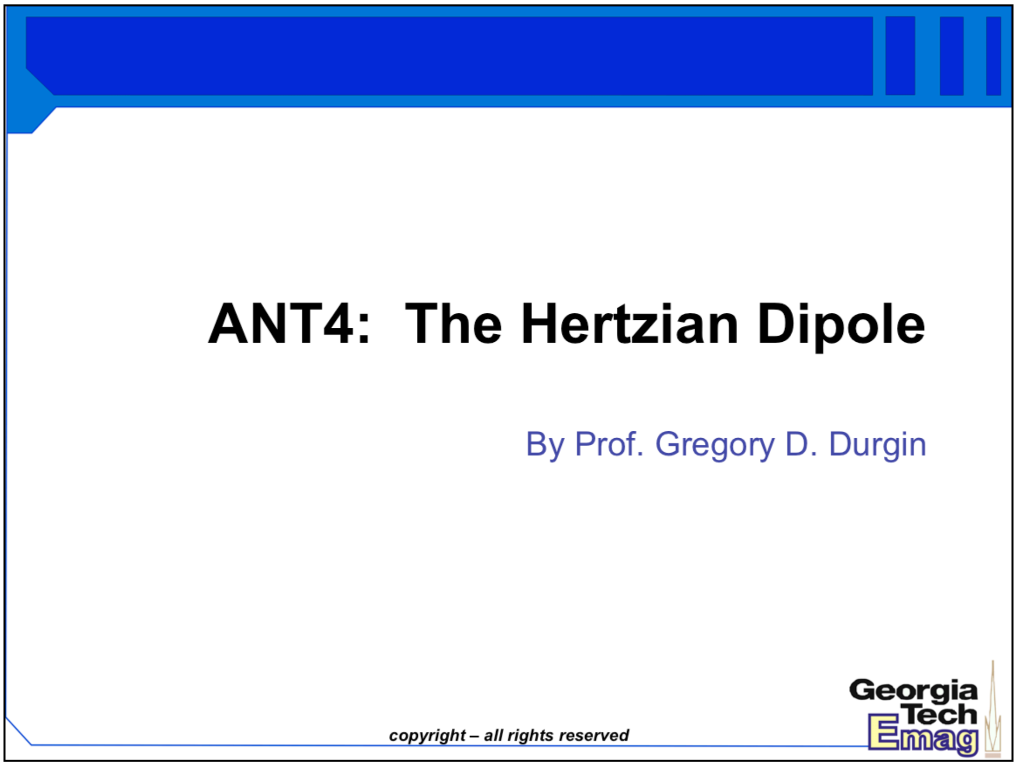

In this module, we study the solution to the simplest radiative system – the Hertzian dipole. Once solved, this simplest of radiating systems is used to introduce the concept of directivity, gain, radiation impedance, and half-power beamwidth.

Example: Ideal (Hertzian) Dipole

Green's Theorem applied to vector magnetic pot:

$$\tilde{A}_z(x,y,z) = \frac{\mu}{4\pi}\int_{\mathcal{V}} \tilde{J}_z(x',y',z')\,\frac{\exp(-jkR)}{R}\,dv'$$

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

point of observation: $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

variables of integration: $\vec{r}' = x'\hat{x} + y'\hat{y} + z'\hat{z}$

$$\tilde{A}_z(x,y,z) = \frac{\mu}{4\pi}\iiint_{\mathcal{V}} \tilde{J}_z(x',y',z')\,\frac{\exp\left(-jk\left\|\vec{r}'-\vec{r}\right\|\right)}{\left\|\vec{r}'-\vec{r}\right\|}\,dx'\,dy'\,dz'$$

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This simple solution for Az given a possible source at the origin can be used to synthesize a field from any z-directed spatial distribution of current. Thus, plug any given Jz into the equation above and solve for Az. Recall that the standard quantities of electromagnetics – E-field, H-field, and Power – may all be derived from Az after this integration is complete.

Example: Ideal (Hertzian) Dipole

Hertzian Dipole

$$\tilde{J}_z(x, y, z) = I\,dl\,\delta(x)\,\delta(y)\,\delta(z) = I\,dl\,\delta(\vec{\mathrm{r}})$$

This is a small, infinitessimal amount of current at the origin of length dl, which can equivalently be collapsed into delta functions.

$$\tilde{A}_z(r) = \frac{\mu I\,dl}{4\pi}\hat{\mathrm{z}}\frac{\exp(-jkr)}{r}$$

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The simplest radiating system is the Hertzian dipole, resulting from an infinitesimal current element at the origin flowing in the z-direction. This can be described mathematically as a Jz with three delta functions w.r.t. x, y, and z. Plugging this into our formula, we get the above expression for Az.

