ANT7: Vertical Dipole Arrays

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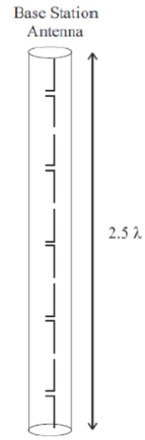
In this lecture, we study the general case of radiation from z-directed spatial currents. The far-field radiation equations that result from this treatment form some of the foundational principles of all antenna engineering. In fact, after this lecture, a student should be able to look at most types of antennas and, regardless of type or construction specifics, be able to infer the basic radiation pattern from the size and shape.

In the later section of the talk, we simplify the analysis to include the special (but very important) case of the general wire antenna. Concentrating on results for the half-wave dipole, we demonstrate how a radiator more realistic than the ideal Hertzian dipole operates. We close with a thorough summary of the most common types of wire antennas and their radiation and electrical parameters.

Example Analysis: Cellular Base Station Antenna

Cellular base station antennas tend to focus power along the horizon where all the paying customers operate. To do this, they manufacture antennas with stacked, in-phase half-wave dipoles (see the 5-element example in the figure to the right) such that the total radiating current may be represented by

$$\tilde{I}(z) = I_0 \left|\cos\left(\frac{2\pi z}{\lambda}\right)\right| u\left(\frac{N\lambda}{4} - |z|\right)$$



Where N is the integer number of stacked dipoles and u() is the unit step function. Assuming ideal efficiency, make a dB-polar plot of the θ-pol elevation-cut gain pattern of a base station antenna for the cases of N = 4, 6, 8 and 10. Graphically estimate the peak gain and half-power beamwidth in θ for each case. Multiply the linear value of the peak gain and the HPBW angle for each case. What do you notice?

Field Solution for the Half-Wave Dipole

Recall our procedure for calculating line current fields:

$$\tilde{A}_z(\vec{r}) \;=\; \frac{\mu}{4\pi r}\exp\left(-jkr\right)\int_{-\infty}^{+\infty}\tilde{I}_z(z')\exp(+jkz'\cos\theta)\;dz'$$

Current on a Half-Wave Dipole

$$\tilde{I}_z(z') = I\cos\left(kz'\right)\,\mathrm{u}\left(\frac{\lambda}{4}-|z'|\right)$$

$$\tilde{A}_z(\vec{r}) \;=\; \frac{\mu}{4\pi r}\exp\left(-jkr\right)\int_{-\lambda/4}^{+\lambda/4}I\cos(kz')\exp(jkz'\cos\theta)\;dz'$$

So let's simplify this expression for a case of current distribution I(z) that exists only on the z-axis. This corresponds to the case of a wire antenna, which is one of the most common instances in basic antennas. The most common of these common antennas is the half-wave dipole (HWDP), because it is a compact, efficient radiator with many different implementations in practice. It may be used by itself or as the radiative element in a reflector (dish) based antenna.

Note that we can start by defining the z-directed current density Jz in terms of the simpler 1-D current distribution I(z) with units of Amps by "collapsing" the current density onto the z-axis with two delta functions with respect to x and y. The simplified expression for magnetic potential is a single integration of this current with respect to a single complex exponent kernal. Here more than before is the very straightforward "Fourier Transform" relationship between current distribution and pattern.

For a HWDP, the current is non-zero over a $\lambda/2$ region, where it is in-phase and sinusoidally-tapered in amplitude. This is basically the standing-wave current pattern at the end of an open-circuited transmission line whose last $\lambda/4$ ends have been bent backwards.

Field Solution for the Half-Wave Dipole

$$|\tilde{A}_z(\vec{r})| = \frac{\mu}{4\pi r}\left|\int_{-\lambda/4}^{+\lambda/4} I\cos(kz')\exp(jkz'\cos\theta)\ dz'\right|$$

$$= \frac{\mu I\cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi k\sin^2\theta r}$$

$$\text{Power Density (W/m}^2\text{):} = \frac{k^2\eta\sin^2\theta}{2\mu^2}|\tilde{A}_z|^2$$

$$= \frac{k^2\eta\sin^2\theta}{2\mu^2}\underbrace{\frac{\mu^2 I^2\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{4\pi^2 r^2 k^2\sin^4\theta}}_{|\tilde{A}_z|^2}$$

$$= \frac{I^2\eta}{8\pi^2 r^2}\underbrace{\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}}_{\propto \text{ gain pattern}}$$

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Here is the solution for the HWDP electric and magnetic fields. Note the similarity to the Hertzian/ ideal dipole radiator: the fields are at a maximum along the azimuth (theta = 90 degrees). The fields have a null along the z-axis (theta = 0 or 180 degrees). The antenna pattern is omnidirectional, having no dependence on azimuth angle, phi.

