Name: ____________________

GTID: ____________________

ECE 3065: Electromagnetics
TEST 1 (Spring 2009)

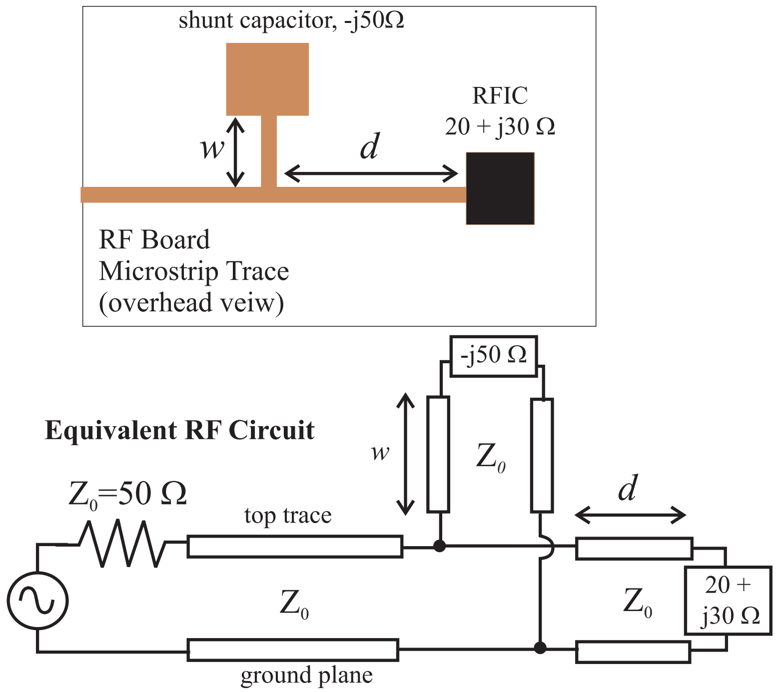
- Please read all instructions before continuing with the test.
- This is a **closed** notes, **closed** book, **closed** friend, **open** mind test. On your desk you should only have writing instruments, a calculator, and a compass+ruler for working Smith chart problems.
- Show all work. (It helps me give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.
- Work intelligently – read through the exam and do the easiest problems first. Save the hard ones for last.
- All necessary mathematical formulas are included either in the problem statements or the last few pages of this test.
- You have 75 minutes to complete this examination. When I announce a “last call” for examination papers, I will leave the room in 5 minutes. The fact that I do not have your examination in my possession will not stop me.
- I will not grade your examination if you fail to 1) put your name and GTID number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.
- Have a nice day!