Solution for the Ideal Dipole

Fields from the Hertzian Dipole

$$\tilde{\vec{\mathrm{H}}}(\vec{\mathrm{r}}) = \frac{\nabla \times (\tilde{A}_z \hat{\mathrm{z}})}{\mu}$$

$$\tilde{\vec{\mathrm{E}}}(\vec{\mathrm{r}}) = \frac{\nabla \times \nabla \times (\tilde{A}_z \hat{\mathrm{z}})}{j2\pi f \mu \epsilon}$$

$$\tilde{\vec{\mathrm{H}}}(\vec{\mathrm{r}}) = \frac{jIdlk\sin\theta}{4\pi r}\exp(-jkr)\left[\ \hat{\phi} - \overbrace{\frac{j\hat{\phi}}{kr}}^{\text{order } \frac{1}{kr} \text{ terms}}\ \right]$$

$$\tilde{\vec{\mathrm{E}}}(\vec{\mathrm{r}}) = \frac{jIdlk\eta\sin\theta}{4\pi r}\exp(-jkr)\left[\ \hat{\theta} - \underbrace{j\frac{\hat{\theta} + 2\cot\theta\hat{\mathrm{r}}}{kr}}_{\text{order } \frac{1}{kr} \text{ terms}} - \underbrace{\frac{\hat{\theta} + 2\cot\theta\hat{\mathrm{r}}}{(kr)^2}}_{\text{order } \frac{1}{(kr)^2} \text{ terms}}\ \right]$$

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Now it's just a matter of straight-forward (but tedious) differential calculus to arrive at the solution for classical E and H field for the Hertzian dipole.

A few comments about the final solution:

1)It's messy! Keep in mind that this is the *simplest possible* radiating system. This attribute of radiation is one reason why antenna engineering is difficult and expertise is often in short demand!

2)If we move our point of observation of the system more than a wavelength (r>lambda) from the source/origin, then only the 1/r terms dominate. These represent propagating waves away from the sourcce.

3)The 1/r^2 and 1/r^3 field terms dominate when observed for r<lambda. These represent stored energy/circulating fields that swirl around the radiated system.

Ideal Dipole: Total Radiated Power

Usually interested in *Power*:

$$\vec{\mathrm{S}}(r,\theta,\phi) = \frac{1}{2}\Re\left\{\tilde{\vec{\mathrm{E}}} \times \tilde{\vec{\mathrm{H}}}^*\right\} = \frac{I^2\, dl^2\, k^2 \sin^2 \eta\theta}{32\pi^2 r^2}\hat{\mathrm{r}}$$

Average power density via *Poynting* vector.
Total Transmitted Power:

$$P_T = \int_{\mathcal{A}} \vec{\mathrm{S}} \cdot d\hat{\mathrm{n}} = \int_0^{\pi}\int_0^{2\pi} \frac{I^2\, dl^2\, k^2 \sin^2 \eta\theta}{32\pi^2 r^2}\hat{\mathrm{r}} \cdot \underbrace{r^2 \sin\theta\, d\theta\, d\phi \hat{\mathrm{r}}}_{d\hat{\mathrm{n}}}$$

$$= \frac{\pi I^2\, dl^2\, \eta}{4\lambda}\int_0^{\pi} \sin^3\theta\, d\theta = \frac{\pi I^2\, dl^2\, \eta}{3\lambda} \qquad \text{*uses} \int \sin^3\theta\, d\theta = -\cos\theta + \frac{1}{3}\cos^3\theta$$

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Let's calculate the total amount of power radiated by the Hertzian Dipole. To do this, we calculate the Poynting vector (units of W/m^2) from E and H. Then we integrate this quantity around 4pi steredians. This gives us total power as a function of current magnitude, I (Amps), wavelength, and infinitesimal length Delta l. Note how the Poynting vector is always pointing away from the origin, regardless of point of observation.

Hertzian/Ideal Dipole: Characteristic Impedance

Impedance for the Hertzian Dipole

$$\tilde{Z}_L = R_A + jX_A$$

Real part of the antenna impedance represents (ideally) power absorbed by the antenna and transformed into traveling waves. In the absence of other losses, we estimate the radiation resistance, R_A, based on input current and power transmission.

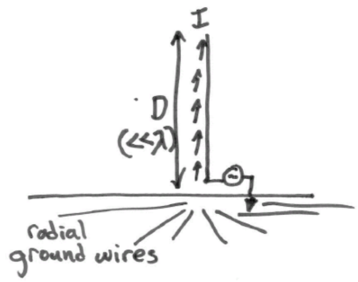
$$P_T = \frac{1}{2} I^2 R_A = \frac{\pi I^2 \, dl^2 \, \eta}{3\lambda} \quad \longrightarrow \quad R_A = \frac{I^2}{2P_T} = \frac{2\pi\eta}{3}\left(\frac{dl}{\lambda}\right)^2$$

The reactive portion of X_A will be large and negative (capacitive), representing an open circuit with slight load transformation along a short transmission line. This makes for a serious mismatch between the Hertzian dipole and any realistic transmission line feed.