Note, however, that the overall elevation cut of the pattern is somewhat more “squinted” than the ideal dipole due to the cos(pi/2cos(theta) term in the expressions. This slightly more complicated expression gives a half-power beamwidth of 78 degrees to the HWDP, as opposed to the 90 degrees for the ideal dipole.

Potential for Array of N Half-wave Dipoles

$$
\begin{aligned}
|\tilde{A}_z(\vec{\mathrm{r}})| &= \frac{\mu}{4\pi r}\left|\int_{-\lambda/4}^{(2N-1)\lambda/4} I|\cos(kz')|\exp(jkz'\cos\theta)\ dz'\right| \\
&= \frac{\mu}{4\pi r}\left|\sum_{n=0}^{N-1}(-1)^n \int_{(2n-1)\lambda/4}^{(2n+1)\lambda/4} I\cos(kz')\exp(jkz'\cos\theta)\ dz'\right| \\
&= \frac{\mu}{4\pi r}\left|\sum_{n=0}^{N-1}\frac{2I\cos\left(\frac{\pi}{2}\cos\theta\right)}{k\sin^2\theta}\exp(jn\pi\cos\theta)\right| \\
&= \frac{\mu}{2\pi r}\frac{I\cos\left(\frac{\pi}{2}\cos\theta\right)}{k\sin^2\theta}\underbrace{\left|\sum_{n=0}^{N-1}\exp(jn\pi\cos\theta)\right|}_{\sin(N\pi/2\cdot\cos\theta)/\sin(\pi/2\cdot\cos\theta)}
\end{aligned}
$$

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Total Radiation Pattern for Dipole Array

Now calculate the total power pattern for the radiating system:

$$\text{Power Density (W/m}^2\text{):} \quad = \frac{k^2 \eta \sin^2\theta}{2\mu^2} |\tilde{A}_z|^2$$

$$= \frac{I^2 \eta}{8\pi^2 r^2} \underbrace{\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}}_{\text{dipole gain pattern}} \underbrace{\frac{\sin^2\left(\frac{N\pi}{2}\cos\theta\right)}{\sin^2\left(\frac{\pi}{2}\cos\theta\right)}}_{\text{array pattern}}$$

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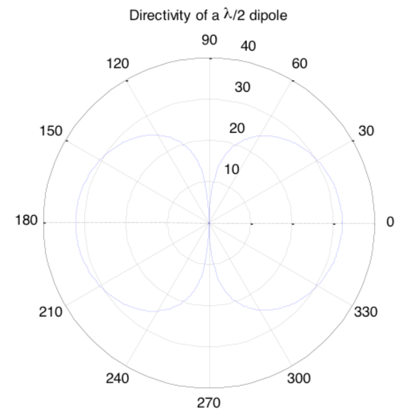
Half-Wave Dipole Analysis




```
Half-wave dipole array of 1 element(s)
Element input current 1.0 A
----------------------------------------------
Total radiated power:    36.4 W/m^2 (15.6 dBW)
Peak gain:               1.6 (2.2 dBi)
Half-power beadwidth:    78.3 deg
Side-lobe level:         Inf dB
Radiation resistance:    72.7 Ohms
```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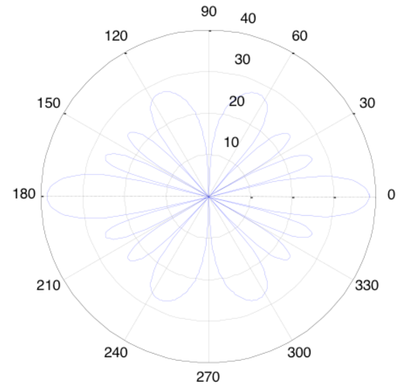
Vertical Array of 4 λ/2-Dipoles