Pledge Signature: ____

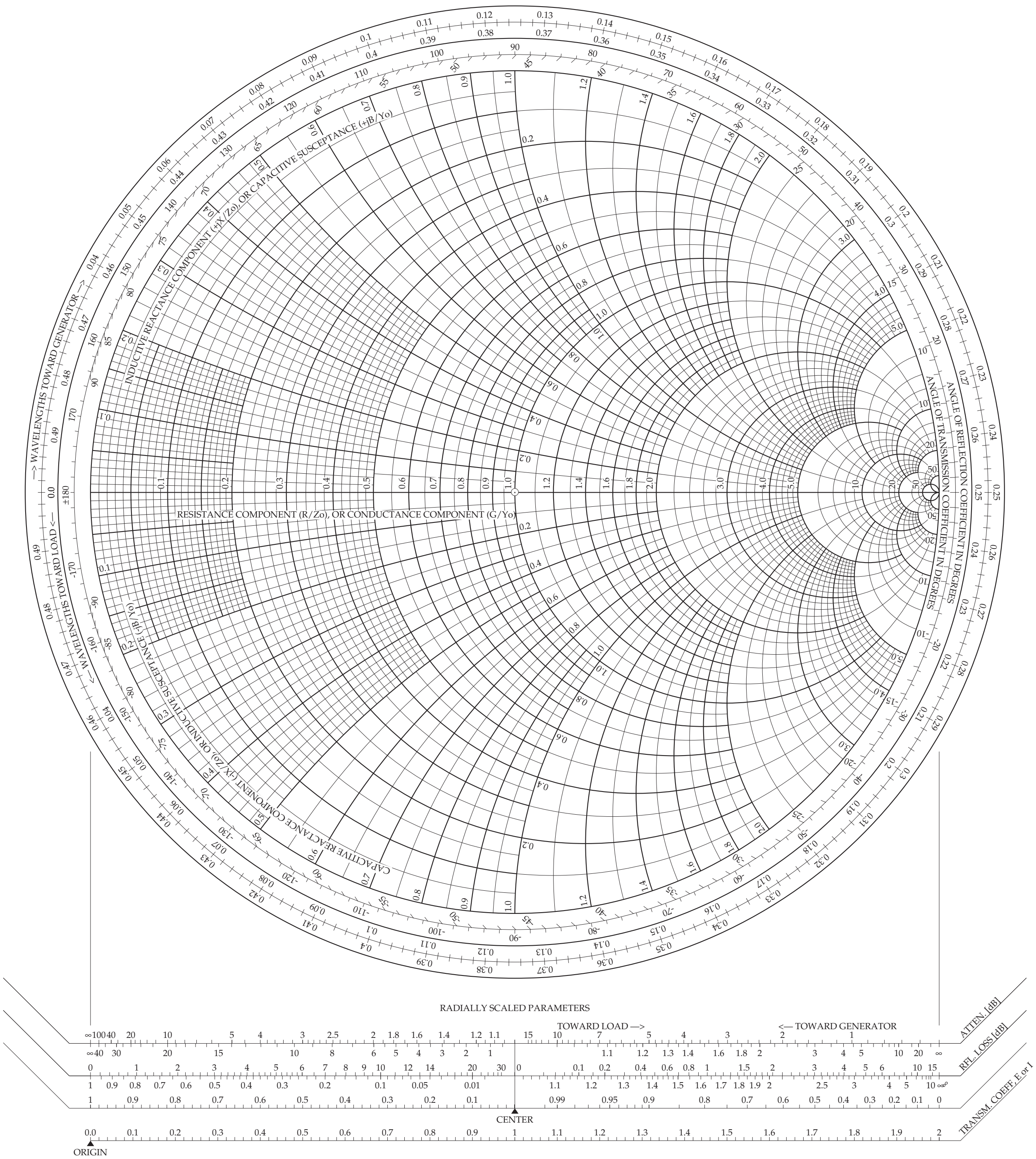
I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.

1. **Parallel Stub Matching (25 points):**

   You must match an RF integrated circuit (RFIC) to a standard 50Ω trace on a thin microstrip printed circuit board. This particular match is difficult because the board must be as small as possible. Thus, at the end of a parallel stub, a patch capacitor of $-j50\Omega$ is placed at the end of the stub line in the hopes of minimizing the stub's length on the printed circuit board. Neglecting the dimensions of the capacitor itself, how much length (in terms of λ) does this design require compared to a conventional open-circuit parallel stub match design? Show your work on the attached Smith Chart.

