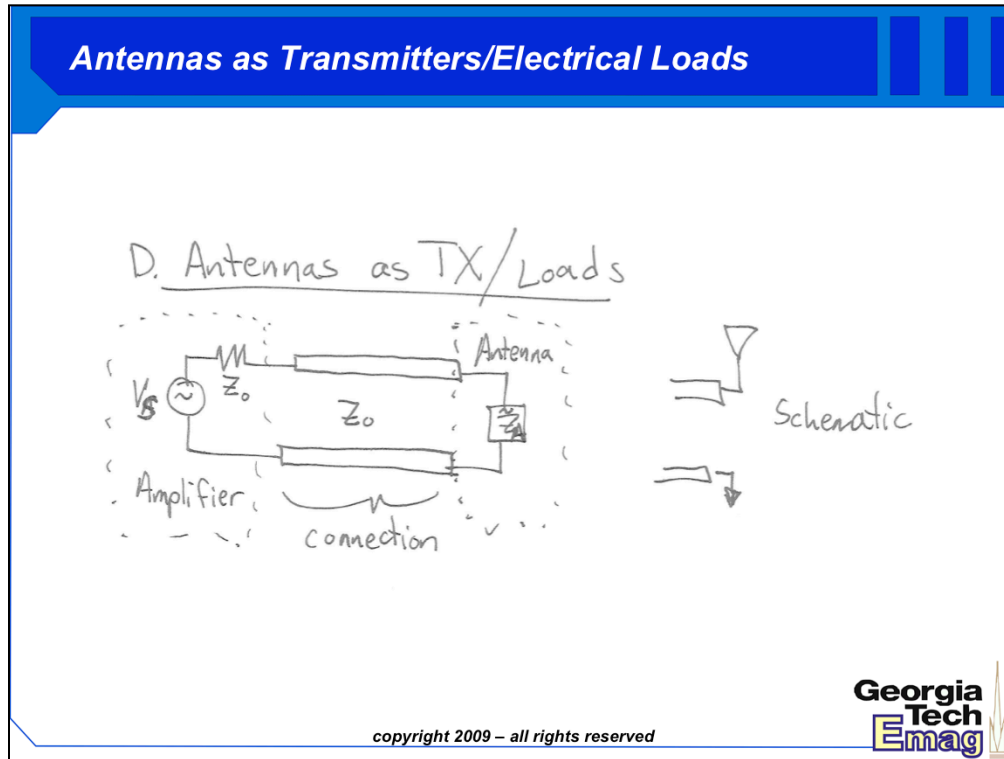


In this module, we look to treat and analyze an antenna in the context of circuit theory. One of the key outcomes of our discussion is to understand how to model antennas with circuit theories, as well as how to translate between the world of voltage and power at the terminals of an antenna.



If we are transmitting with an antenna, it will look like a complex load Z_A at the end of the transmission line with intrinsic impedance Z_0 . The source, say an RF power amplifier, sends a signal to the impedance-modeled antenna through a transmission line. If it is a good antenna, most of the power absorbed by this load represents radiated power.

Review of Electrical Loads

$$P_T = \frac{1}{2} \Re \left\{ \tilde{V}_A \tilde{I}_A \right\} = \frac{V_S^2 R_A}{2 |\tilde{Z}_S + \tilde{Z}_A|^2}$$

$$\tilde{Z}_S = R_S + jX_S$$

$$\tilde{Z}_A = R_A + jX_A$$

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How much power is delivered to an antenna with a source of amplitude V_s ? Here is a basic analysis of the total power delivered to the load. This expression above is for time-averaged, real power based on a phasor representation of voltage and current.