Directivity of 4 stacked λ/2 dipoles




```
Half-wave dipole array of 4 element(s)
Element input current 4.0 A
----------------------------------------------
Total radiated power:    130.9 W/m^2 (21.2 dBW)
Peak gain:               7.3 (8.7 dBi)
Half-power beadwidth:    13.5 deg
Side-lobe level:         11.2 dB
Radiation resistance:    16.4 Ohms
```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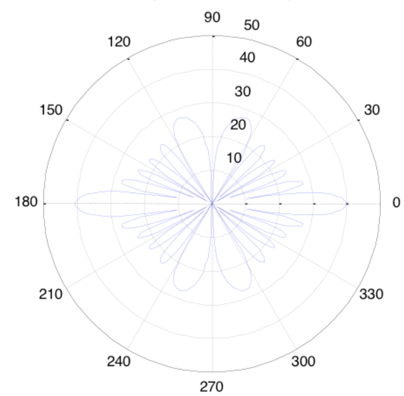
Vertical Array of 6 λ/2-Dipoles

Directivity of 6 stacked λ/2 dipoles




```
Half-wave dipole array of 6 element(s)
Element input current 6.0 A
----------------------------------------------
Total radiated power:    192.0 W/m^2 (22.8 dBW)
Peak gain:               11.2 (10.5 dBi)
Half-power beadwidth:    8.1 deg
Side-lobe level:         12.9 dB
Radiation resistance:    10.7 Ohms
```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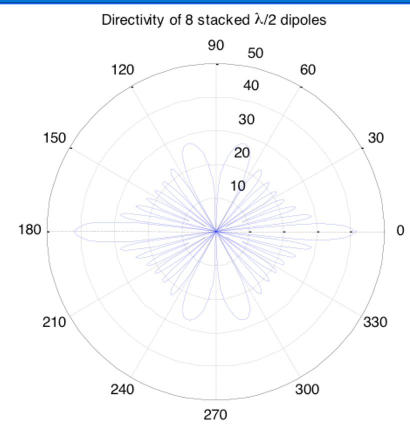
Vertical Array of 8 λ/2-Dipoles




```
Half-wave dipole array of 8 element(s)
Element input current 8.0 A
----------------------------------------------
Total radiated power:   252.8 W/m^2 (24.0 dBW)
Peak gain:              15.2 (11.8 dBi)
Half-power beadwidth:   6.3 deg
Side-lobe level:        13.2 dB
Radiation resistance:   7.9 Ohms
```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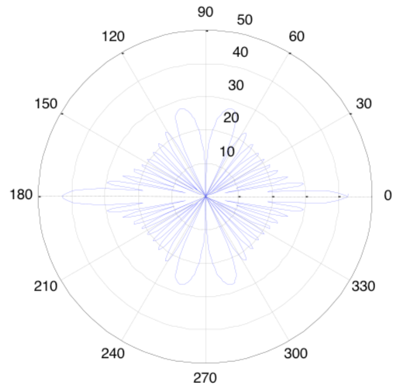
Vertical Array of 10 λ/2-Dipoles

Directivity of 10 stacked λ/2 dipoles




```
Half-wave dipole array of 10 element(s)
Element input current 10.0 A
----------------------------------------------
Total radiated power:    313.3 W/m^2 (25.0 dBW)
Peak gain:               19.1 (12.8 dBi)
Half-power beadwidth:    4.5 deg
Side-lobe level:         13.1 dB
Radiation resistance:    6.3 Ohms
```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Example: Base Station Antenna Spec Sheet

Stella Doradus Ireland Ltd.

24 12008 2.4GHz Base Station antenna

Electrical Specification	
Gain	15dBi
3dB beam Pattern	120° x 8°
Bandwidth	2.4-2.485Ghz
VSWR	1.8 : 1
Front to Back Ratio	33dB
Polarization	Vertical
Power Rating	50W
Impedance	50 ohms
Termination	N-female
Cross Pol. Discrimination	22dB
Surge Protection	In Built

Mechanical Specifications	
Length	100cm
Width	17cm
Depth	10 cm
Weight	2Kg
Windage(at 215km/h)	49kg
Mechanical Tilt	0-25 degrees
Mounting Pipe	5 cm pipe

Materials	
Radiating Element	Beam forming PCB patch array
Radome (feed)	ABS Grey
Clamps	HDG steel + galvanised steel bolts