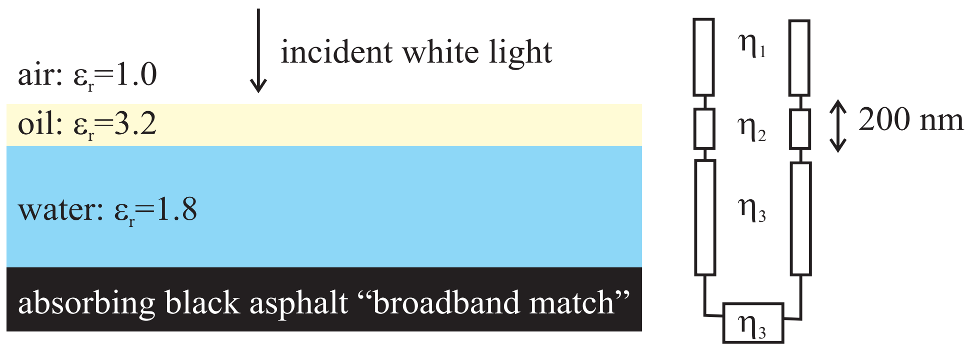


The Complete Smith Chart



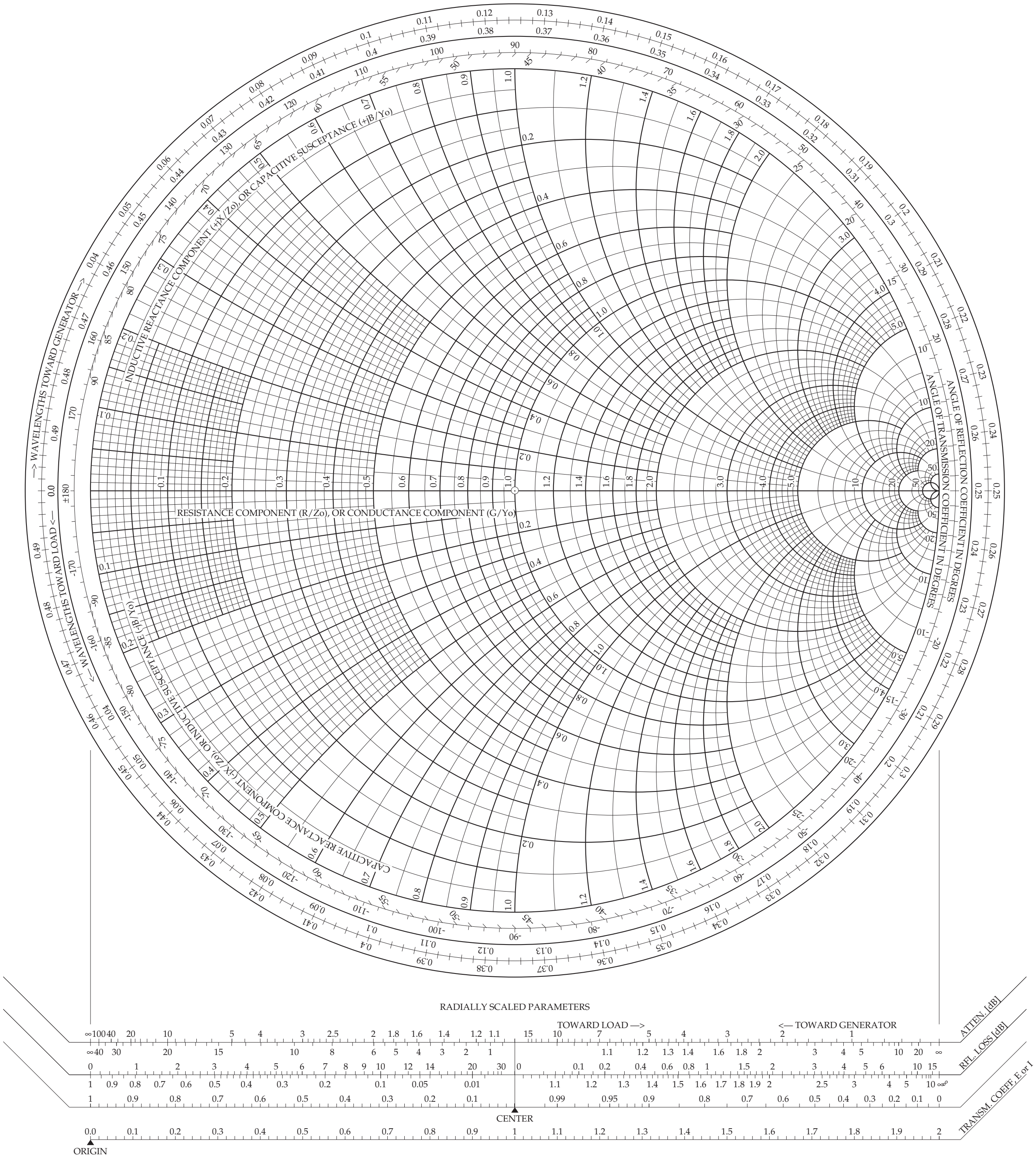
2. **Newton's Rings (25 points):**

   When it begins to rain on asphalt, a thin 200nm layer of oil and petrol (dielectric $\epsilon_r = 3.2$) floats to the surface of water puddles (dielectric $\epsilon_r = 1.8$), according to the diagram shown below. Using the analogy of transmission line theory, predict what will happen when several constituent components of white light – red ($\lambda = 700$nm), green ($\lambda = 550$nm), and violet ($\lambda = 400$nm) – are normal-incident upon this flat dielectric surface. What is the magnitude of the reflection coefficient of each of these colors at the air-oil interface? Show your work on the attached Smith chart (following page). (**20 points**)



In reality, oil thickness will vary across the surface of the water slightly and the white light will be arriving and reflecting at oblique incidences. Explain (without using math) what you might see on the surface of the puddle and why. (**5 points**)

The Complete Smith Chart



3. **Satellite Radio Wave Propagation (50 points):**

A satellite launches a radio wave towards earth where a satellite dish and receiver attempts to pick up this signal at an earth station on the flat, dry desert of New Mexico. Here the ground behaves like a smooth, lossless dielectric interface of $\epsilon_r = 3$. The incident electric field for this wave behaves locally like a plane wave with the following form:

$$\tilde{\vec{\mathrm{E}}}_i(\vec{\mathrm{r}}) = 5\left(\frac{1}{\sqrt{2}}\hat{\mathrm{x}} - \frac{1}{\sqrt{2}}\hat{\mathrm{z}}\right)\exp\left(-j50\left[\frac{1}{\sqrt{2}}\hat{\mathrm{x}} + \frac{1}{\sqrt{2}}\hat{\mathrm{z}}\right]\cdot\vec{\mathrm{r}}\right)\ \mu\mathrm{V/m}$$

