AAT2: Rician Fading

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Rician Wave Model

What causes small-scale fading?

- Frequency: caused by dispersion (radio echoes)
- Time: caused by time variations in TX, RX, or channel
- Space: caused by multipath arriving from different directions

First, break sum-of-waves model into *specular* and *diffuse* waves:

$$R = \left|\tilde{V}\right| = \left| \underbrace{V_1 \exp(j\Phi_1)}_{\text{specular}} + \underbrace{\sum_{i=2}^{N} V_i \exp(j\Phi_i)}_{\text{diffuse}} \right|$$

the largest specular wave contributes most of the power, no longer Rayleigh.

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Rician Derivation

What causes *Rician* fading?

- a line-of-sight from TX to RX location
- a strong specular reflection or transmission
- a time-varying channel for otherwise fixed TX and RX

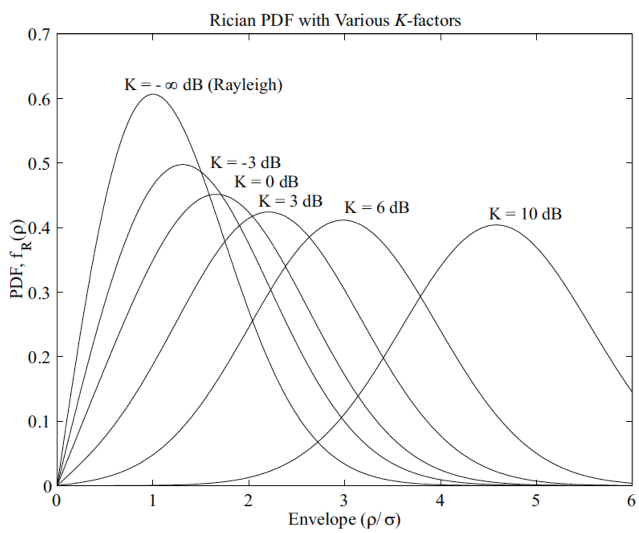
Rician probability density function (PDF):

$$f_R(\rho) = \frac{\rho}{\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2} - K\right) I_0\left(\frac{\rho}{\sigma}\sqrt{2K}\right) \mathrm{u}(\rho)$$

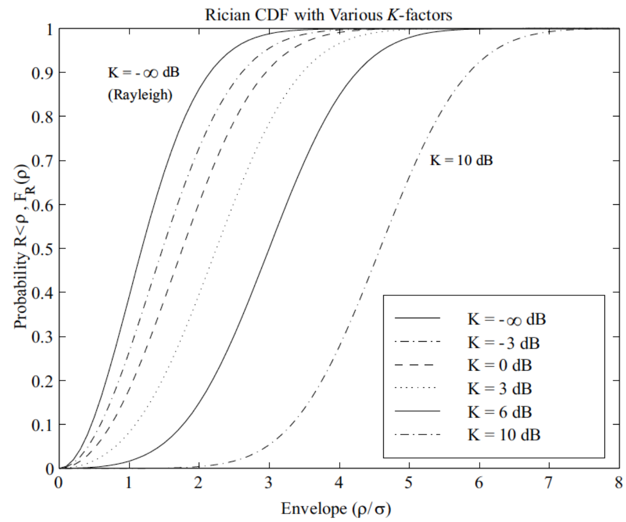
Specular Power: V_1^2 Average Diffuse Power: $2\sigma^2 = \sum_{i=2}^{N} V_i^2$

$$\text{Rician K-factor: } K = \frac{\text{Specular Power}}{\text{Diffuse Power}} = \frac{V_1^2}{2\sigma^2}$$

