Name: $\qquad$

GTID: $\qquad$

## ECE 3065: Electromagnetics

TEST 2 (Spring 2009)

- Please read all instructions before continuing with the test.
- This is a closed notes, closed book, closed friend, open mind test. On your desk you should only have writing instruments and a calculator.
- Show all work. (It helps me give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.
- Work intelligently - read through the exam and do the easiest problems first. Save the hard ones for last.
- All necessary mathematical formulas are included either in the problem statements or the last few pages of this test.
- You have 75 minutes to complete this examination. When I announce a "last call" for examination papers, I will leave the room in 5 minutes. The fact that I do not have your examination in my possession will not stop me.
- I will not grade your examination if you fail to 1) put your name and GTID number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.
- Have a nice day!

Pledge Signature:
$I$ acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.

1. Path Loss Modeling: (15 points) A wireless ad-hoc sensor network consists of many small 2.45 GHz transceiver nodes that communicate with one another using 100 mW of transmit power and 0 dBi transmit and receive antennas. If the minimum received power for link operation is -97 dBm , what kind of radio environment (i.e. what is the maximum path loss exponent?) that results in at least 100 m of range between sensors?
2. Diffraction: (25 points) Below is a terrain diffraction problem for a horizontally-polarized UHF broadcast digital TV antenna tower. Calculate the received power to the home if the transmitter EIRP is 5000 Watts, the receiver antenna has a gain of 10 dBi , and the frequency of the broadcast is 500 MHz . Estimate the effect of the peak obstruction by treating it as a diffracting metal screen.

3. Poynting-Vector (25 points): Below is a generic formula for the total field solution of the $\mathrm{TE}_{10}$ mode of a rectangular waveguide with dimensions $a \times b$ (in $x$ and $y$, respectively). Use these formulas to estimate, in Watts, the mode's total power propagating in the z-direction in terms of the constants in the equation below. (Hint: Poynting vector is in units of Watts $/ \mathrm{m}^{2}$, not Watts.)

$$
\begin{aligned}
\tilde{E}_{x}(x, y, z) & =0 \\
\tilde{E}_{y}(x, y, z) & =\frac{-j \omega \mu \pi}{b h^{2}} H_{0} \sin \left(\frac{\pi}{a} x\right) \exp (-j \beta z) \\
\tilde{H}_{x}(x, y, z) & =\frac{j \pi \beta}{b h^{2}} H_{0} \sin \left(\frac{\pi}{a} x\right) \exp (-j \beta z) \\
\tilde{H}_{y}(x, y, z) & =0 \\
\tilde{H}_{z}(x, y, z) & =H_{0} \cos \left(\frac{\pi}{a} x\right) \exp (-j \beta z)
\end{aligned}
$$

4. Waveguide Propagation (15 points): You are using circular HVAC ducts at 915 MHz to provide wireless LAN coverage to your office environment. At this frequency, the access point antenna excites 6 modes in the HVAC waveguide antenna with the following cut-off frequencies: $400,620,730,780,820$, and 880 MHz . If the farthest coverage point lies in a room 100 m of duct-length away from the access point, what is the maximum dispersion (difference in time between earliest arriving signal and latest arriving signal) that this HVAC waveguide introduces into the transmitted signal?

## 5. Transmission Line Resonator: (20 points)

You must design a critically-coupled stripline resonator at 4 GHz coupled to a $50 \Omega$ line. The velocity of propagation on the stripline is $2.0 \times 10^{8} \mathrm{~m} / \mathrm{s}$.

