Name: ______________________________

GTID: ______________________________

ECE 3065: Electromagnetics
TEST 1 (Spring 2005)

- Please read all instructions before continuing with the test.
- This is a **closed** notes, **closed** book, **closed** friend, **open** mind test. On your desk you should only have writing instruments, a calculator, and a compass+ruler for working Smith chart problems.
- Show all work. (It helps me give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.
- Work intelligently – read through the exam and do the easiest problems first. Save the hard ones for last.
- All necessary mathematical formulas are included either in the problem statements or the last few pages of this test.
- You have 75 minutes to complete this examination. When I announce a "last call" for examination papers, I will leave the room in 5 minutes. The fact that I do not have your examination in my possession will not stop me.
- I will not grade your examination if you fail to 1) put your name and GTID number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.
- Have a nice day!

Pledge Signature: ____

I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.

1. **Short Answer Section (20 points)**

   (a) ________________
   A plane wave experiences total reflection for all incident angles above the *Answer* angle.

   (b) ________________ (1) ________________ (2) ________________ (3)
   A simple medium has the following four properties: it must be *Answer 1*, *Answer 2*, *Answer 3*, and linear.

   (c) ________________
   A transmission line terminated with a pure inductor will have a VSWR of *Answer*.

   (d) ________________
   (True or False) Every antenna has one type of plane wave polarization that it cannot receive at all.

   (e) ________________
   If the load of a transmission line is complex, it may be matched with a quarter-wavelength *Answer*.

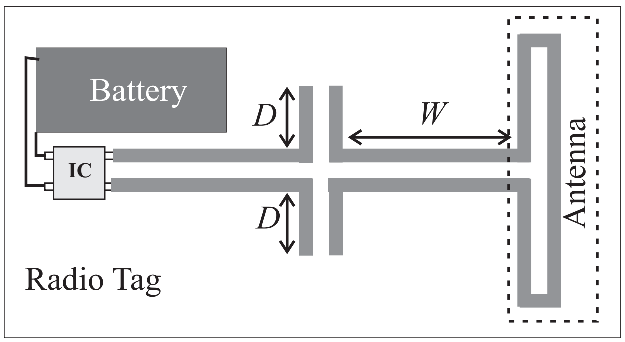
   (f) ________________ (1) ________________ (2) ________________ (3)
   Simply by changing its length, an RF engineer can make an unterminated microstrip stub line appear to behave as the following 4 circuit components: a *Answer 1*, a *Answer 2*, a *Answer 3*, or an open circuit.

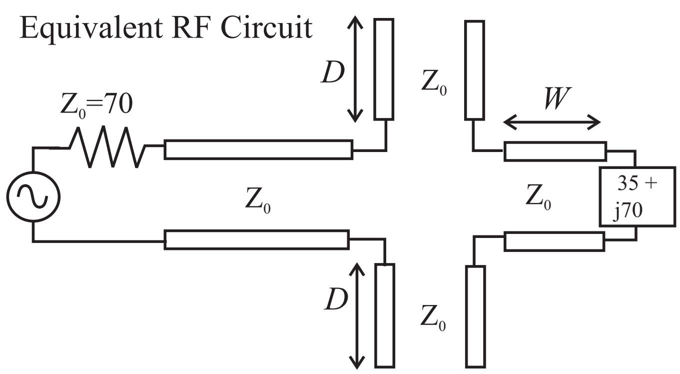
2. **Supermarket Radio Price Tags (10 points):**

   Here is a true story. A very big company in the business of electronic checkout technology developed a system of cheap, battery-operated liquid crystal display price tags that could be placed on supermarket shelves and updated by a low-powered, radio-frequency transmitter mounted in the ceiling in each aisle of the store. By synchronizing the price of goods on the shelf with the checkout database, lots of frustration and millions of dollars were going to be saved. The system worked well until the tags were placed on the shelves inside the glass freezer cases. The doors of freezer cases have a glass with a thin (optically translucent) metallic coating on the inside. (The reason for the coating: a DC voltage drop is applied across the freezer door, so that the metallic coating acts as a giant resistor, slightly warming the inside of the glass and keeping it from fogging.) Using first principles from electromagnetics, explain in writing why the freezer tags are not working.

3. **Tracking Tag (25 points):**

Below is a schematic for a thin, flexible radio tag that is designed to be placed flat on a crate and emit an RF signal at 2.45 GHz for the purpose of identifying and tracking the crate. The circuit traces are all 70Ω coplanar strip metallic pathways that have been screen-printed onto a sheet of clear plastic. Driven by a discrete flat battery, an *application-specific integrated circuit* (ASIC) serves as the sinusoidal voltage source while a folded dipole antenna serves as the load, which appears to be a $35 + j70\Omega$ impedance *when the tag is placed upon a crate*. You must match the antenna to the source with two symmetrical, open-circuited stub lines connected in series as shown in the diagram below. Solve for D and W (in terms of λ) and show all of your work on the Smith Chart on the following page.