We can estimate the impedance of a very short dipole at the end of a transmission line by using this result from the Hertzian dipole. Say we take a tiny dl-length of line and bend it back. We know that this load would appear highly capacitive and can calculate the result from the Smith chart or the load transformation formula.

There will also be a small, real-valued resistance that represents power delivered to the antenna and radiated into space. This is called the radiation impedance. Assuming an ideal system where 100% of the power into the radiation resistance makes it into traveling waves, this I^2 R radiated power must be equal to the total power found on the previous slide. That actually gives us a method for solving the radiation resistance. See above that the radiation resistance is a function of (dl/lambda). As the length of the dipole increases, radiation resistance increases. Keep in mind that this expression is only valid for very small values of dl.

Examples of Short Dipoles and Mismatch Problems



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One example of a short dipole is the AM radio transmit tower. AM radio waves have very long wavelengths (> 300m), so that even the tallest towers can only support radiating wires that are a fraction of this length. In many cases, the radiator behaves effectively like a short dipole with large capacitive mismatches.

Besides matching the source discretely, one strategy is also to "mirror" the current on the ground plane so that the dipole looks effectively larger. This only works when the ground is conductive, which is a poor approximation for some regions on earth … especially those with dry and rocky soils.

Directivity and Gain

Definition of Directivity

$$D(\theta, \phi) = \frac{\text{Radiated Power Denisty}}{\text{Equivalent Isotropic Power Density}} = \frac{\|\vec{S}(r, \phi, \theta)\|}{P_T/(4\pi r^2)}$$

Directivity of Hertzian Dipole

$$D(\theta, \phi) = \frac{3}{2}\sin^2\theta$$

Directivity should always integrate to 4π over 4π steradians. Gain pattern, $G(\theta, \phi)$, is related to directivity multiplied by radiation efficiency, which includes realistic losses. Thus, gain pattern should always integrate to less than 4π over 4π steradians.

$$G(\theta, \phi) = D(\theta, \phi) \times \underbrace{\eta_{\text{rad}}}_{\text{radiation efficiency}}$$

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Another quantity that we can calculate from the Hertzian dipole fields is the directivity. Directivity is defined as the ratio of power density radiated in a particular direction to the average power density radiated in the system. Under this definition, directivity will always integrate to 1 over the full 4pi steradians of space. An example calculation for our Hertzian dipole is shown above.

The *gain* of a radiating system is equal to its directivity times an efficiency factor. This factor can account for additional power losses in the radiating system, such as Ohmic losses on the antenna's conductive material, complex permittivity losses in the surrounding dielectric medium, and mismatch losses at the junction of the antenna. When all of these things are accounted for, the gain is technically "Realized Gain", although standards bodies for antennas simply refer to this as "Gain"; this is the value reported on antenna spec sheets.

Note that under this definition, Gain pattern of a passive radiative system should *never* integrate to a value above 1 over 4pi steradians. Gain > 1 is possible for *specific* directions, but this always comes at the cost of gain in other directions.

Half-Power Beamwidth

Step 1: Find Peak Gain/Directivity

Hertzian dipole has peak gain at $\theta = 90°$

Step 2: Find Half-Power Points

$$D(90°) = \frac{3}{2} \quad D(\theta_{1/2}) = \frac{3}{4} \quad \longrightarrow \quad \theta_{1/2} = 45°,\ 135°$$

Step 3: θ_{HPBW} is difference between half-power points

$$\theta_{\mathrm{HPBW}} = \left(\theta_{1/2}\right)_2 - \left(\theta_{1/2}\right)_1 = 90°$$

There are several useful values that come out of a gain pattern:

Maximum or Peak gain shows the maximum amount of power transferred when radiated power is observed from the direction of maximum pattern focusing. For the Hertzian dipole, this is anywhere along the horizon (theta=90) and has a value of 3/2 (1.8 dBi in log scale)

Half-Power Beamwidth is the contiguous width, in degrees, of the radiation pattern about its peak corresponding to gains that do not drop below 50% of the peak gain. Thus, the Hertzian dipole pattern falls to a gain of ¾ at theta = 45 degrees and 135 degrees, corresponding to a half-power beamwidth in elevation of 90 degrees, centered at theta = 90 degrees. In azimuth, the half-power beamwidth is essentially 360 degrees since the pattern is omnidirectional about the horizon.