## ECE 3065: Electromagnetics Applications

## TEST 1 Solutions (Spring 2004)

## 1. Short Answer Section (24 points)

(a) lossy
(b) $100 \Omega$
(c) linear (1), isotropic (2), homogeneous (3), source-free (4)
(d) boundary
(e) short circuit (1), open circuit (2), inductor (3), capacitor (4)
(f) fiber optics

## 2. Descriptive Answer Section

Note: Students were not penalized for incorrect answers on this section due to a point of confusion that was apparent in the problem statement.
(a) Glare: From the graphs we can see that $\Gamma_{\perp} \geq \Gamma_{\|}$for all values of dielectric permittivity. Thus, horizontal polarization (resulting from perpendicular incidence) will likely dominate in the reflected wave, regardless of incident angle.
(b) Polarizing Sunglasses: We know that a medium that has conductivity will result in lossy plane waves (waves that attenuate exponentially in the direction of propagation). Since this medium has conductivity for electric field in the $z$-direction, plane waves with vertical polarization will be lossy as they travel through the lens. Waves with horizontal polarization, however, see a lossless medium. The vertical polarization is the one filtered out.
3. Splice Match: You can get 10 really easy points on this problem by recognizing that all splice problems we study in this class are quarter-wavelength. Thus, $D_{M}=0.25 \lambda$. In order to use this quarter-wavelength match, the line must be placed a distance $D_{S}=0.088 \lambda$ from the load to make the equivalent impedance purely real (see Smith Chart calculation in Figure 1). This section of transmission line should have an impedance of $Z_{M}=\sqrt{(50 \Omega)(350 \Omega)}=132 \Omega$. According to our formula, this implies that:

$$
132 \Omega=\frac{d}{w} \sqrt{\frac{\mu_{0}}{16 \epsilon_{0}}} \quad \longrightarrow \quad w=0.71 \mathrm{~mm}
$$



Figure 1: Smith chart calculation for the quarter-wavelength splice.

## 4. Circular-Polarized Radio Wave:

(a) We recognize from the table on the back that this wave is already broken into nice parallel and perpendicular components:

$$
\tilde{\overrightarrow{\mathrm{E}}}_{i}(\overrightarrow{\mathrm{r}})=\frac{E_{i}}{\sqrt{2}}(\underbrace{\cos \theta_{i} \hat{\mathrm{x}}-\sin \theta_{i} \hat{\mathrm{z}}}_{\hat{\mathrm{e}}_{\|}}+\underbrace{j \hat{\mathrm{y}}}_{\hat{\mathrm{e}}_{\perp}}) \exp \left(-j k\left[\sin \theta_{i} \hat{\mathrm{x}}+\cos \theta_{i} \hat{\mathrm{z}}\right] \cdot \overrightarrow{\mathrm{r}}\right)
$$

Following directly from the tables, we can write

$$
\tilde{\overrightarrow{\mathrm{E}}}_{r}(\overrightarrow{\mathrm{r}})=\frac{E_{r}}{\sqrt{2}}\left(\cos \theta_{i} \hat{\mathrm{x}}+\sin \theta_{i} \hat{\mathrm{z}}+j \hat{\mathrm{y}}\right) \exp \left(-j k\left[\sin \theta_{i} \hat{\mathrm{x}}-\cos \theta_{i} \hat{\mathrm{z}}\right] \cdot \overrightarrow{\mathrm{r}}\right)
$$

Furthermore, we know that grazing reflection coefficient for both polarizations is -1 , so we finally write:

$$
\tilde{\overrightarrow{\mathrm{E}}}_{r}(\overrightarrow{\mathrm{r}})=-\frac{E_{i}}{\sqrt{2}}\left(\cos \theta_{i} \hat{\mathrm{x}}+\sin \theta_{i} \hat{\mathrm{z}}+j \hat{\mathrm{y}}\right) \exp \left(-j k\left[\sin \theta_{i} \hat{\mathrm{x}}-\cos \theta_{i} \hat{\mathrm{z}}\right] \cdot \overrightarrow{\mathrm{r}}\right)
$$

(b) The easiest way to do this section is to evaluate the incident and reflected fields on the surface of the dielectric (I did not penalize heavily for using incorrect answers from part a). The surface fields lie at $\overrightarrow{\mathrm{r}}_{S}=x \hat{\mathrm{x}}+y \hat{\mathrm{y}}+0 \hat{\mathrm{z}}$. Plugging this in and adding up the fields produces a much-simplified result:

$$
\tilde{\overrightarrow{\mathrm{E}}}\left(\overrightarrow{\mathrm{r}}_{s}\right)=\tilde{\overrightarrow{\mathrm{E}}}_{i}\left(\overrightarrow{\mathrm{r}}_{s}\right)+\tilde{\overrightarrow{\mathrm{E}}}_{r}\left(\overrightarrow{\mathrm{r}}_{s}\right)=-E_{i} \sqrt{2} \sin \theta_{i} \hat{\mathrm{z}} \exp \left(-j k x \sin \theta_{i}\right)
$$

This problem was not easy and I was encouraged by the number of people that got it all or nearly-all correct.

