Name: ______________________

GTID: ______________________

ECE 3065: Electromagnetics
TEST 1 (Spring 2006)

- Please read all instructions before continuing with the test.

- This is a **closed** notes, **closed** book, **closed** friend, **open** mind test. On your desk you should only have writing instruments, a calculator, and a compass+ruler for working Smith chart problems.

- Show all work. (It helps me give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.

- Work intelligently – read through the exam and do the easiest problems first. Save the hard ones for last.

- All necessary mathematical formulas are included either in the problem statements or the last few pages of this test.

- You have 75 minutes to complete this examination. When I announce a "last call" for examination papers, I will leave the room in 5 minutes. The fact that I do not have your examination in my possession will not stop me.

- I will not grade your examination if you fail to 1) put your name and GTID number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.

- Have a nice day!

Pledge Signature: ____

I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.

1. **Short Answer Section** (**14 points**)

   (a) ____________

   For a microstrip transmission line topology, it is very difficult to match loads with *Answer* stubs.

   (b) ____________

   A radio tower that wishes to confine its VHF signal into a tight "footprint" should transmit with *Answer* polarization.

   (c) ____________

   For a critical angle to exist at a dielectric interface, the *Answer* in the incident wave's medium must be larger than the transmitted wave's medium.

   (d) ____________

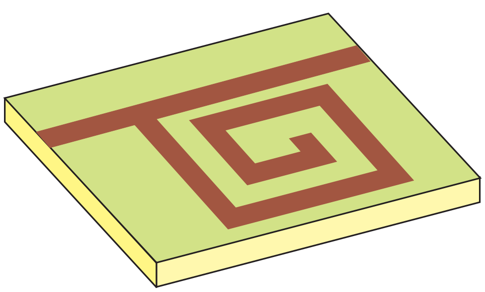
   The phase of the wave is sometimes called the *Answer*.

   (e) ____________ (1) ____________ (2) ____________ (3)

   A simple medium has the following four properties: it must be *Answer 1*, *Answer 2*, *Answer 3*, and homogeneous.

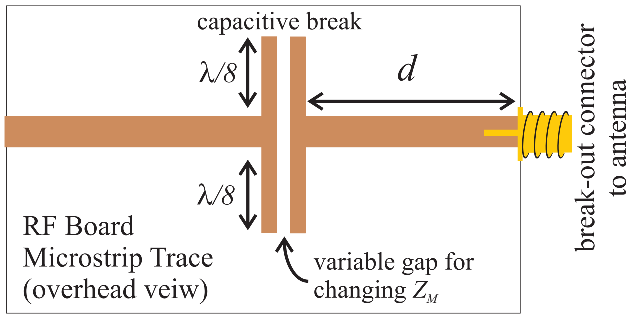
2. **Mystery RF Circuit Structure (11 points):**

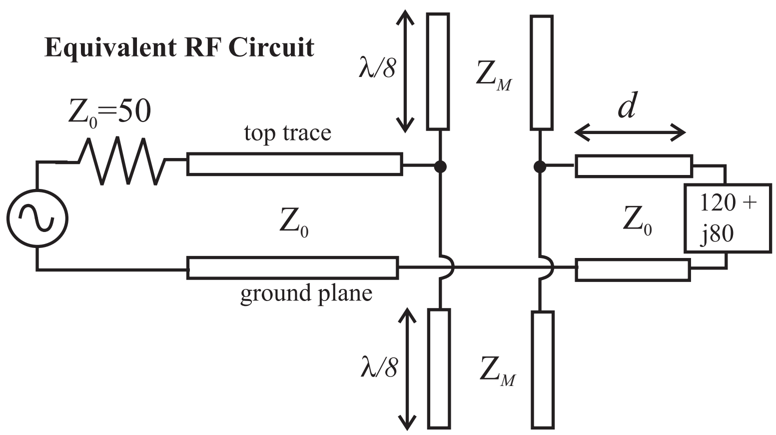
   You are asked to reverse engineering a competitor's RF board. When you open the enclosure to look at the board, you notice a strange squiggly structure (see below) protruding from the side of a microstrip line. The squiggle is somewhere between 0.25λ and 0.5λ in length. What equivalent circuit element is the RF designer trying to recreate here? Why not use a ceramic circuit component instead?