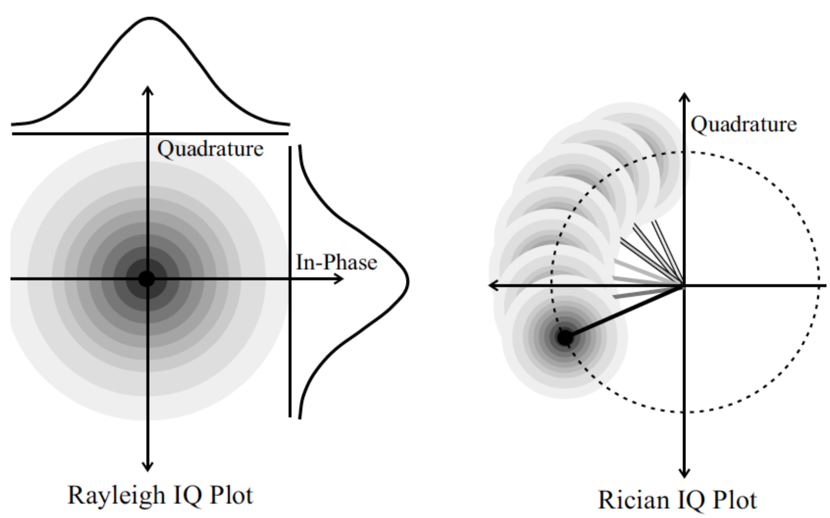
Rician PDF with Various K-factors



Rician CDF with Various K-factors



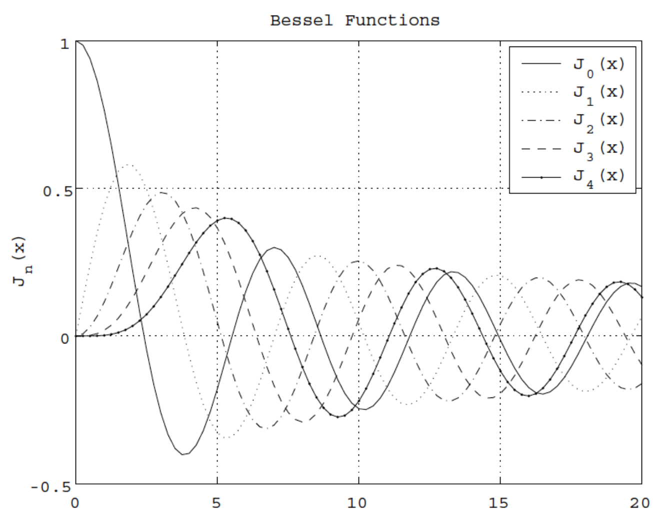
Geometry of Rayleigh/Rician PDFs



Stephen Oswald Rice

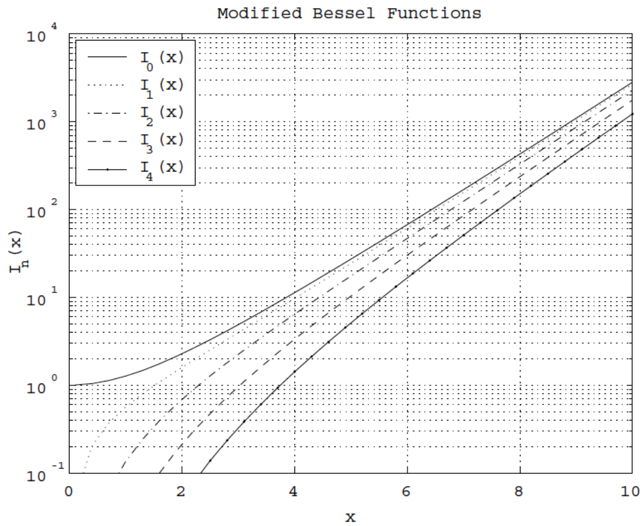
- 1907-1986
- Famous scientist and mathematician from Bell Labs
- won the 1983 IEEE Alexander Graham Bell Medal "For his contributions to the fundamental understanding of communications systems and to the underlying mathematics, and for inspiring younger scientists and engineers. ”
- Originally applied the Rician distribution to large-carrier AM detection problems in famous 1944 paper "Mathematical Analysis of Random Noise"