where $z = 0$ corresponds to the flat dielectric surface of earth (uses same coordinate system as formula sheet). Answer the following questions based on this information.

(a) Write the corresponding magnetic field phasor that completes this incident plane wave solution. (**15 points**)

(b) Which direction in azimuth and elevation should you point the dish antenna in order to capture this signal? (Measure elevation angle from the *horizon*). (**10 points**)

(c) At what frequency should you tune your earth station receiver in order to capture the signal on this radio wave? (**5 points**)

(d) What percentage of the power is reflected off the surface of the ground? (**10 points**)

(e) If your satellite receiver dish antenna has a diameter of 2m, estimate the amount of power that your earth station receiver "grabs" from the plane wave. Hint: this is just magnitude of the incident-wave Poynting vector ($\mathrm{W/m^2}$) multiplied by the circular cross-sectional area ($\mathrm{m^2}$) of the dish. (**10 points**)

Cheat Sheet

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \qquad \mu_0 = 4\pi \times 10^{-7} \text{H/m}$$

$$\lambda f = v_p \qquad \omega = 2\pi f \qquad \beta = \frac{2\pi}{\lambda} \qquad D = T v_p \qquad c = \sqrt{\frac{\mu_o}{\epsilon_o}} = 3 \times 10^8 \text{m/s}$$

$$\text{Reflection: } \Gamma_{L,G} = \frac{Z_{L,G} - Z_0}{Z_{L,G} + Z_0} \qquad \text{Transmission: } 1 + \Gamma_{L,G}$$

$$\text{Phasor Transform: } A\cos(2\pi f t + \phi) \longrightarrow A\exp(j\phi)$$

$$\text{Reverse Transform: } \tilde{x} \longrightarrow \text{Real}\left\{\tilde{x}\exp(j2\pi f t)\right\}$$

$$\text{Poynting Vector: } \vec{\mathrm{S}}(\vec{\mathrm{r}}) = \frac{1}{2}\text{Real}\{\tilde{\vec{\mathsf{E}}}(\vec{\mathrm{r}}) \times \tilde{\vec{\mathsf{H}}}^*(\vec{\mathrm{r}})\}$$