Power Delivery to a Load

$$\text{Conjugate Match } \tilde{Z}_A = \tilde{Z}_S^*: \quad P_S = \frac{V_S^2}{8R_S}$$

In terms of the total available source power, P_S , we can express the transmitted power as

$$P_T = \frac{4R_S R_A}{\underbrace{\left| \tilde{Z}_S + \tilde{Z}_A \right|^2}_{\text{mismatch losses}}} P_S$$

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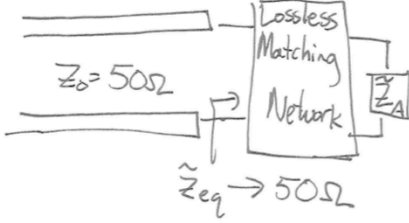
The above expression illustrates how much power is absorbed by a load impedance.

Of course, maximum power transfer occurs when the load is conjugate-matched with the source impedance. Thus, when the imaginary portion of the load impedance is opposite valued from the imaginary value of the source impedance and the real value of the impedances are identical, the maximum amount of power is transferred to the load. The expression for this is shown above.

Antenna Matching

Most T-lines are (hopefully) lossless \therefore
 Z_0 is real; matching antenna to this
 results in zero reflection AND max power
 delivered.

Classic Antenna Problem:



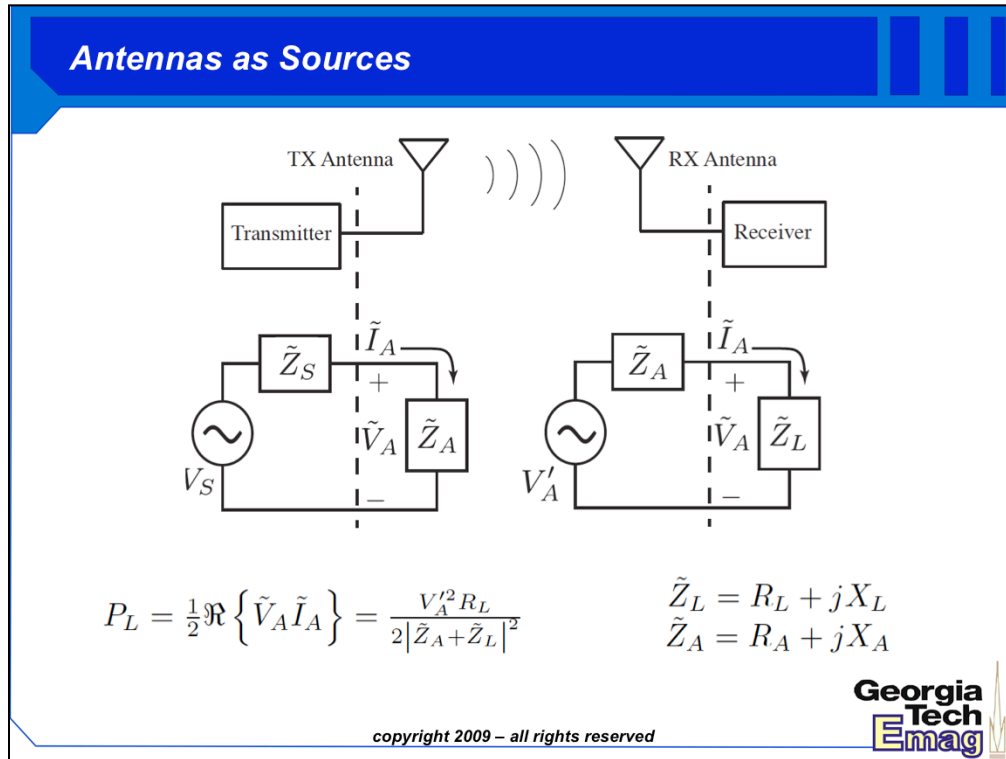
How do we design
 a lossless matching
 network to maximize
 power delivery to
 an antenna? (6)

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This illustrates one of the classical engineering problems in antenna engineering: matching an antenna to a lossless transmission line (real impedance). Usually, antennas are designed for pattern, gain, polarization, and a variety of other radiation characteristics. The resulting impedance is a secondary concern; thus, the impedance off the design table is rarely matched to the actual feed line without additional engineering work.