(a) The ohmic loss of the transmission line results in an attenuation of $0.1 \mathrm{~dB} / \mathrm{m}$. What is the value of $\alpha$ in $\mathrm{Np} / \mathrm{m}$ for this line? ( 5 points)
(b) What length $D$ (in centimeters) should the stripline resonator be? (5 points)
(c) What should the value of the coupling capacitor $C$ be? (5 points)
(d) If this whole circuit is used as a filter, what will be its bandwidth in MHz ? (5 points)

## Cheat Sheet

$$
\begin{gathered}
\epsilon_{0}=8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m} \quad \mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m} \quad c=3.0 \times 10^{8} \mathrm{~m} / \mathrm{s} \\
\lambda f=v_{p} \quad \omega=2 \pi f \quad \beta=\frac{2 \pi}{\lambda} \quad D=T v_{p}
\end{gathered}
$$

## Link Budget Formula (Logarithmic)

$$
\mathrm{P}_{R}=\mathrm{P}_{T}+G_{T}+G_{R}-20 \log _{10}(4 \pi / \lambda)-10 n \log _{10}(r)
$$

## Waveguide Formulas

$$
v_{g}=\frac{1}{\sqrt{\epsilon \mu}} \sqrt{1-\left(\frac{f_{c}}{f}\right)^{2}} \quad \lambda_{g}=\frac{\lambda}{\sqrt{1-\left(\frac{f_{c}}{f}\right)^{2}}}
$$

Rectangular Waveguide: $\left(f_{c}\right)_{m n}=\frac{1}{2 \sqrt{\mu \epsilon}} \sqrt{\left(\frac{m}{a}\right)^{2}+\left(\frac{n}{b}\right)^{2}}$

$$
\text { Poynting Vector: } \overrightarrow{\mathrm{S}}=\frac{1}{2} \operatorname{Real}\left\{\tilde{\overrightarrow{\mathrm{E}}} \times \tilde{\overrightarrow{\mathrm{H}}}^{*}\right\}
$$

## Unloaded Resonance of a Transmission Line

Open-Open/Short-Short: $D=n \frac{\lambda}{2} \quad$ Open-Short: $D=\left(n+\frac{1}{2}\right) \frac{\lambda}{2} \quad$ Ring: $D=n \lambda$

## Design of a Critically-Coupled Transmission Line Resonator

$$
\begin{gathered}
D=\frac{1}{\beta}\left[\pi-\tan ^{-1}(\sqrt{\alpha D})\right] \quad \omega_{0} C Z_{0}=-\tan (\beta D) \quad Q_{U}=\frac{\beta}{2 \alpha} \\
\text { Loss }(\mathrm{dB} / \mathrm{m})=8.7 \alpha \quad \alpha=\alpha_{c}+\alpha_{d} \\
Q=\frac{\text { Total Energy Stored }}{\text { Power Lost }}=\frac{\text { Resonant/Center Frequency }}{\text { Bandwidth }} \quad \frac{1}{Q_{L}}=\frac{1}{Q_{U}}+\frac{1}{Q_{e x t}} \\
\left.\begin{array}{|l|lll}
\text { SoLUTION TYPE } & \text { DIFFRACTION COEFFICIENT } \\
\hline \text { Sommerfeld- } \perp & D_{\perp}\left(\phi, \phi_{i}\right) & =\frac{-\exp \left(-j \frac{\pi}{4}\right)}{2 \sqrt{2 \pi}}\left[\sec \left(\frac{\phi-\phi_{i}}{2}\right)-\sec \left(\frac{\phi+\phi_{i}}{2}\right)\right] \\
\text { Sommerfeld- } \| & D_{\|}\left(\phi, \phi_{i}\right) & =\frac{-\exp \left(-j \frac{\pi}{4}\right)}{2 \sqrt{2 \pi}}\left[\sec \left(\frac{\phi-\phi_{i}}{2}\right)+\sec \left(\frac{\phi+\phi_{i}}{2}\right)\right]
\end{array}\right] \\
P_{R}=\operatorname{EIRP} \frac{G_{R} \lambda^{3}\left|D\left(\phi_{2}, \phi_{1}\right)\right|^{2}}{32 \pi^{3} r_{1} r_{2}\left(r_{1}+r_{2}\right)} \\
\text { Cross Product: } \vec{a} \times \vec{b}=\operatorname{det}\left|\begin{array}{ccc}
\hat{x} & \hat{y} & \hat{z} \\
a_{x} & a_{y} & a_{z} \\
b_{x} & b_{y} & b_{z}
\end{array}\right|=\|\vec{a}\|\|\vec{b}\| \sin \theta_{a b} \hat{n}
\end{gathered}
$$

