ECE 3065: Electromagnetics
Solutions to TEST 2 (Spring 2005)

1. **Short Answer Section (20 points)**

   (a) dispersion

   (b) Q-factor

   (c) TM

   (d) small-scale (1) Rayleigh (2)

   (e) cut-off

   (f) decrease

   (g) Q-factor

   (h) velocity

   (i) Nepers/m

2. **Rectangular Waveguide Solution (15 points):** Using our standard geometry where $\hat{z}$ points along the axis of the waveguide, one plane wave would propagate in the direction ($\varphi = 0°$, $\theta = \theta_0$) and the other would propagate in the direction ($\varphi = 180°$, $\theta = \theta_0$):

$$\begin{aligned}
\tilde{\vec{E}}(x,y,z) &= \underbrace{E_0 \exp\left(-jk\left[x\cos\theta_0 + z\sin\theta_0\right]\right)\hat{y}}_{\text{Plane Wave 1}} - \underbrace{E_0 \exp\left(-jk\left[-x\cos\theta_0 + z\sin\theta_0\right]\right)\hat{y}}_{\text{Plane Wave 2}} \\
&= E_0 \exp\left(-jkz\sin\theta_0\right)\left[\exp\left(-jkx\cos\theta_0\right) - \exp\left(jkx\cos\theta_0\right)\right]\hat{y} \\
&= -j2E_0 \exp\left(-jkz\sin\theta_0\right)\left[\frac{\exp\left(jkx\cos\theta_0\right) - \exp\left(-jkx\cos\theta_0\right)}{j2}\right]\hat{y} \\
&= \underbrace{-j2E_0}_{E_{m0}}\sin(\underbrace{k\cos\theta_0}_{\frac{m\pi}{a}}x)\exp(-j\underbrace{k\sin\theta_0}_{\beta}z)\hat{y}
\end{aligned}$$

   Compare this to the TE_{10} solution:

$$\tilde{\vec{E}}(x,y,z) = E_{m0}\sin\left(\frac{m\pi x}{a}\right)\hat{y}\exp(-j\beta z)\ \text{V/m}$$

3. **Radio Link Budget:**

   (a) First let's calculate the noise power in the receiver:

$$P_N = kTB = (1.3807\times10^{-23}\ \text{J K}^{-1})(180K)(2\times10^6 Hz) = -4.907\times10^{-15}\ \text{W} \rightarrow -113.0\ \text{dBm}$$

The received signal level must be at least 12 dB above this threshold. Plugging all of the values into the link budget equation with a path loss exponent of 7.0 results in a distance of 18.2 meters. Common mistakes in this problem included confusing dBW with dBm, using a path loss coefficient of 2.0, and trying to do something with the Shannon Limit instead.

(b) This is simple plug-and-chug with the 12 dB minimum signal-to-noise ratio – just remember to convert to linear scale before plugging it in:

Shannon Limit: $C = B\log_2(1+\text{SINR}) = (2\times10^6 \text{ Hz})\log_2(1+10^{12/10}) = 8.15$ Mbit/s

The stated operating bit rate of 2 Mbit/s is only 12.3% of this value.

(c) Another way to formulate this question: if the average power of this local area is -76 dBm, what is the probability of dropping below the minimum threshold (-101 dBm from part (a) – I did not penalize students for using a bogus answer from part (a) in an otherwise correct analysis):

$$\Pr\{P \le P_{\min}\} = \int_0^{P_{\min}=10^{-101/10}} \frac{1}{10^{-76/10}} \exp\left(-\frac{p}{10^{-76/10}}\right) dp = 0.0032$$

(d) The room in (c) has plenty of average power for communications ... the correct answer is to move just a few centimeters in any direction to get out of the small-scale fade.

(e) There are 4 waves of equal power and the total power is -76 dBm. Each wave must then carry -82 dBm or 6.3×10^{-9} mW.

(f) The point of this question: this is an electromagnetically large cavity so it will have *lots* of modes. I accepted any estimate in the range between 100 and 1,000,000 modes. There are a lot of ways for crudely estimating this number (what a great engineering problem!) One example by a student: The maximum mode index number for m is 131, for n is 82, and for p is 49. Thus, an upper-bound of modes for the waveguide is 131 × 82 × 49 or 526,358 modes, which would be higher than the actual number. For anyone who might be interested, a short computer program will reveal that this cavity supports 273,569 resonant modes.

4. **Circular vs. Rectangular Waveguide (20 points):**

(a) For a given cut-off frequency f, the perimeter of the circular waveguide is:

$$r = \frac{0.293}{f\sqrt{\mu\epsilon}} \longrightarrow \text{Perimeter} = 2\pi r = \frac{1.84}{f\sqrt{\mu\epsilon}}$$

For a square wave with the same cut-off frequency:

$$a = \frac{0.5}{f\sqrt{\mu\epsilon}} \longrightarrow \text{Perimeter} = 4a = \frac{2.00}{f\sqrt{\mu\epsilon}}$$

Clearly the circular guide is cheaper.

(b) The ratio between the second and the first mode cut-offs for the circular guide is:

$$\frac{\text{Cut-off for TM}_{01}}{\text{Cut-off for TE}_{11}} = 1.31$$

The ratio between the second and the first mode cut-offs for the square waveguide is:

$$\frac{\text{Cut-off for TE/M}_{11}}{\text{Cut-off for TE}_{10}} = 1.41$$

For the same initial cut-off frequency, the square guide should have more single-mode bandwidth.