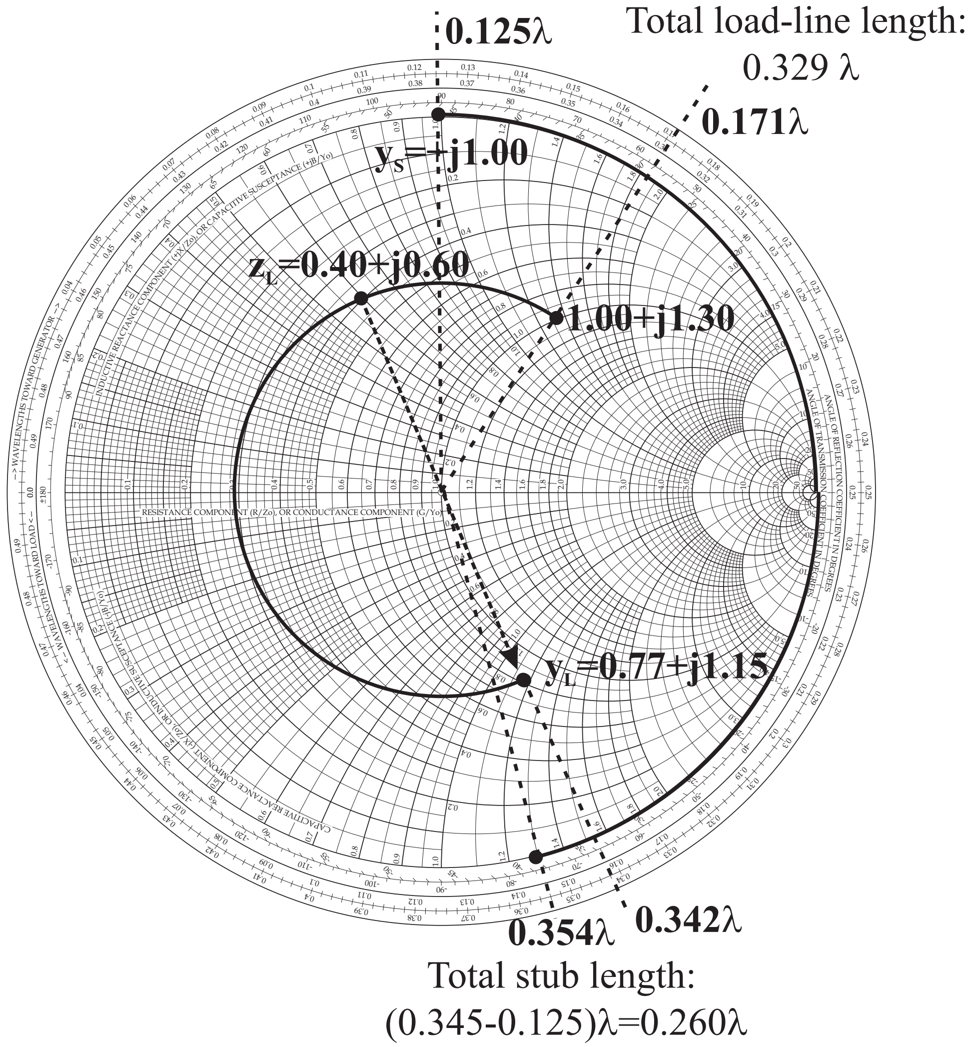
ECE 3065: Electromagnetics
Solutions to TEST 1 (Spring 2009)

1. **Parallel Stub Matching (25 points):**



This problem follows the same formulaic approach of all parallel stub problems. Follow the algorithm:

(a) Calculate the normalized load impedance ($z_L = 0.40 + j0.60$) and plot on the Smith Chart.

(b) Rotate the load point 180° about the center to achieve the normalized *admittance* ($y_L = 0.77 - j1.15$).

(c) Translate this point down the transmission line until the equivalent load admittance has a matched real component. This occurs after a distance of $d = 0.329\lambda$, which results in a 1.0 + j1.3 effective load.

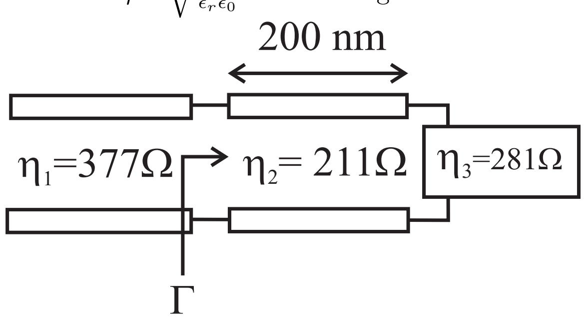
(d) Since our stub terminates in a capacitor, we start out with a normalized admittance of $+j1.0$ (not on the $+j0.0$ point, which would be the admittance of an open circuit).

(e) Translate the stub down a length of line until the effective stub admittance is $-j1.3$ to cancel the reactive component of our load at point d. This occurs after a rotation of $w = 0.220\lambda$.