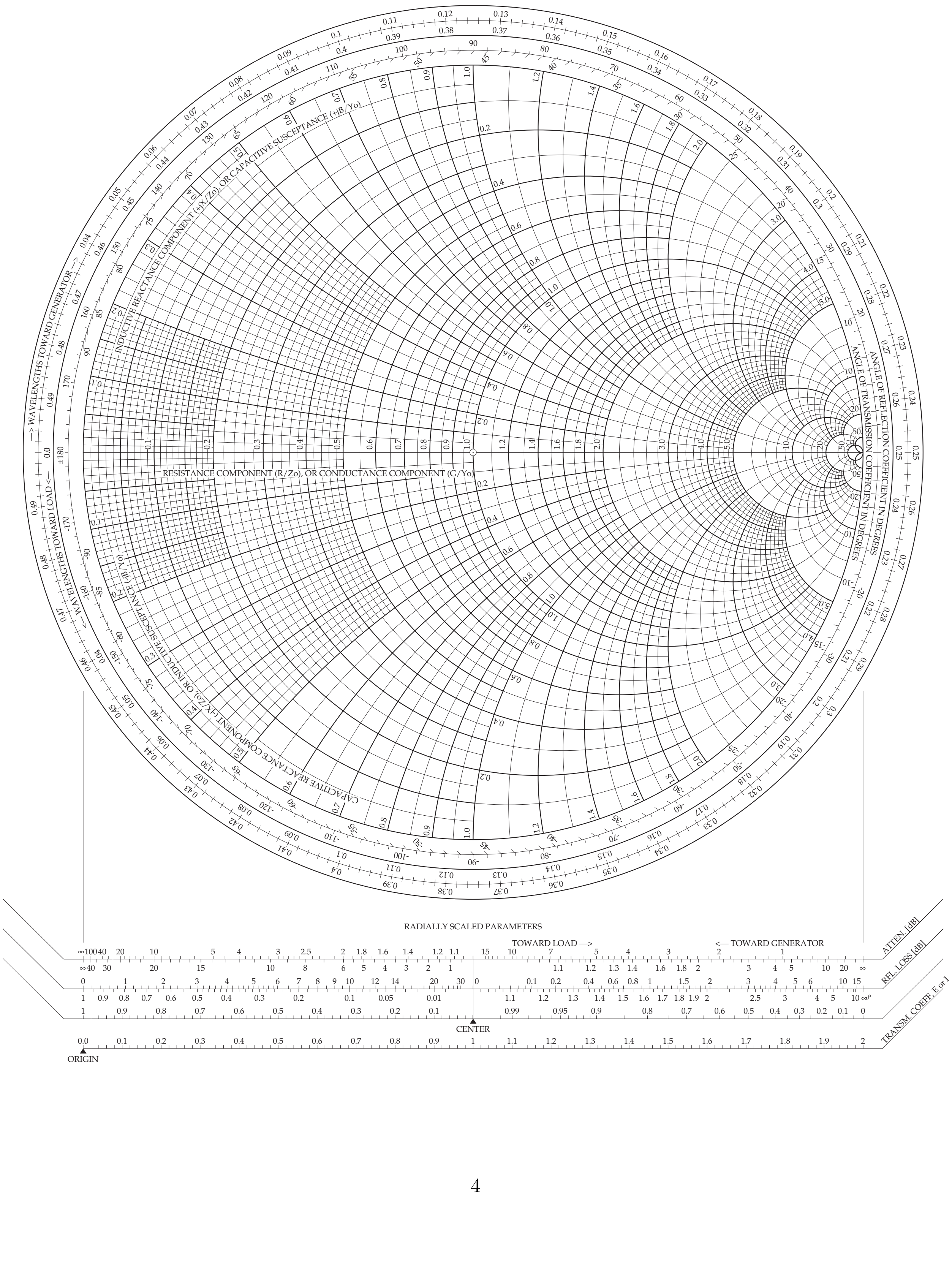
3. **Novel Matching Scheme (30 points):**

