Name: ______________________________

GTID: ______________________________

ECE 3065: Electromagnetics
TEST 2 (Spring 2005)

- Please read all instructions before continuing with the test.
- This is a **closed** notes, **closed** book, **closed** friend, **open** mind test. On your desk you should only have writing instruments and a calculator.
- Show all work. (It helps me give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.
- Work intelligently – read through the exam and do the easiest problems first. Save the hard ones for last.
- All necessary mathematical formulas are included either in the problem statements or the last few pages of this test.
- You have 75 minutes to complete this examination. When I announce a "last call" for examination papers, I will leave the room in 5 minutes. The fact that I do not have your examination in my possession will not stop me.
- I will not grade your examination if you fail to 1) put your name and GTID number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.
- Have a nice day!

Pledge Signature: ____

I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.

1. **Short Answer Section (20 points)**

(a) ____________

One problem with a multi-mode waveguide is that echoes of the same propagating waveform will arrive at the end of the guide; this effect leads to [*Answer*].

(b) ____________

The [*Answer*] of an RF device is the ratio between its stored energy and its power loss (integrated over a period).

(c) ____________

A [*Answer*] mode in a waveguide has a field solution with a magnetic field component that is always perpendicular to the direction of propagation.

(d) ____________ (1) ____________ (2)

If there are many small-valued waves at the terminals of an antenna moving through space, the received signal envelope will experience [*Answer 1*] fading that follows a [*Answer 2*] distribution.

(e) ____________

A waveguide mode will not propagate if excited below the [*Answer*] frequency.

(f) ____________

The loss of a TE_{10} mode in a rectangular waveguide in dB/m will [*increase/decrease/remain unchanged*] if, while keeping all other variables the same, the conductivity of the metallic walls increases.

(g) ____________

The [*Answer*] of an RF device is the ratio between its center frequency and bandwidth.

(h) ____________

A waveguide mode consists of a collection of plane waves that travel in the z-direction at the same [*Answer*].

(i) ____________

[*Answer*] are the units we use for reporting values of α, the attenuation constant.

2. **Rectangular Waveguide Solution (15 points):** Demonstrate that the electric field solution (shown below) of the TE_{m0} mode of a rectangular waveguide can be written as the sum of two ŷ-polarized plane waves of equal amplitude. Hint: using our standard geometry where ẑ points along the axis of the waveguide, one plane wave would propagate in the direction ($\varphi = 0°$, $\theta = \theta_0$) and the other would propagate in the direction ($\varphi = 180°$, $\theta = \theta_0$).

$$\tilde{\vec{\mathrm{E}}}(x, y, z) = E_{m0} \sin\left(\frac{m\pi x}{a}\right) \hat{\mathrm{y}} \exp(-j\beta z) \, \mathrm{V/m}$$

3. **Radio Link Budget (45 points):** You are deploying an IEEE 802.11b wireless internet access point in the center of the 3rd floor of Van Leer. This link operates at about 2.45 GHz, an unlicensed Industrial-Scientific-Medical (ISM) band. Here are some useful facts about the radio link:

   - The bandwidth of the link is (effectively) 2 MHz.
   - The transmit power into the access point antenna is about 27 dBm.
   - The noise temperature of a typical wireless card in a computer is 180 Kelvin.
   - The longest TR separation distance is 40m to the far corner of the building.
   - Van Leer has a path loss exponent of 7.00.
   - The base station antenna has a gain value of 2 dBi. The smaller, less-efficient computer antennas usually have a gain value of -1 dBi.
   - Reliable transmission at the target data rate of 1 Mbit/sec requires a minimum SINR of 12 dB from access point to computer.

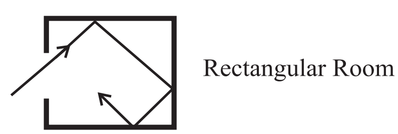
   (a) Based on the above specifications, what is the maximum TR-separation distance that communications can still be supported in this link? (**15 points**)

(b) What percentage of the Shannon Channel Capacity Limit are we achieving when operating at the very fringe of the link in part (a)? (**5 points**)

(c) If you are surfing the web on your laptop in a nearby room with a average, local area received signal strength of -76 dBm, estimate the probability that you are in a small-scale fade that will lead to a link outage. Assume a single-antenna receiver. (**10 points**)

(d) What should you do if the outage in (c) occurs? (**5 points**)

(e) In the room of part (c), the propagation consists of 4 equal-amplitude plane waves that result from the resonance of specular reflections from the four walls of the room (see diagram below). What power does each wave carry individually? (**5 points**)



(f) If we model the room in part (e) as an 8m × 5m × 3m rectangular cavity resonator, make an order-of-magnitude estimate of the number of resonant TE modes supported at 2.45 GHz? (Full credit for anyone within a factor of 10 of the right answer.) (**5 points**)

4. **Circular vs. Rectangular Waveguide (20 points):** You have been hired by small engineering company that specializes in making super-cheap microwave components. The first product that you are asked to design is a new ultra-cheap, air-filled metallic waveguide. Sections of this waveguide are manufactured by taking long rectangular sheets of tin and bending them around a cross-section of your choosing. You must decide between a square cross section or a circular cross section to produce a waveguide with a given low-frequency cut-off.