One key task of the antenna engineer is to design a matching network to transmission line/source. The matching network is usually a low-loss portion of circuitry consisting of either discrete capacitors and inductors or stubs and other resonant t-line structures that provides a Z_0 -equivalent impedance at the end of a transmission line ... all with the intention of providing maximum power transfer to the antenna. This is important, since any reflected power in such a system represents a waste that burns unnecessary power or reduces signal integrity at a receiver.



We may also view the antenna in a circuit when it is used to receive a signal. In this case, by virtue of the reciprocity theorem in electromagnetics, the antenna has the same intrinsic impedance whether it functions as a transmitter or a receiver. The power that an antenna delivers to a complex load is shown above.

Voltage Amplitudes on Antennas

$$\text{Conjugate Match } \tilde{Z}_L = \tilde{Z}_A^*: \quad P_R = \frac{V_A'^2}{8R_A}$$

$$\text{Antenna Eq.} \quad V_A' = 2\sqrt{2R_A P_R}$$

Source Voltage

Magnitude at Terminals of a RX Antenna

$$P_L = \underbrace{\frac{4R_L R_A}{|\tilde{Z}_A + \tilde{Z}_L|^2}}_{\text{mismatch losses}} P_R \quad V_A = |\tilde{V}_A| = \frac{2|\tilde{Z}_L| \sqrt{2R_A P_R}}{|\tilde{Z}_A + \tilde{Z}_L|}$$

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Here, the maximum power delivered to the conjugate-matched load is shown in the top formula. From this, we can back-solve the voltage of the terminals of a matched antenna if given the power. This is particularly useful, since the received power of a link budget usually assumes matched antennas. What if a different load is attached to the matched antenna? If so, the amplitude of the voltage at that terminal will change. The bottom equation shows how to compute this amplitude given the impedance of the load and antenna as well as the ideal “link budget” received power.

Example: Amplitudes on a Coaxial Cable

Example 2.1: Voltage on a 50Ω Coaxial Cable

Problem: A 50Ω coaxial cable connected to an antenna is receiving 20 dBm when connected to a matched load. What is the peak amplitude of its output voltage?

Solution: We first convert 20 dBm into linear Watts by using the following formula:

$$\text{Watts} = 10^{(\text{dBm}-30)/10} \rightarrow P_R = 0.10 \text{ Watts}$$

The amplitude follows from Equation (2.1.8) under the condition that $\tilde{Z}_A = \tilde{Z}_L = 50\Omega$. The peak voltage amplitude is 3.16 V.

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Example 1: Amplitude on a Charge Pump

Example

A $50\text{-}\Omega$ antenna receives 10 dBm of power and is connected to a microwave energy-harvesting circuit with impedance $30 - j100\text{ }\Omega$. What must the threshold voltage be (max) of the energy-harvesting diodes in this circuit?

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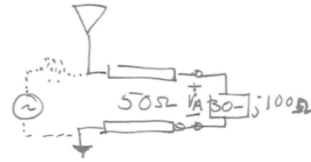
Solution 1: Amplitude on a Charge Pump

Ans

$$R_A = 50\Omega \quad X_A = 0$$

$$R_L = 30\Omega \quad X_L = -j100\Omega$$

$$P_R = 10 \text{ dBm} = 10 \text{ mW}$$



$$V_A = \frac{2 \left[30^2 + 100^2 \right]^{1/2} \sqrt{2 \cdot 50 \cdot 0.01 \text{ W}}}{\left[(50+30)^2 + (0+100)^2 \right]^{1/2}}$$

$$= \frac{2 |\tilde{Z}_L| \sqrt{2 R_L P_R}}{|\tilde{Z}_L + \tilde{Z}_A|} = 0.16 \text{ V}$$

What would make this higher?

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