Note on Bessel Functions



$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \nu^2)y = 0$$

Note on Bessel Functions



$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - (x^2 + \nu^2) y = 0$$

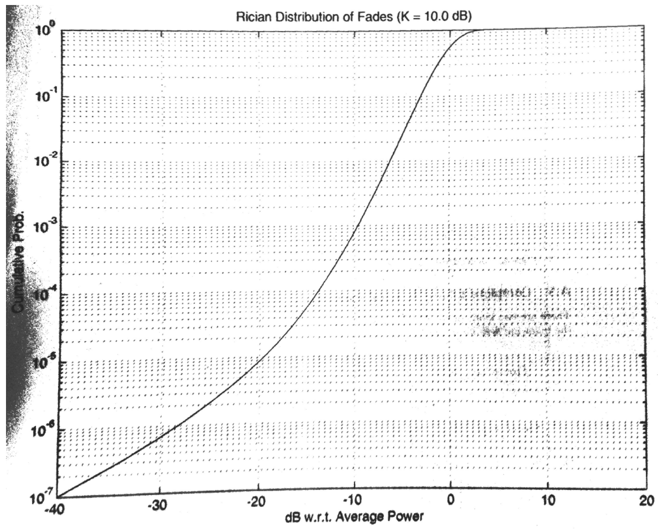
Example of Rician Fading Problem

Rician fading in satellite links

- Scintillation (temporal, spatial) due to atmospheric inhomogeneities
- Weather (temporal, spatial) due to rain attenuation and scattering
- Mobile ground scatter (spatial, frequency)

Example: A particular 30 GHz satellite link requries -105 dBm of received instantaneous power to properly decode a digital signal. A rain storm moves into the area and attenuates the line-of-sight power to the satellite's earth station to -100 dBm. Additionally, the moving raindrops introduce a diffuse, time-varying component of -110 dBm. How often will this link be unusable?

Solution Using Rician CDFs



Prob 5 dB drop is about 0.02 from graph

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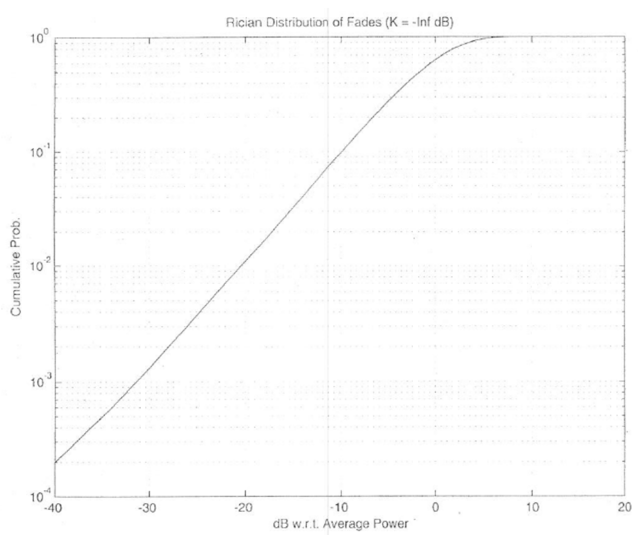
Solution Using Rician CDFs



Prob 5 dB drop is about 0.3 from graph

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Rician Parameter Estimation

Given a collection of N measured envelopes $\{R_1, R_2, R_3, \cdots R_N\}$ in a local area, what is the best K factor that fits the data?

Method 1: Use mean and variance of envelope to solve for K:

$$\frac{[\overline{R}]^2}{\overline{R^2}} = \frac{\left[\frac{1}{N}\sum_{i=1}^{N} R_i\right]^2}{\frac{1}{N}\sum_{i=1}^{N} R_i^2} = \frac{\pi \exp(-K)}{4(K+1)}\left[(K+1)I_0\left(\frac{K}{2}\right) + KI_1\left(\frac{K}{2}\right)\right]^2$$

This method works best for smaller sample sizes ($N < 1000$), but requires solving a very complicated nonlinear equation.

Rician Parameter Estimation

Given a collection of N measured power levels $\{P_1, P_2, P_3, \cdots P_N\}$ in a local area ($P = R^2$), what is the best K factor that fits the data?

Method 2: Use mean and variance of power to solve for K:

$$\frac{[\overline{P}]^2}{\overline{P^2}} = \frac{\left[\frac{1}{N}\sum_{i=1}^{N} P_i\right]^2}{\frac{1}{N}\sum_{i=1}^{N} P_i^2} = \frac{(K+1)^2}{K^2 + 4K + 2}$$

This method is not as robust as method 1, but requires only a simple quadratic calculation.

What Happens for 2 Big Specular Waves?