$$k = \frac{2\pi}{\lambda} \qquad v_p = \frac{1}{\sqrt{LC}} \qquad Z_0 = \sqrt{\frac{L}{C}} \qquad Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta D}{Z_0 + jZ_L \tan\beta D}$$

$$\text{Quarter Wave Match: } Z_M = \sqrt{Z_0 Z_L}$$

$$\|\text{-Brewster Angle: } \theta_B = \sin^{-1}\frac{1}{\sqrt{1 + \epsilon_1/\epsilon_2}} \quad (\mu_1 = \mu_2)$$

$$\begin{aligned} \tilde{\vec{\mathsf{E}}}(\vec{\mathrm{r}}) &= E_0 \hat{\mathrm{e}} \exp(j[\phi - k\hat{\mathrm{k}} \cdot \vec{\mathrm{r}}]) \\ \tilde{\vec{\mathsf{H}}}(\vec{\mathrm{r}}) &= H_0 \hat{\mathrm{h}} \exp(j[\phi - k\hat{\mathrm{k}} \cdot \vec{\mathrm{r}}]) \end{aligned}$$

$$H_0 = \frac{E_0}{\eta} \qquad \eta = \sqrt{\frac{\mu}{\epsilon}} \qquad v_p = \frac{1}{\sqrt{\mu\epsilon}} \qquad \hat{\mathrm{e}} \times \hat{\mathrm{h}}^* = \hat{\mathrm{k}} \qquad \hat{\mathrm{h}} = (\hat{\mathrm{k}} \times \hat{\mathrm{e}})^*$$

$$\text{Cross Product: } \vec{\mathrm{a}} \times \vec{\mathrm{b}} = \det \begin{vmatrix} \hat{\mathrm{x}} & \hat{\mathrm{y}} & \hat{\mathrm{z}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

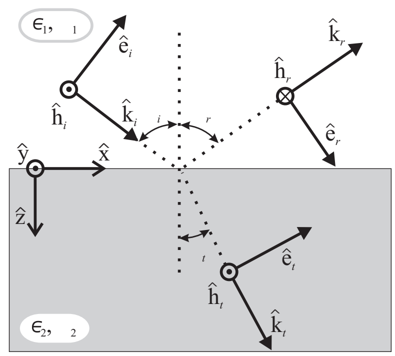
AOA Equations (Elevation measured from horizon)

$$-\hat{\mathrm{k}} = \cos\varphi\cos\theta\,\hat{\mathrm{x}} + \sin\varphi\cos\theta\,\hat{\mathrm{y}} + \sin\theta\,\hat{\mathrm{z}}$$

$$\theta = \tan^{-1}\frac{-k_z}{\sqrt{k_x^2 + k_y^2}} \qquad \varphi = \tan^{-1}\frac{k_y}{k_x} \quad (\text{add } \pi \text{ if } k_x > 0)$$

Fresnel Reflection Coefficients for a Dielectric Interface

$\parallel$ Polarization



$$\Gamma_{\parallel} = -\frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

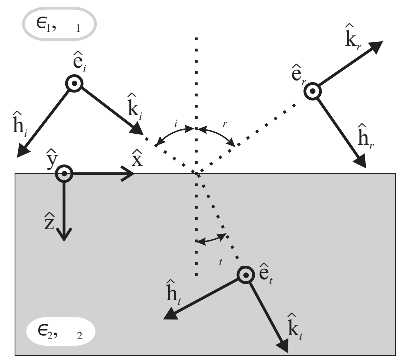
$$\tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} = \frac{\cos\theta_i}{\cos\theta_t}\left(1 + \Gamma_{\parallel}\right)$$

$$\begin{aligned}\hat{\mathrm{k}}_i &= \sin\theta_i \hat{\mathrm{x}} + \cos\theta_i \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_i &= \cos\theta_i \hat{\mathrm{x}} - \sin\theta_i \hat{\mathrm{z}} \\ \hat{\mathrm{h}}_i &= \hat{\mathrm{y}}\end{aligned}$$

