ECE 3065: Electromagnetics
Solutions to TEST 2 (Spring 2009)

1. **Path Loss Modeling:** Start with the basic link-budget equation:

$$P_R = P_T + G_T + G_R + 20\log_{10}\left(\frac{\lambda}{4\pi}\right) - 10n\log_{10} r$$

This may be re-arranged in terms of n:

$$n = \frac{P_T - P_R + G_T + G_R + 20\log_{10}\left(\frac{\lambda}{4\pi}\right)}{10\log_{10} r}$$

Solving for the sensor nodes' wireless parameters (-97 dBm received power, +20 dBm transmit power, 0 dBi antenna gains, 100m range) implies $n = 3.8$.

2. **Diffraction:** The geometry of the problem leads to an incidence angle of ϕ_1 and an observation angle of ϕ_2:

$$\phi_1 = 90^\circ - \sin^{-1}(50/2000) = 88.6^\circ$$
$$\phi_2 = 270^\circ + \sin^{-1}(100/1000) = 275.7^\circ$$

If the transmitter is horizontally polarized, this corresponds to the $\perp$-incidence case. The solution is a straightforward evaluation of:

$$P_R = \text{EIRP}\frac{G_R \lambda^3 |D_\perp(\phi_2, \phi_1)|^2}{32\pi^3 r_1 r_2 (r_1 + r_2)}$$

which leads to 1.6×10^{-8} W.

3. **Poynting-Vector:** Starting with the field solution:

$$\begin{aligned}
\tilde{E}_x(x,y,z) &= 0 \\
\tilde{E}_y(x,y,z) &= \frac{-j\omega\mu\pi}{bh^2} H_0 \sin\left(\frac{\pi}{a}x\right)\exp(-j\beta z) \\
\tilde{H}_x(x,y,z) &= \frac{j\pi\beta}{bh^2} H_0 \sin\left(\frac{\pi}{a}x\right)\exp(-j\beta z) \\
\tilde{H}_y(x,y,z) &= 0 \\
\tilde{H}_z(x,y,z) &= H_0 \cos\left(\frac{\pi}{a}x\right)\exp(-j\beta z)
\end{aligned}$$

We note that the Poynting vector is given by

$$\begin{aligned}
\vec{\text{S}}(x,y,z) &= \frac{1}{2}\text{Real}\left\{\tilde{E}_y(x,y,z)\hat{\text{y}} \times \left[\tilde{H}_x(x,y,z)\hat{\text{x}} + \tilde{H}_z(x,y,z)\hat{\text{z}}\right]^*\right\} \\
&= \frac{1}{2}\text{Real}\left\{-\tilde{E}_y(x,y,z)\tilde{H}_x^*(x,y,z)\hat{\text{z}} + \tilde{E}_y(x,y,z)\tilde{H}_z^*(x,y,z)\hat{\text{x}}\right\} \\
&= \frac{1}{2}\cdot\frac{\omega\mu\pi}{bh^2} H_0 \sin\left(\frac{\pi}{a}x\right)\cdot\frac{\pi\beta}{bh^2} H_0 \sin\left(\frac{\pi}{a}x\right)\hat{\text{z}} \\
&= \frac{\omega\mu\pi^2\beta H_0^2}{2b^2h^4}\sin^2\left(\frac{\pi}{a}x\right)\hat{\text{z}}
\end{aligned}$$

This result is not quite the answer, however, since the problem asked for *total power* – the Poynting vector provides the power density, but must be integrated over the $a \times b$ waveguide aperture to provide power. The final result is

$$P_T = \frac{a\omega\beta\pi^2 H_0^2}{4bh^4}$$

Note: in the original problem statement, the term $j\beta$ was expressed using the propagation constant γ, which, while technically correct, gave the appearance of being a real-valued constant. Without knowledge of this (and there was no helpful formula on the test to inform the student) an full evaluation of the Poynting vector would result in a zero-field across the entire waveguide aperture. Some students saw this and reported 0 total power. Others used their intuition and reasoned that the power propagating in the z-direction must be real in the Poynting vector; they kept that term and performed calculations accordingly. Full credit was given for both methods, provided the student showed all work and was consistent with their assumptions.

4. **Waveguide Propagation:** The fastest propagating waveguide mode is the first one with 400 MHz cut-off and the slowest is the last one with 880 MHz. Using the formula

$$v_g = c\sqrt{1 - \left(\frac{f_{c1}}{f}\right)^2}$$

we find that, over 100m length, the difference between the two propagation times is 846 ns.

5. **Transmission Line Resonator:**

   (a) $\alpha = 0.0115$ Np/m

   (b) 0.0249 m

   (c) 1.35×10^{-14} F

   (d) 1.5 MHz