ECE 3065: Electromagnetics
Solutions to TEST 1 (Spring 2006)

1. **Short Answer Section (14 points)**

   (a) series

   (b) horizontal (perpendicular also accepted)

   (c) index of refraction (permittivity also accepted)

   (d) eikonal

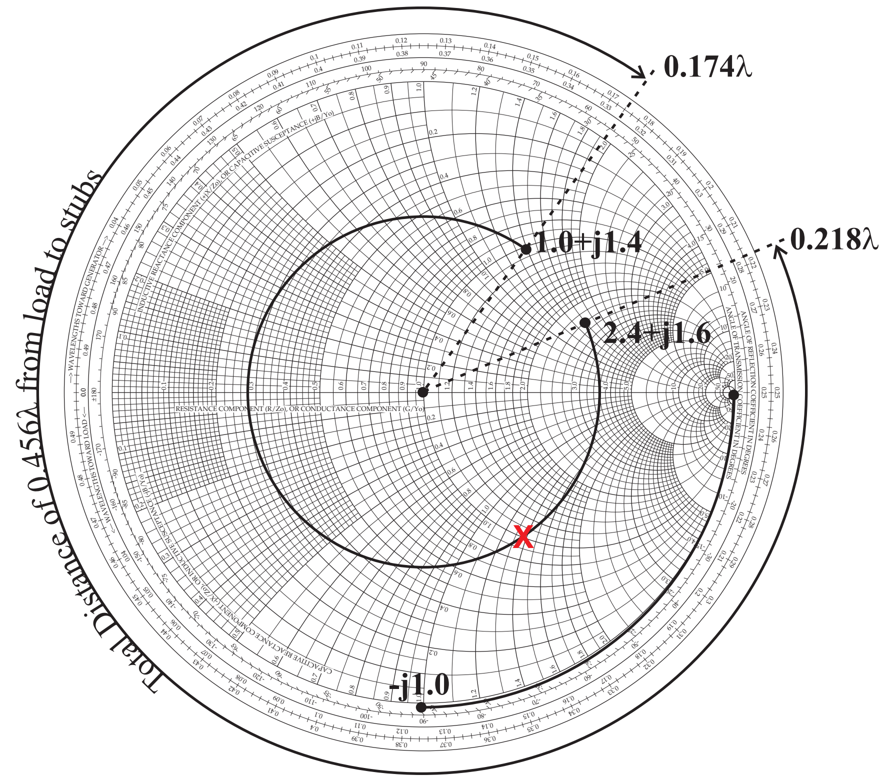
   (e) linear (1) isotropic (2) source-free (3)

2. **Mystery RF Circuit Structure (11 points):**

   This is an inductor (check the Smith chart: an open-circuited stub transforms to a positive imaginary impedance for lengths between 0.25λ and 0.5λ). Anyone who even brushed up against the following benefits in their description was given credit: 1) the squiggle is cheaper than a discrete inductor, 2) less lossy, 3) higher Q, 4) requires less packaging/assembly, 5) more stable/predictable inductor value.

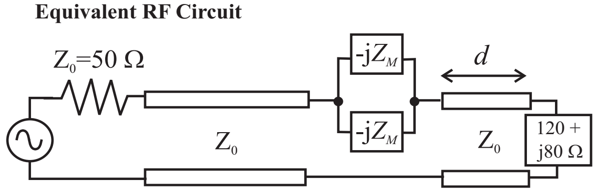
3. **Novel Matching Scheme (30 points):**

   Start by plotting the normalized load impedance, $2.4 + j1.6$, on the Smith chart. The stubs are in series with this load line (though parallel with each other), so this may be worked with impedances. There are two possible points on the Smith chart where the equivalent impedance of the load line appears to be $1.0 \pm jX$ (remember Homework 3):

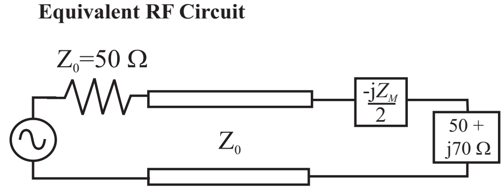


We must choose the second one because the stubs that we will use to remove the imaginary portion are *capacitive* because they are fixed, short lengths. Therefore, the equivalent impedance of the load line 0.456λ down the line is $50 + j70\Omega$, unnormalized. The two $\lambda/8$ open-circuit stub lines each have an equivalent impedance of $-jZ_m$; collectively, the two stubs behave as $-j\frac{Z_m}{2}$ in the circuit:

Equivalent RF Circuit



Equivalent RF Circuit



Thus, $Z_m = 140\Omega$ to remove the $+j70\Omega$ when placed in series.

4. **Slant-Polarized Radio Wave (45 points):**

   (a) Write the corresponding magnetic field phasor that completes this plane wave solution. (**15 points**)

$$\tilde{\vec{\mathrm{H}}}_i(\vec{\mathrm{r}}) = \frac{10}{377\sqrt{2}}\left(-\frac{\sqrt{2}}{4}\hat{\mathrm{x}} + \frac{\sqrt{2}}{2}\hat{\mathrm{y}} + \frac{\sqrt{6}}{4}\hat{\mathrm{z}}\right)\exp\left(-j20\left[\frac{\sqrt{3}}{2}\hat{\mathrm{x}} + \frac{1}{2}\hat{\mathrm{z}}\right]\cdot\vec{\mathrm{r}}\right)\ \mu\mathrm{V/m}$$

   (b) What is the wavelength and angle-of-incidence for this particular plane wave? (Measure elevation angle from the *horizon*.) (**10 points**)

$$\lambda = \frac{\pi}{10} \qquad \theta_i = \sin^{-1}\frac{k_z}{k_x} = 30^\circ$$

Angles-of-incidence of -30°, -60°, and $+60^\circ$ were all accepted, depending on the convention you used in your calculation and coordinate system.

(c) Show that the angle of incidence for this wave is actually a special case that we studied in class. What is the name for this angle? (**5 points**)

$$\|\text{-Brewster Angle: } \theta_B = \sin^{-1}\frac{1}{\sqrt{1+\epsilon_1/\epsilon_2}} = \sin^{-1}\frac{1}{\sqrt{1+\frac{1}{3}}} = \sin^{-1}\frac{\sqrt{3}}{2} = 60^\circ$$

as measured from the surface normal, 30° as measured from the horizon.

(d) Write a numerical expression for the *reflected electric* field in this problem. (**15 points**)

$$\tilde{\vec{\mathrm{E}}}_r(\vec{\mathrm{r}}) = -5\hat{\mathrm{y}}\exp\left(-j20\left[\frac{\sqrt{3}}{2}\hat{\mathrm{x}} - \frac{1}{2}\hat{\mathrm{z}}\right]\cdot\vec{\mathrm{r}}\right)\ \mu\mathrm{V/m}$$

This is the Brewster angle, so $\Gamma_{\|} = 0$. The reflection coefficient for $\perp$ is *not* 0 – it evaluates to $-\frac{1}{2}$.