Below is a schematic for a microstrip trace which contains a break-out to an antenna that is mismatched to the 50Ω feed line. At the point of the break-out, the equivalent impedance of the load is $120 + j80\Omega$. This load is matched with a symmetrical pair of $\lambda/8$-stubs that form a capacitive break in the upper plane of the microstrip line. An equivalent transmission line circuit that models this break is shown below. By changing the separation distance of the break, you may change the intrinsic impedance Z_M of the two line sections. Calculate the Z_M and the electromagnetic distance d between the capacitive break and the load. Show any work on the Smith chart (following page).





The Complete Smith Chart



4. **Slant-Polarized Radio Wave (45 points):**

A NASA deep space exploration satellite sends a probing radar waveform down to the surface of Saturn's moon Titan. The atmosphere of this moon is like free space and the surface is a slushy, icy, non-conductive, non-magnetic mix that has $\epsilon_r = 3$. The electric field from the probe's wave has the following form:

$$\tilde{\vec{\mathrm{E}}}_i(\vec{\mathrm{r}}) = 10\left(\frac{1}{2}\hat{\mathrm{x}} - \frac{\sqrt{3}}{2}\hat{\mathrm{z}} + \hat{\mathrm{y}}\right)\exp\left(-j20\left[\frac{\sqrt{3}}{2}\hat{\mathrm{x}} + \frac{1}{2}\hat{\mathrm{z}}\right]\cdot\vec{\mathrm{r}}\right)\ \mu\mathrm{V/m}$$

where $z = 0$ corresponds to the (approximately) flat dielectric surface of the planet. Answer the following questions based on this information.

(a) Write the corresponding magnetic field phasor that completes this plane wave solution. (**15 points**)

(b) What is the wavelength and angle-of-incidence for this particular plane wave? (Measure elevation angle from the *horizon.*) (**10 points**)

(c) Show that the angle of incidence for this wave is actually a special case that we studied in class. What is the name for this angle? (**5 points**)

(d) Write a numerical expression for the *reflected electric* field in this problem. (**15 points**)

Cheat Sheet

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \qquad \mu_0 = 4\pi \times 10^{-7} \text{H/m}$$

$$\lambda f = v_p \qquad \omega = 2\pi f \qquad \beta = \frac{2\pi}{\lambda} \qquad D = T v_p$$

$$\text{Reflection: } \Gamma_{L,G} = \frac{Z_{L,G} - Z_0}{Z_{L,G} + Z_0} \qquad \text{Transmission: } 1 + \Gamma_{L,G}$$

$$\text{Phasor Transform: } A\cos(2\pi f t + \phi) \longrightarrow A\exp(j\phi)$$

$$\text{Reverse Transform: } \tilde{x} \longrightarrow \text{Real}\left\{\tilde{x}\exp(j2\pi f t)\right\}$$

$$v(z) = V^{+}\exp(-jkz) + V^{-}\exp(+jkz)$$

$$i(z) = \frac{V^{+}}{Z_0}\exp(-jkz) - \frac{V^{-}}{Z_0}\exp(+jkz)$$

$$k = \frac{2\pi}{\lambda} \qquad v_p = \frac{1}{\sqrt{LC}} \qquad Z_0 = \sqrt{\frac{L}{C}} \qquad Z_{in} = Z_0\frac{Z_L + jZ_0\tan\beta D}{Z_0 + jZ_L\tan\beta D}$$