   (a) Manufacturing cost of the waveguide is proportional to the perimeter of the cross-section. Calculate which cross section is cheaper to build. (**10 points**)

   (b) Calculate which of these cross sections has more single-mode bandwidth (bandwidth between dominant and second-highest mode). (**10 points**)

Cheat Sheet

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \qquad \mu_0 = 4\pi \times 10^{-7} \text{H/m}$$

$$\lambda f = v_p \qquad \omega = 2\pi f \qquad \beta = \frac{2\pi}{\lambda} \qquad D = T v_p$$

Phasor Transform: $A\cos(2\pi f t + \phi) \longrightarrow A\exp(j\phi)$

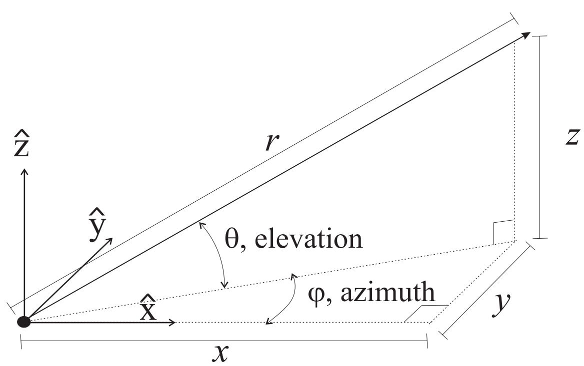
Reverse Transform: $\tilde{x} \longrightarrow \text{Real}\left\{\tilde{x}\exp(j2\pi f t)\right\}$

$$\begin{aligned}\tilde{\vec{\mathsf{E}}}(\vec{\mathrm{r}}) &= E_0 \hat{\mathrm{e}} \exp(j[\phi - k\hat{\mathrm{k}} \cdot \vec{\mathrm{r}}]) \\ \tilde{\vec{\mathsf{H}}}(\vec{\mathrm{r}}) &= H_0 \hat{\mathrm{h}} \exp(j[\phi - k\hat{\mathrm{k}} \cdot \vec{\mathrm{r}}])\end{aligned}$$

$$H_0 = \frac{E_0}{\eta} \qquad \eta = \sqrt{\frac{\mu}{\epsilon}} \qquad v_p = \frac{1}{\sqrt{\mu\epsilon}} \qquad \hat{\mathrm{e}} \times \hat{\mathrm{h}}^* = \hat{\mathrm{k}} \qquad \hat{\mathrm{h}} = (\hat{\mathrm{k}} \times \hat{\mathrm{e}})^*$$

$$-\hat{\mathrm{k}} = \cos\varphi\cos\theta\,\hat{\mathrm{x}} + \sin\varphi\cos\theta\,\hat{\mathrm{y}} + \sin\theta\,\hat{\mathrm{z}}$$

$$\theta = \tan^{-1}\frac{-k_z}{\sqrt{k_x^2 + k_y^2}} \qquad \varphi = \tan^{-1}\frac{k_y}{k_x} \quad (\text{add } \pi \text{ if } k_x > 0)$$



Cross Product: $\vec{\mathrm{a}} \times \vec{\mathrm{b}} = \det \begin{vmatrix} \hat{\mathrm{x}} & \hat{\mathrm{y}} & \hat{\mathrm{z}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$

Euler's Relationships: $\cos x = \dfrac{\exp(jx) + \exp(-jx)}{2} \qquad \sin x = \dfrac{\exp(jx) - \exp(-jx)}{j2}$

Link Budget Formulas

$$\mathrm{P}_R = \mathrm{P}_T + G_T + G_R - 20\log_{10}(r) - 20\log_{10}(f) + 20\log_{10}(c/4\pi) - \text{Extra Loss}$$

$$\mathrm{P}_R = \mathrm{P}_T + G_T + G_R - 20\log_{10}(f) + 20\log_{10}(c/4\pi) - 10n\log_{10}\left(\frac{r}{1\text{m}}\right) + X_\sigma$$

$$P_N = kTB \qquad k = 1.3807 \times 10^{-23} \text{ J K}^{-1} \qquad \text{SINR} = \frac{P_R}{P_N + P_I}$$

Shannon Limit: $C = B \log_2 (1 + \text{SINR}) \qquad \log_2 x = \frac{\ln x}{\ln 2}$

Waveguide Formulas

$$v_g = \frac{1}{\sqrt{\epsilon\mu}} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \qquad \lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

Loss in a Rectangular Waveguide

$$(\alpha_c)_{\text{TE}_{10}} = \frac{1}{\eta b} \sqrt{\frac{\pi f \mu_c}{\sigma_c [1 - (f_c/f)^2]}} \left[1 + \frac{2b}{a} \left(\frac{f_c}{f}\right)^2\right] \qquad \alpha_d = \frac{\sigma\eta}{2\sqrt{1 - (f_c/f)^2}}$$

Circular Waveguide: $(f_c)_{\text{TM}_{01}} = \frac{0.383}{a\sqrt{\epsilon\mu}} \qquad (f_c)_{\text{TE}_{11}} = \frac{0.293}{a\sqrt{\epsilon\mu}}$

Planar Waveguide: $(f_c)_m = \frac{m}{2a\sqrt{\mu\epsilon}}$

Rectangular Waveguide: $(f_c)_{mn} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

Rectangular Cavity Resonator: $f_{mnp} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$

Small-Scale Fading Formulas

Envelope: $f_r(r) = \frac{2r}{P_T} \exp\left(-\frac{r^2}{P_T}\right) \mathrm{u}(r) \qquad$ Power: $f_P(p) = \frac{1}{P_T} \exp\left(-\frac{p}{P_T}\right) \mathrm{u}(p)$

$$P_T \text{ (units of Volts}^2) = \sum_{i=1}^{N} V_i^2$$