$$\begin{aligned}\hat{\mathrm{k}}_r &= \sin\theta_r \hat{\mathrm{x}} - \cos\theta_r \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_r &= \cos\theta_r \hat{\mathrm{x}} + \sin\theta_r \hat{\mathrm{z}} \\ \hat{\mathrm{h}}_r &= -\hat{\mathrm{y}}\end{aligned}$$

$$\begin{aligned}\hat{\mathrm{k}}_t &= \sin\theta_t \hat{\mathrm{x}} + \cos\theta_t \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_t &= \cos\theta_t \hat{\mathrm{x}} - \sin\theta_t \hat{\mathrm{z}} \\ \hat{\mathrm{h}}_t &= \hat{\mathrm{y}}\end{aligned}$$

$\perp$ Polarization



$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\tau_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = 1 + \Gamma_{\perp}$$

$$\begin{aligned}\hat{\mathrm{k}}_i &= \sin\theta_i \hat{\mathrm{x}} + \cos\theta_i \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_i &= \hat{\mathrm{y}} \\ \hat{\mathrm{h}}_i &= -\cos\theta_i \hat{\mathrm{x}} + \sin\theta_i \hat{\mathrm{z}}\end{aligned}$$

$$\begin{aligned}\hat{\mathrm{k}}_r &= \sin\theta_r \hat{\mathrm{x}} - \cos\theta_r \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_r &= \hat{\mathrm{y}} \\ \hat{\mathrm{h}}_r &= \cos\theta_r \hat{\mathrm{x}} + \sin\theta_r \hat{\mathrm{z}}\end{aligned}$$

$$\begin{aligned}\hat{\mathrm{k}}_t &= \sin\theta_t \hat{\mathrm{x}} + \cos\theta_t \hat{\mathrm{z}} \\ \hat{\mathrm{e}}_t &= \hat{\mathrm{y}} \\ \hat{\mathrm{h}}_t &= -\cos\theta_t \hat{\mathrm{x}} + \sin\theta_t \hat{\mathrm{z}}\end{aligned}$$

General Plane Wave Solution

$$\vec{\mathsf{E}}_\diamond(\vec{\mathrm{r}}) = E_\diamond\, \hat{\mathrm{e}}_\diamond \exp\left(j\left[\phi - k\hat{\mathrm{k}}_\diamond \cdot \vec{\mathrm{r}}\right]\right) \qquad \vec{\mathsf{H}}_\diamond(\vec{\mathrm{r}}) = \frac{E_\diamond}{\eta}\hat{\mathrm{h}}_\diamond \exp\left(j\left[\phi - k\hat{\mathrm{k}}_\diamond \cdot \vec{\mathrm{r}}\right]\right)$$

$\vec{\mathrm{r}} = x\hat{\mathrm{x}} + y\hat{\mathrm{y}} + z\hat{\mathrm{z}}$ $\quad \eta = \sqrt{\frac{\mu}{\epsilon}}$ $\quad k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_p}$ $\quad \diamond \to i$ (incident) **or** r (reflected) **or** t (transmitted)

Snell's Law of Reflection

$$\theta_i = \theta_r$$

Snell's Law of Refraction

$$\frac{\sin\theta_i}{v_{p1}} = \frac{\sin\theta_t}{v_{p2}} \quad \text{where } v_p = \frac{1}{\sqrt{\epsilon\mu}}$$

Physical Quantities

θ_i	angle of incidence	E	electric field amplitude (V/m)
θ_r	angle of reflection	$\Gamma_{\parallel,\perp}$	reflection coefficient ($\frac{E_r}{E_i}$)
θ_t	angle of transmission	$\tau_{\parallel,\perp}$	transmission coefficient ($\frac{E_t}{E_i}$)
$\hat{\mathrm{e}}$	electric field polarization	η	intrinsic impedance (Ω, Ohms)
$\hat{\mathrm{h}}$	magnetic field polarization	v_p	velocity of propagation (m/s)
$\hat{\mathrm{k}}$	direction of propagation	k	wavenumber (radians/m)
μ	magnetic permeability (H/m)	λ	wavelength (m)
ϵ	electric permittivity (F/m)	f	frequency (Hz)