Solutions to ECE 3065 Practice Test 1

1. **Short Answer Section**

   (a) lossless

   (b) half

   (c) -1

   (d) isotropic

   (e) homogeneous

   (f) linear

   (g) source-free

   (h) equi-phase (1), λ (2)

   (i) False

   (j) magnetic

   (k) homogeneous (or uniform)

   (l) inhomogeneous (or evanescent)

   (m) evanescent

   (n) polarization

2. **Descriptive Answer Section**
   Write a **concise** answer to each question in the spaces provided beneath each problem statement. **Note:** Correct answers that are extremely verbose will be penalized.

   (a) **Plane Wave Polarization:**

      a. Real, linear combinations of polarizations (c) and (d) may be used to describe any arbitrary linear polarization.

      b. Polarization (b) is left-handed circular.

      c. Polarization (a) is horizontal.

d. Elliptical polarization occurs when the horizontal and vertical components are out-of-phase. Circular polarization is a special case of elliptical polarization.

(b) **Optics Experiment:** Since the dielectric material is not magnetic, it will have a Brewster angle for parallel polarization where the reflection coefficient is zero. The perpendicular polarization, however, has no Brewster angle. Therefore, if you allow the laser beam of unknown polarization to strike the dielectric at the Brewster angle, only the perpendicular polarization will be present in the reflected wave.

(c) **Transmission Line Analogy:**

Transmission Line	Plane Wave
Voltage, V	Electric field, $\vec{\mathrm{E}}$
Current, I	Magnetic field, $\vec{\mathrm{H}}$
Impedance, Z_0	Impedance, η
Reflection Coefficient, Γ_L	Reflection Coefficients, $\Gamma_{\perp,\parallel}$
per unit length Inductance, L	permeability, μ
per unit length Capacitance, C	permittivity, ϵ
wavenumber, β	wavenumber, k
VSWR, $\frac{V_{max}}{V_{min}}$	Standing Wave Ratio, $\frac{E_{\max}}{E_{\min}}$

3. **Circular Polarization:** We start with the expression for circular polarization:

$$\tilde{\vec{\mathrm{E}}}_i(\vec{\mathrm{r}}) = E_i\left(\frac{1}{\sqrt{2}}\hat{\mathrm{x}} + \frac{j}{\sqrt{2}}\hat{\mathrm{y}}\right)\exp(-jk\hat{\mathrm{z}}\cdot\vec{\mathrm{r}})$$

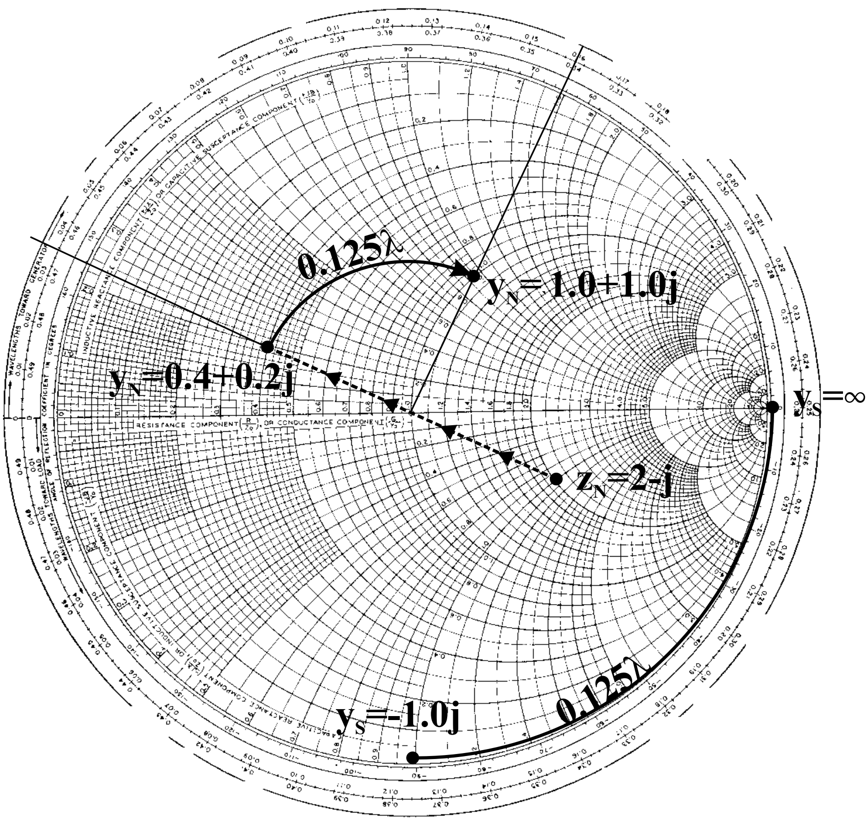
At the PEC, the reflection coefficients for both $\perp$ and $\parallel$ components is -1. The reflected wave vector will travel in the -ẑ direction. Thus we can write this reflected wave as