$$\text{Quarter Wave Match: } Z_M = \sqrt{Z_0 Z_L}$$

$$\|\text{-Brewster Angle: } \theta_B = \sin^{-1}\frac{1}{\sqrt{1+\epsilon_1/\epsilon_2}} \qquad (\mu_1 = \mu_2)$$

$$\tilde{\vec{\mathsf{E}}}(\vec{\mathrm{r}}) = E_0\hat{\mathrm{e}}\exp(j[\phi - k\hat{\mathrm{k}}\cdot\vec{\mathrm{r}}])$$
$$\tilde{\vec{\mathsf{H}}}(\vec{\mathrm{r}}) = H_0\hat{\mathrm{h}}\exp(j[\phi - k\hat{\mathrm{k}}\cdot\vec{\mathrm{r}}])$$

$$H_0 = \frac{E_0}{\eta} \qquad \eta = \sqrt{\frac{\mu}{\epsilon}} \qquad v_p = \frac{1}{\sqrt{\mu\epsilon}} \qquad \hat{\mathrm{e}}\times\hat{\mathrm{h}}^* = \hat{\mathrm{k}} \qquad \hat{\mathrm{h}} = (\hat{\mathrm{k}}\times\hat{\mathrm{e}})^*$$

$$\text{Cross Product: } \vec{\mathrm{a}}\times\vec{\mathrm{b}} = \det\begin{vmatrix} \hat{\mathrm{x}} & \hat{\mathrm{y}} & \hat{\mathrm{z}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

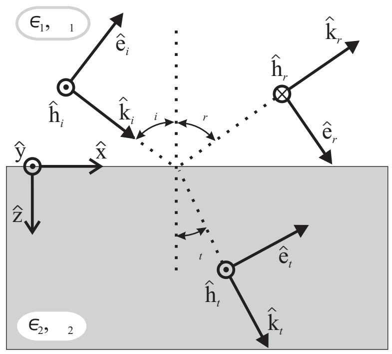
AOA Equations (Elevation measured from horizon)

$$-\hat{\mathrm{k}} = \cos\varphi\cos\theta\,\hat{\mathrm{x}} + \sin\varphi\cos\theta\,\hat{\mathrm{y}} + \sin\theta\,\hat{\mathrm{z}}$$

$$\theta = \tan^{-1}\frac{-k_z}{\sqrt{k_x^2 + k_y^2}} \qquad \varphi = \tan^{-1}\frac{k_y}{k_x} \quad (\text{add } \pi \text{ if } k_x > 0)$$

Fresnel Reflection Coefficients for a Dielectric Interface

$\parallel$ Polarization



$$\Gamma_{\parallel} = -\frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

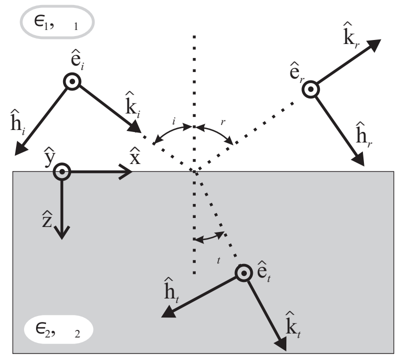
$$\tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} = \frac{\cos\theta_i}{\cos\theta_t}\left(1 + \Gamma_{\parallel}\right)$$

$$\begin{aligned} \hat{\mathrm{k}}_i &= \sin\theta_i \hat{\mathrm{x}} + \cos\theta_i \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_i &= \cos\theta_i \hat{\mathrm{x}} - \sin\theta_i \hat{\mathrm{z}} \\ \hat{\mathrm{h}}_i &= \hat{\mathrm{y}} \end{aligned}$$

$$\begin{aligned} \hat{\mathrm{k}}_r &= \sin\theta_r \hat{\mathrm{x}} - \cos\theta_r \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_r &= \cos\theta_r \hat{\mathrm{x}} + \sin\theta_r \hat{\mathrm{z}} \\ \hat{\mathrm{h}}_r &= -\hat{\mathrm{y}} \end{aligned}$$

$$\begin{aligned} \hat{\mathrm{k}}_t &= \sin\theta_t \hat{\mathrm{x}} + \cos\theta_t \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_t &= \cos\theta_t \hat{\mathrm{x}} - \sin\theta_t \hat{\mathrm{z}} \\ \hat{\mathrm{h}}_t &= \hat{\mathrm{y}} \end{aligned}$$

