\|}=\frac{2 \eta_{2} \cos \theta_{i}}{\eta_{1} \cos \theta_{i}+\eta_{2} \cos \theta_{t}}=\frac{\cos \theta_{i}}{\cos \theta_{t}}\left(1+\Gamma_{\\|}\right) \end{gathered}$ | $\perp$ Polarization $\begin{gathered} \Gamma_{\perp}=\frac{\eta_{2} \cos \theta_{i}-\eta_{1} \cos \theta_{t}}{\eta_{2} \cos \theta_{i}+\eta_{1} \cos \theta_{t}} \\ \tau_{\perp}=\frac{2 \eta_{2} \cos \theta_{i}}{\eta_{2} \cos \theta_{i}+\eta_{1} \cos \theta_{t}}=1+\Gamma_{\perp} \end{gathered}$ |
| :---: | :---: |
| $\begin{aligned} \hat{\mathrm{k}}_{i} & =\sin \theta_{i} \hat{\mathrm{x}}+\cos \theta_{i} \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_{i} & =\cos \theta_{i} \hat{\mathrm{x}}-\sin \theta_{i} \hat{\mathrm{z}} \\ \hat{\mathrm{~h}}_{i} & =\hat{\mathrm{y}} \end{aligned}$ | $\begin{aligned} \hat{\mathrm{k}}_{i} & =\sin \theta_{i} \hat{\mathrm{x}}+\cos \theta_{i} \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_{i} & =\hat{\mathrm{y}} \\ \hat{\mathrm{~h}}_{i} & =-\cos \theta_{i} \hat{\mathrm{x}}+\sin \theta_{i} \hat{\mathrm{z}} \end{aligned}$ |
| $\begin{aligned} \hat{\mathrm{k}}_{r} & =\sin \theta_{r} \hat{\mathrm{x}}-\cos \theta_{r} \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_{r} & =\cos \theta_{r} \hat{\mathrm{x}}+\sin \theta_{r} \hat{\mathrm{Z}} \\ \hat{\mathrm{~h}}_{r} & =-\hat{\mathrm{y}} \end{aligned}$ | $\begin{aligned} \hat{\mathrm{k}}_{r} & =\sin \theta_{r} \hat{\mathrm{x}}-\cos \theta_{r} \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_{r} & =\hat{\mathrm{y}} \\ \hat{\mathrm{~h}}_{r} & =\cos \theta_{r} \hat{\mathrm{x}}+\sin \theta_{r} \hat{\mathrm{z}} \end{aligned}$ |
| $\begin{aligned} \hat{\mathrm{k}}_{t} & =\sin \theta_{t} \hat{\mathrm{x}}+\cos \theta_{t} \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_{t} & =\cos \theta_{t} \hat{\mathrm{x}}-\sin \theta_{t} \hat{\mathrm{z}} \\ \hat{\mathrm{~h}}_{t} & =\hat{\mathrm{y}} \end{aligned}$ | $\begin{aligned} \hat{\mathrm{k}}_{t} & =\sin \theta_{t} \hat{\mathrm{x}}+\cos \theta_{t} \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_{t} & =\hat{\mathrm{y}} \\ \hat{\mathrm{~h}}_{t} & =-\cos \theta_{t} \hat{\mathrm{x}}+\sin \theta_{t} \hat{\mathrm{z}} \end{aligned}$ |

## General Plane Wave Solution

| $\begin{gathered} \overrightarrow{\mathrm{E}}_{\diamond}(\overrightarrow{\mathrm{r}})=E_{\diamond} \hat{\mathrm{e}}_{\diamond} \exp \left(j\left[\phi-k \hat{\mathrm{k}}_{\diamond} \cdot \overrightarrow{\mathrm{r}}\right]\right) \\ \overrightarrow{\mathrm{r}}=x \hat{\mathrm{x}}+y \hat{\mathrm{y}}+z \hat{\mathrm{z}} \quad \eta=\sqrt{\frac{\mu}{\epsilon}} \quad k=\frac{2 \pi}{\lambda}=\frac{2 \pi f}{v_{p}} \end{gathered}$ | $\overrightarrow{\mathrm{H}}_{\diamond}(\overrightarrow{\mathrm{r}})=\frac{E_{\diamond}}{\eta} \hat{\mathrm{h}}_{\diamond} \exp \left(j\left[\phi-k \hat{\mathrm{k}}_{\diamond} \cdot \overrightarrow{\mathrm{r}}\right]\right)$ <br> $\diamond \rightarrow i$ (incident) or $r$ (reflected) or $t$ (transmitted) |
| :---: | :---: |
| Snell's Law of Reflection $\theta_{i}=\theta_{r}$ | $\begin{gathered} \text { Snell's Law of Refraction } \\ \frac{\sin \theta_{i}}{v_{p 1}}=\frac{\sin \theta_{t}}{v_{p 2}} \quad \text { where } v_{p}=\frac{1}{\sqrt{\epsilon \mu}} \end{gathered}$ |
| Physical Quantities |  |
| $\theta_{i}$ angle of incidence | $E \quad$ electric field amplitude ( $\mathrm{V} / \mathrm{m}$ ) |
| $\theta_{r}$ angle of reflection | $\Gamma_{\\|, \perp} \quad$ reflection coefficient $\left(\frac{E_{r}}{E_{i}}\right)$ |
| $\theta_{t}$ angle of transmission | $\tau_{\\|, \perp} \quad$ transmission coefficient $\left(\frac{E_{t}}{E_{i}}\right)$ |
| ê electric field polarization | $\eta$ intrinsic impedance ( $\Omega$, Ohms) |
| $\hat{\mathrm{h}}$ magnetic field polarization | $v_{p} \quad$ velocity of propagation ( $\mathrm{m} / \mathrm{s}$ ) |
| $\hat{\mathrm{k}}$ direction of propagation | $k \quad$ wavenumber (radians/m) |
| $\mu$ magnetic permeability (H/m) | $\lambda$ wavelength (m) |
| $\epsilon \quad$ electric permittivity ( $\mathrm{F} / \mathrm{m}$ ) | $f$ frequency (Hz) |