$$\begin{aligned}\tilde{\vec{\mathrm{E}}}_r(\vec{\mathrm{r}}) &= E_i\left(-\frac{1}{\sqrt{2}}\hat{\mathrm{x}} - \frac{j}{\sqrt{2}}\hat{\mathrm{y}}\right)\exp(+jk\hat{\mathrm{z}}\cdot\vec{\mathrm{r}}) \\ &= E_i\left(\frac{1}{\sqrt{2}}\hat{\mathrm{x}} + \frac{j}{\sqrt{2}}\hat{\mathrm{y}}\right)\exp(j[\pi + k\hat{\mathrm{z}}\cdot\vec{\mathrm{r}}])\end{aligned}$$

Note that y-polarization leads the x-polarization by 90° just like the original wave, but the direction of propagation is reversed. Thus, the rotation is the same but our opposite (left) hand now curls around the rotation to point in the direction of propagation.

4. **Parallel Stub Match:**

Start with a normalized impedance of 2-j. This converts to a normalized admittance of 0.4+0.2j. We see from the Smith Chart that the real part of the equivalent admittance becomes 1 at $d_M = 0.125\lambda$ and the imaginary portion is $+1.0j$. Thus, we need a short-circuit stub (normalized impedance of ∞) of length $d_S = 0.125\lambda$ to achieve a cancelling $-1.0j$ in parallel. See chart below for graphical analysis.



5. **Plane Wave Equation:** A homogeneous plane wave is traveling in a simple, sourceless dielectric medium in the (ϕ, θ) direction. The phasor-form E-field and H-field expressions are given by the following system of equations:

$$\tilde{\vec{E}}(\vec{r}) = \overbrace{(E_x\hat{\mathrm{x}} + E_y\hat{\mathrm{y}} + E_z\hat{\mathrm{z}})}^{E_\mathrm{o}\hat{\mathrm{e}}} \exp\Big(j[\phi - k\hat{\mathrm{k}}\cdot\vec{r}]\Big)$$

$$\tilde{\vec{H}}(\vec{r}) = \underbrace{(H_x\hat{\mathrm{x}} + H_y\hat{\mathrm{y}} + H_z\hat{\mathrm{z}})}_{\frac{E_\mathrm{o}}{\sqrt{\mu/\epsilon}}\hat{\mathrm{h}}} \exp\Big(j[\phi - k\hat{\mathrm{k}}\cdot\vec{r}]\Big)$$

$$\hat{\mathrm{e}} \times \hat{\mathrm{h}}^* = \hat{\mathrm{k}} \quad \hat{\mathrm{k}} = \cos\phi\sin\theta\hat{\mathrm{x}} + \sin\phi\sin\theta\hat{\mathrm{y}} + \cos\theta\hat{\mathrm{z}} \quad k = \frac{2\pi}{\lambda} = 2\pi f\sqrt{\mu\epsilon} \quad \vec{r} = x\hat{\mathrm{x}} + y\hat{\mathrm{y}} + z\hat{\mathrm{z}}$$

Answer the questions below based on these equations:

(a) Circle E_o.

(b) Box $\hat{\mathrm{h}}$.

(c) In the space below, show that the x-component of the E-field satisfies the scalar wave equation.

$$
\begin{aligned}
(\nabla^2 + k^2)(\hat{\mathrm{x}} \cdot \tilde{\vec{E}}) &= 0 \\
(\nabla^2 + k^2) E_x \exp\left(j[\phi - k\hat{\mathrm{k}} \cdot \vec{r}]\right) &= \\
E_x \exp(j\phi) \left[\nabla^2 \exp\left(-jk\hat{\mathrm{k}} \cdot \vec{r}\right) + k^2 \exp\left(-jk\hat{\mathrm{k}} \cdot \vec{r}\right)\right] &= \\
E_x \exp(j\phi) \left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \exp\left(-jk\hat{\mathrm{k}} \cdot \vec{r}\right) + k^2 \exp\left(-jk\hat{\mathrm{k}} \cdot \vec{r}\right)\right] &= \\
E_x \exp\left(j[\phi - k\hat{\mathrm{k}} \cdot \vec{r}]\right) \left[-k^2 \underbrace{\left(\cos^2\phi \sin^2\theta + \sin^2\phi \sin^2\theta + \cos^2\theta\right)}_{\hat{\mathrm{k}} \cdot \hat{\mathrm{k}} = 1} + k^2\right] &= \\
E_x \exp\left(j[\phi - k\hat{\mathrm{k}} \cdot \vec{r}]\right) \cdot 0 &= \\
0 &=
\end{aligned}
$$