$\perp$ Polarization



$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\tau_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = 1 + \Gamma_{\perp}$$

$$\begin{aligned} \hat{\mathrm{k}}_i &= \sin\theta_i \hat{\mathrm{x}} + \cos\theta_i \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_i &= \hat{\mathrm{y}} \\ \hat{\mathrm{h}}_i &= -\cos\theta_i \hat{\mathrm{x}} + \sin\theta_i \hat{\mathrm{z}} \end{aligned}$$

$$\begin{aligned} \hat{\mathrm{k}}_r &= \sin\theta_r \hat{\mathrm{x}} - \cos\theta_r \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_r &= \hat{\mathrm{y}} \\ \hat{\mathrm{h}}_r &= \cos\theta_r \hat{\mathrm{x}} + \sin\theta_r \hat{\mathrm{z}} \end{aligned}$$

$$\begin{aligned} \hat{\mathrm{k}}_t &= \sin\theta_t \hat{\mathrm{x}} + \cos\theta_t \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_t &= \hat{\mathrm{y}} \\ \hat{\mathrm{h}}_t &= -\cos\theta_t \hat{\mathrm{x}} + \sin\theta_t \hat{\mathrm{z}} \end{aligned}$$

General Plane Wave Solution

$$\vec{\mathsf{E}}_\diamond(\vec{\mathrm{r}}) = E_\diamond\, \hat{\mathrm{e}}_\diamond \exp\left(j\left[\phi - k\hat{\mathrm{k}}_\diamond \cdot \vec{\mathrm{r}}\right]\right) \qquad \vec{\mathsf{H}}_\diamond(\vec{\mathrm{r}}) = \frac{E_\diamond}{\eta} \hat{\mathrm{h}}_\diamond \exp\left(j\left[\phi - k\hat{\mathrm{k}}_\diamond \cdot \vec{\mathrm{r}}\right]\right)$$

$\vec{\mathrm{r}} = x\hat{\mathrm{x}} + y\hat{\mathrm{y}} + z\hat{\mathrm{z}}$ $\eta = \sqrt{\frac{\mu}{\epsilon}}$ $k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_p}$ $\diamond \rightarrow i$ (incident) **or** r (reflected) **or** t (transmitted)

Snell's Law of Reflection

$$\theta_i = \theta_r$$

Snell's Law of Refraction

$$\frac{\sin\theta_i}{v_{p1}} = \frac{\sin\theta_t}{v_{p2}} \quad \text{where } v_p = \frac{1}{\sqrt{\epsilon\mu}}$$

Physical Quantities

θ_i	angle of incidence	E	electric field amplitude (V/m)
θ_r	angle of reflection	$\Gamma_{\parallel,\perp}$	reflection coefficient ($\frac{E_r}{E_i}$)
θ_t	angle of transmission	$\tau_{\parallel,\perp}$	transmission coefficient ($\frac{E_t}{E_i}$)
$\hat{\mathrm{e}}$	electric field polarization	η	intrinsic impedance (Ω, Ohms)
$\hat{\mathrm{h}}$	magnetic field polarization	v_p	velocity of propagation (m/s)
$\hat{\mathrm{k}}$	direction of propagation	k	wavenumber (radians/m)
μ	magnetic permeability (H/m)	λ	wavelength (m)
ϵ	electric permittivity (F/m)	f	frequency (Hz)