4. **Circular-Polarized Radio Wave:** A UHF satellite is transmitting a circularly polarized wave to a boat on the surface of a calm lake. In the immediate area around the boat, the radio wave behaves like a plane wave and has an angle-of-incidence with respect to the water surface that happens to be the Brewster angle for $\|$ polarization. The equation for its electric field is

$$\tilde{\vec{E}}_i(\vec{r}) = \frac{377}{\sqrt{2}}\left(\cos\theta_i\hat{x} - \sin\theta_i\hat{z} + j\hat{y}\right)\exp(-j4.2[\sin\theta_i\hat{x} + \cos\theta_i\hat{z}]\cdot\vec{r})\ \mu\text{V/m}$$

where the direction $\hat{z}$ points into the water. The water has a permittivity of $\epsilon_r = 81$ and is, of course, non-magnetic. Answer the following questions accordingly. (**45 points**)

(a) What is the wavelength for this radio wave? (**5 points**)

(b) What is the angle of incidence for this radio wave? (**10 points**)

(c) What is the corresponding magnetic field for the incident plane wave? (**10 points**)

(d) The boat on the surface of the water is measuring the angles-of-arrival for this radio wave (and, hence, the position in the sky of the satellite). If the x-direction corresponds to $0°$ azimuth, what is the azimuth and elevation angles-of-arrival for this radio wave? (**10 points**)

(e) Write the expression for reflected wave, $\tilde{\vec{E}}_r(\vec{r})$, substituting numbers for any known variables: (**10 points**)

Cheat Sheet

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \qquad \mu_0 = 4\pi \times 10^{-7} \text{H/m}$$

$$\lambda f = v_p \qquad \omega = 2\pi f \qquad \beta = \frac{2\pi}{\lambda} \qquad D = T v_p$$

$$\text{Reflection: } \Gamma_{L,G} = \frac{Z_{L,G} - Z_0}{Z_{L,G} + Z_0} \qquad \text{Transmission: } 1 + \Gamma_{L,G}$$

$$\text{Phasor Transform: } A\cos(2\pi f t + \phi) \longrightarrow A\exp(j\phi)$$

$$\text{Reverse Transform: } \tilde{x} \longrightarrow \text{Real}\left\{\tilde{x}\exp(j2\pi f t)\right\}$$

$$v(z) = V^{+}\exp(-jkz) + V^{-}\exp(+jkz)$$

$$i(z) = \frac{V^{+}}{Z_0}\exp(-jkz) - \frac{V^{-}}{Z_0}\exp(+jkz)$$

$$k = \frac{2\pi}{\lambda} \qquad v_p = \frac{1}{\sqrt{LC}} \qquad Z_0 = \sqrt{\frac{L}{C}} \qquad Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta D}{Z_0 + jZ_L \tan\beta D}$$

$$\text{Quarter Wave Match: } Z_M = \sqrt{Z_0 Z_L}$$

$$\|\text{-Brewster Angle: } \theta_B = \sin^{-1}\frac{1}{\sqrt{1+\epsilon_1/\epsilon_2}} \quad (\mu_1 = \mu_2)$$

$$\tilde{\vec{\mathsf{E}}}(\vec{\mathrm{r}}) = E_0 \hat{\mathrm{e}} \exp(j[\phi - k\hat{\mathrm{k}}\cdot\vec{\mathrm{r}}])$$
$$\tilde{\vec{\mathsf{H}}}(\vec{\mathrm{r}}) = H_0 \hat{\mathrm{h}} \exp(j[\phi - k\hat{\mathrm{k}}\cdot\vec{\mathrm{r}}])$$

$$H_0 = \frac{E_0}{\eta} \qquad \eta = \sqrt{\frac{\mu}{\epsilon}} \qquad v_p = \frac{1}{\sqrt{\mu\epsilon}} \qquad \hat{\mathrm{e}} \times \hat{\mathrm{h}}^* = \hat{\mathrm{k}} \qquad \hat{\mathrm{h}} = (\hat{\mathrm{k}} \times \hat{\mathrm{e}})^*$$

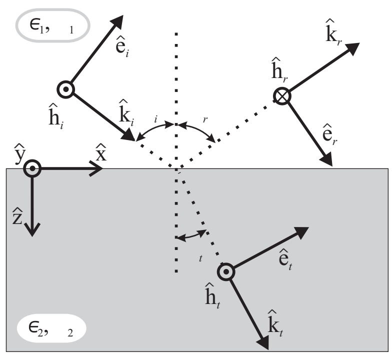
$$-\hat{\mathrm{k}} = \cos\varphi\cos\theta\,\hat{\mathrm{x}} + \sin\varphi\cos\theta\,\hat{\mathrm{y}} + \sin\theta\,\hat{\mathrm{z}}$$

$$\theta = \tan^{-1}\frac{-k_z}{\sqrt{k_x^2 + k_y^2}} \qquad \varphi = \tan^{-1}\frac{k_y}{k_x} \quad (\text{add } \pi \text{ if } k_x > 0)$$

$$\text{Cross Product: } \vec{\mathrm{a}} \times \vec{\mathrm{b}} = \det \begin{vmatrix} \hat{\mathrm{x}} & \hat{\mathrm{y}} & \hat{\mathrm{z}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Fresnel Reflection Coefficients for a Dielectric Interface