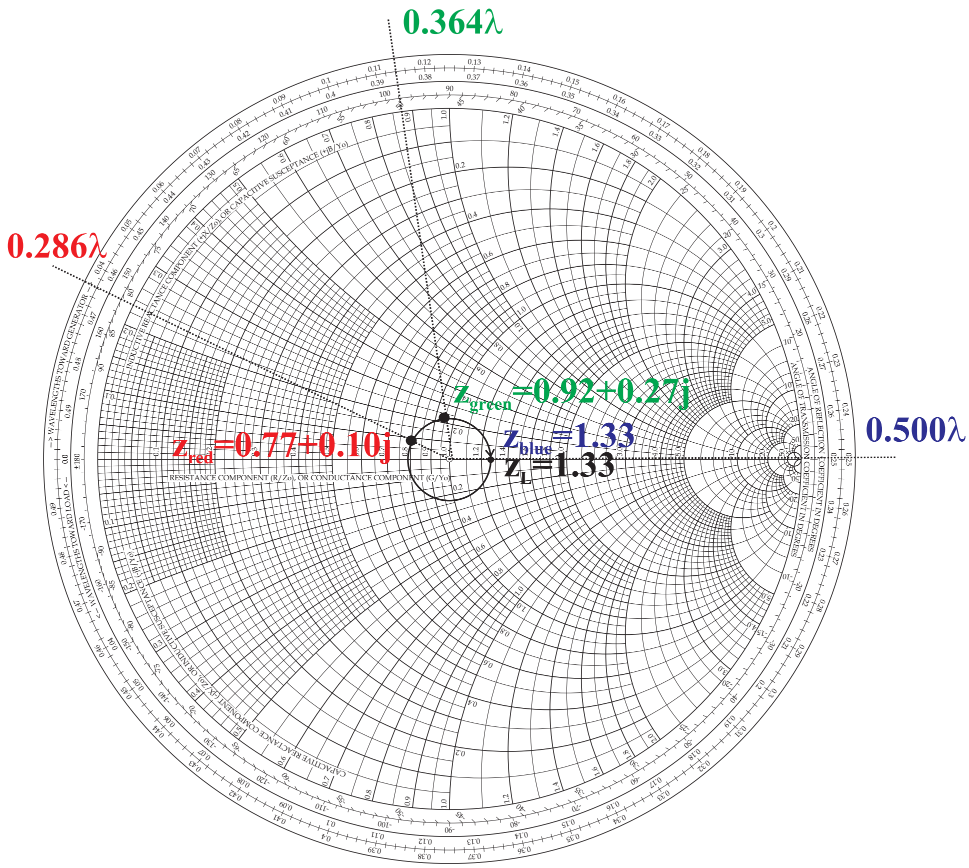
This clever maneuver does save some space compared to an open circuit stub design, which would have the same design except we would be required to start at $+j0.0$ – introducing an extra eight-wavelength to the stub length for a total of $w = 0.345\lambda$.

2. **Newton's Rings (25 points):**

Our equivalent "circuit" for finding the reflection coefficient at the surface of the oil is shown below, keeping in mind that $\eta = \sqrt{\frac{\mu_0}{\epsilon_r \epsilon_0}}$ for non-magnetic media:



The reflection coefficient is found by plotting the normalized impedance $z_L = 281/211 = 1.33$ and rotating it 200nm. Keep in mind for each different wavelength of light (which corresponds to a different frequency), the "electrical" length of this oil section is different than the "physical" length of the medium. For the red light component (700 nm), this load only rotates 0.286λ around the Smith Chart; for the green light component (550 nm), the rotation is 0.364λ; for the blue light component (400 nm), the rotation is 0.5λ – exactly one revolution around the Smith Chart. The equivalent impedances at the oil interface for these three cases are plotted on the Smith Chart and result in total reflection coefficients of $-0.305 + 0.128j$ (red), $-0.397 + 0.053j$ (green), and -0.146 (blue).



Given this model, white light incident onto this surface will have a color-selective reflection-based filter that absorbs some colors efficiently and reflects others back to your eye. Since this effect is angle-dependent and surface tension effects keep the oil from uniform thickness on a water puddle, different parts of the puddle will reflect different color combinations. The results are the tinted pastel rings on the surface of oily water – a phenomenon known as *Newton's rings*.

3. **Satellite Radio Wave Propagation (50 points):**

(a) The magnetic field is

$$\tilde{\vec{\mathrm{H}}}_i(\vec{\mathrm{r}}) = 13.3\hat{\mathrm{y}} \exp\left(-j50\left[\frac{1}{\sqrt{2}}\hat{\mathrm{x}} + \frac{1}{\sqrt{2}}\hat{\mathrm{z}}\right]\cdot\vec{\mathrm{r}}\right) \text{ nA/m}$$

(b) This wave is arriving at an incident angle of 45°, which, thanks to the kindness of your instructor, is the same angle regardless of whether you measure from the z-axis (electromagnetics convention) or the xy-plane (radiowave propagation convention). Measured from the x-axis, the wave is arriving from an azimuthal angle of 180°.

(c) Since $k = 50 = 2\pi/\lambda$, then $\lambda = 0.126$m. This corresponds to a frequency of 2.39 GHz.

(d) The fresnel reflection coefficient is 0.146 at this angle; power density will be proportional to $|\gamma|^2$, which means that 2.1% of the power is reflected from the surface.

(e) The power density of the plane wave is $\frac{E_o^2}{2\eta_0}$ or 33.2 femtoWatts/m^2. If the dish has an area of πm^2, then a total of 104 femtoWatts will be collected.