ECE 3065: Electromagnetics

Solutions to TEST 2 (Spring 2006)

1. **Path Loss Modeling:** The path loss with respect to free space at the ith location, PL_i, follows this basic form:

$$\mathrm{PL}_i = 20\log_{10} r_i + a_i X_a + b_i X_b$$

where r_i is the transmitter-receiver separation distance, a_i is the number of hard partitions and b_i is the number of soft partitions; the values X_a and X_b are the corresponding loss factors for each partition. This equation may be rearranged to

$$a_i X_a + b_i X_b = \mathrm{PL}_i - 20\log_{10} r_i$$

Making crude estimates for distance, we may describe the propagation with the following set of matrices:

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 4 & 0 \\ 1 & 0 \\ 4 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} X_a \\ X_b \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 35 - 20\log_{10} 10 \\ 72 - 20\log_{10} 30 \\ 56 - 20\log_{10} 40 \\ 76 - 20\log_{10} 50 \end{bmatrix}}_{\vec{b}}$$

Anything remotely resembling this was given full credit.

2. **Stealing WiFi:**

(a) To start this problem, we recognize that there are two different radio links in this system. In the linear scale, we can write two different link budget equations:

$$\text{Town to Passive Repeater: } P_{R1} = P_{T1}\frac{\lambda^2 G_T G_D}{(4\pi r_1)^2}$$

$$\text{Passive Repeater to Albert's House: } P_{R2} = P_{T2}\frac{\lambda^2 G_D^2}{(4\pi r_2)^2}$$

where G_D is the gain of the dish antenna (transmit or receive), $r_1 = r_2 = 5000$ meters for this link, $G_T = 5$ dBi, $P_{T1} = 30$ dBm, and the power received by the passive repeater P_{R1} becomes the transmit power for the second link P_{T2} (provided there are no additional losses. Thus, we can write one cumulative link budget:

$$P_{R2} = P_{T1}\frac{\lambda^4 G_T G_D^3}{(4\pi r)^4} \quad \text{or} \quad G_D = \left(\frac{P_{R2}(4\pi r)^4}{G_T P_{T1}\lambda^4}\right)^{\frac{1}{3}}$$

If P_{R2} must be greater than 3.2×10^{-10} mW (-95 dBm) to operate correctly, then dish gain must be at least 32.8 dBi. This problem could have been just as easily solved with the dB-link budget equations.

(b) Using the following relationship (which is linear-scale):

$$G = \frac{4\pi}{\lambda^2}A_{em}$$

we see that the minimum electromagnetic area is about 2.27 m^2. A circle of radius 0.85m will make this dish.

3. **Small-Scale Fading:** Sugimoto-san is calling his wife on his *keitai denwa* (mobile phone) while riding at 200 km/hr (55.6 m/s) on the *Shinkansen* (Japanese bullet train) through Tokyo. His NTT DoCoMo phone is communicating with an 1800 MHz base station. You may approximate the angular distribution of plane waves arriving at his handset with the uniform azimuth spectrum ($p(\phi) = \frac{P_R}{2\pi}$). Answer the following questions based on this scenario. (Hint: you should not have to do any math to compute shape factors.)

   (a) We use a Rayleigh PDF to solve this problem:

$$\begin{aligned}
\Pr\{P_{dB} \leq -100 \text{ dBm}\} &= \int_0^{1\times10^{-10}} \frac{1}{1\times10^{-9}} \exp\left(-\frac{p}{1\times10^{-9}}\right) dp \\
&= 1 - \exp\left(\frac{1\times10^{-10}}{1\times10^{-9}}\right) \\
&= 0.095
\end{aligned}$$

   This link fails 9.5% of the time.

   (b) Recognize that the omnidirectional channel has maximum spread ($\Lambda = 1$) and minimum constriction ($\gamma = 0$) – so there is no need to compute Fourier coefficients. For the given threshold, $\rho^2 = \frac{1\times10^{-10}}{1\times10^{-9}} = 0.1$. Thus,

$$LCR = \frac{\sqrt{2\pi}\rho v}{\lambda} \exp\left(-\rho^2\right) = \frac{\sqrt{2\pi}\sqrt{0.1}(55.6 \text{ m/s})}{(0.0167 \text{ m})} \exp(-0.1) = 239.2 \text{ crossings/seconds}$$

   That's a lot of fading!

   (c) Using the same numbers as in the previous problem

$$AFD = \frac{\lambda\left[\exp\left(\rho^2\right) - 1\right]}{\sqrt{2\pi}v\rho} = \frac{(0.0167 \text{ m})\left[\exp(0.1) - 1\right]}{\sqrt{2\pi}(55.6 \text{ m/s})\sqrt{0.1}} = 4.0 \times 10^{-4} \text{ seconds}$$

   Those are some short fades! Note: there was a typo in the equation on the test cheat sheet that modestly skewed the answer. The equations above are correct; no one was penalized for using the typo equation.

   (d) This one is even easier:

$$\text{De-correlation Length} = \frac{\lambda}{\sqrt{23}} = 0.035 \text{ meters}$$

   (e) This one requires a little probabilistic reasoning, but I could not resist putting it on the test to see if someone could get it. The probability of outage in a two-antenna link is the probability that *both* antennas are simultaneously in a -10 dB fade. If the fading on the two antennas is uncorrelated (as it would be in the previous question) then the fades experienced by each antenna are independent events. The probability of two simultaneous fades is the *square* of your answer in part (a), since the probability of independent events is multiplicative. The correct answer is 0.9% of the time – a dramatic improvement over the single antenna case.

4. **Waveguide Propagation:**

   (a) Below is the cut-off equation with the tunnel parameters substituted in:

$$(f_c)_{x0} = \frac{1}{2\sqrt{\mu_0 \epsilon_0}} \sqrt{\left(\frac{x}{a}\right)^2 + \left(\frac{0}{b}\right)^2} = x \frac{1.5 \times 10^8 \text{ m/s}}{10 \text{ m}} = 850 \text{ MHz}$$

   The value of x that satisfies this relationship exactly is 56.67. Since x represents a modal number, it must be an integer. Thus, the maximum value is 56 (57 would be cut-off).

   (b) First, let us calculate the cut-off frequencies for the TE_{10} and TM_{77} modes:

$$(f_c)_{10} = \frac{1}{2\sqrt{\mu_0 \epsilon_0}} \sqrt{\left(\frac{1}{10 \text{ m}}\right)^2 + \left(\frac{0}{6 \text{ m}}\right)^2} = 15 \text{ MHz}$$

$$(f_c)_{77} = \frac{1}{2\sqrt{\mu_0 \epsilon_0}} \sqrt{\left(\frac{7}{10 \text{ m}}\right)^2 + \left(\frac{7}{6 \text{ m}}\right)^2} = 204 \text{ MHz}$$

   We know that the group velocity for these modes will be given by:

$$v_g = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

   which produces a group velocity of 3.00×10^8 m/s for the TE_{10} mode and 2.91×10^8 the TM_{77} mode. After traveling the maximum distance of 1 kilometer (from the base station in the center of the tunnel to a user at the end), this will result in a transit time of 3.33 μs for the TE_{10} mode and a transit time of 3.43 μs for the TM_{77} mode. The difference, 100 ns, is the dispersion between the two modes.

   (c) You are assuming that the walls of the tunnel approximate a perfect electric conductor (PEC).