$\parallel$ Polarization



$$\Gamma_{\parallel} = -\frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

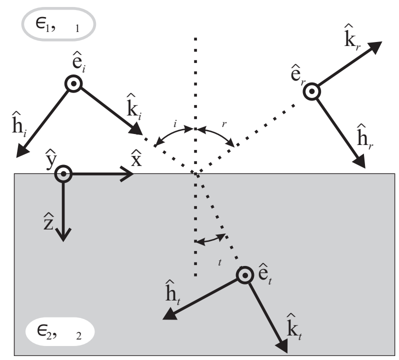
$$\tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} = \frac{\cos\theta_i}{\cos\theta_t}\left(1 + \Gamma_{\parallel}\right)$$

$$\hat{k}_i = \sin\theta_i \hat{x} + \cos\theta_i \hat{z}$$
$$\hat{e}_i = \cos\theta_i \hat{x} - \sin\theta_i \hat{z}$$
$$\hat{h}_i = \hat{y}$$

$$\hat{k}_r = \sin\theta_r \hat{x} - \cos\theta_r \hat{z}$$
$$\hat{e}_r = \cos\theta_r \hat{x} + \sin\theta_r \hat{z}$$
$$\hat{h}_r = -\hat{y}$$

$$\hat{k}_t = \sin\theta_t \hat{x} + \cos\theta_t \hat{z}$$
$$\hat{e}_t = \cos\theta_t \hat{x} - \sin\theta_t \hat{z}$$
$$\hat{h}_t = \hat{y}$$

$\perp$ Polarization



$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\tau_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = 1 + \Gamma_{\perp}$$

$$\hat{k}_i = \sin\theta_i \hat{x} + \cos\theta_i \hat{z}$$
$$\hat{e}_i = \hat{y}$$
$$\hat{h}_i = -\cos\theta_i \hat{x} + \sin\theta_i \hat{z}$$

$$\hat{k}_r = \sin\theta_r \hat{x} - \cos\theta_r \hat{z}$$
$$\hat{e}_r = \hat{y}$$
$$\hat{h}_r = \cos\theta_r \hat{x} + \sin\theta_r \hat{z}$$

$$\hat{k}_t = \sin\theta_t \hat{x} + \cos\theta_t \hat{z}$$
$$\hat{e}_t = \hat{y}$$
$$\hat{h}_t = -\cos\theta_t \hat{x} + \sin\theta_t \hat{z}$$

General Plane Wave Solution

$$\vec{\mathsf{E}}_\diamond(\vec{r}) = E_\diamond\, \hat{e}_\diamond \exp\left(j\left[\phi - k\hat{k}_\diamond \cdot \vec{r}\right]\right) \qquad \vec{\mathsf{H}}_\diamond(\vec{r}) = \frac{E_\diamond}{\eta}\hat{h}_\diamond \exp\left(j\left[\phi - k\hat{k}_\diamond \cdot \vec{r}\right]\right)$$

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ $\quad \eta = \sqrt{\frac{\mu}{\epsilon}}$ $\quad k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_p}$ $\quad \diamond \rightarrow i$ (incident) **or** r (reflected) **or** t (transmitted)

Snell's Law of Reflection

$$\theta_i = \theta_r$$

Snell's Law of Refraction

$$\frac{\sin\theta_i}{v_{p1}} = \frac{\sin\theta_t}{v_{p2}} \quad \text{where } v_p = \frac{1}{\sqrt{\epsilon\mu}}$$

Physical Quantities

θ_i	angle of incidence	E	electric field amplitude (V/m)
θ_r	angle of reflection	$\Gamma_{\parallel,\perp}$	reflection coefficient ($\frac{E_r}{E_i}$)
θ_t	angle of transmission	$\tau_{\parallel,\perp}$	transmission coefficient ($\frac{E_t}{E_i}$)
$\hat{e}$	electric field polarization	η	intrinsic impedance (Ω, Ohms)
$\hat{h}$	magnetic field polarization	v_p	velocity of propagation (m/s)
$\hat{k}$	direction of propagation	k	wavenumber (radians/m)
μ	magnetic permeability (H/m)	λ	wavelength (m)
ϵ	electric permittivity (F/m)	f	frequency (Hz)