The 24 12008 is a base station point to point antenna designed for use in high density RF environments
The excellent radiation characteristics are the distinguishing features of these antennas

http://www.stelladoradus.com/pdfs/2.4Base/24%2012008%20(14-05-08).pdf

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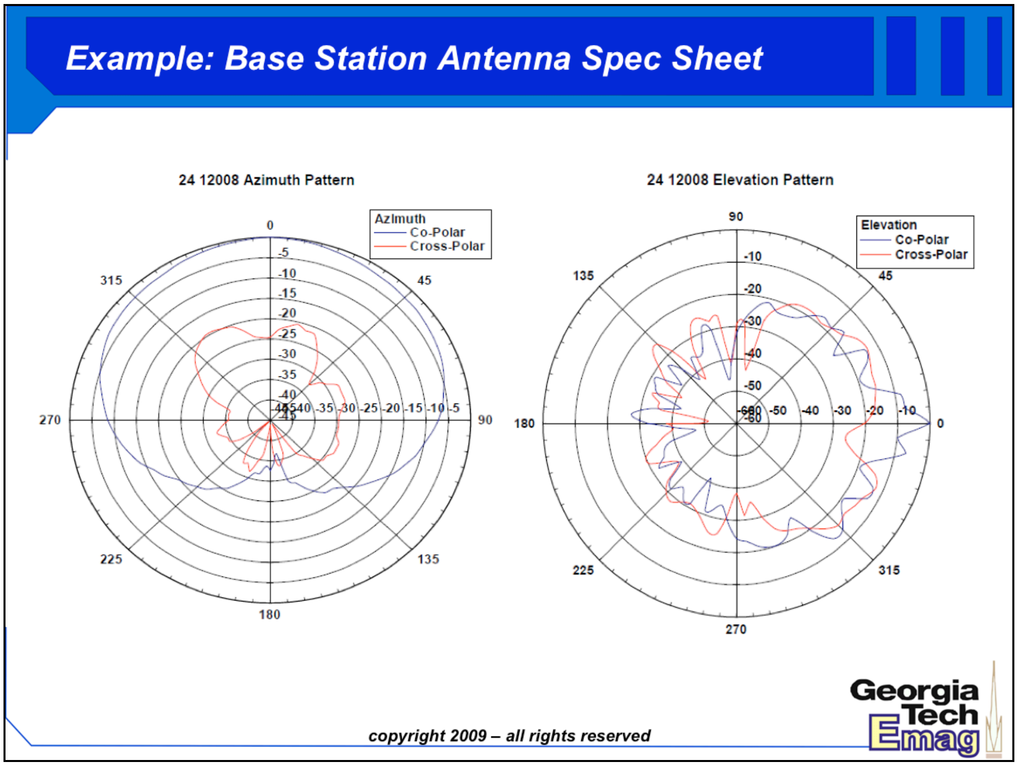
Example: Base Station Antenna Spec Sheet
24 12008 Azimuth Pattern
Azimuth
Co-Polar
Cross-Polar
0
45
90
135
180
225
270
315
-5
-10
-15
-20
-25
-30
-35
-40
24 12008 Elevation Pattern
Elevation
Co-Polar
Cross-Polar
90
45
0
315
270
225
180
135
-10
-20
-30
-40
-50
-60
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Example: Base Station Antenna Spec Sheet

WIRELESS EDGE LTD.

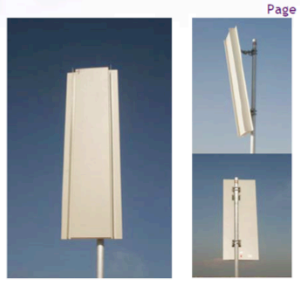
Antenna Data Sheet
MT-243015/NH
Page 1 of 2

MT-243015/NH

902-928 MHz 10dBi 180°

Horizontal Pol.

Base Station Antenna



Specifications

MTI PART NUMBER	MT - 243015/NH
ELECTRICAL	
REGULATORY COMPLIANCE	RoHS, CE 0682
FREQUENCY RANGE	902-928 MHz
GAIN	10 dBi (min)
VSWR	1.7 : 1 (typ), 2 : 1 (max)
AZIMUTH BEAMWIDTH	180° (typ)
POLARIZATION	Linear Horizontal
CROSS POLARIZATION	-18 dB (max) -20 dB (typ)
F/B RATIO	-5 dB (typ)
INPUT IMPEDANCE	50 (ohm)
INPUT POWER	20 W (max)
LIGHTNING PROTECTION	DC Grounded

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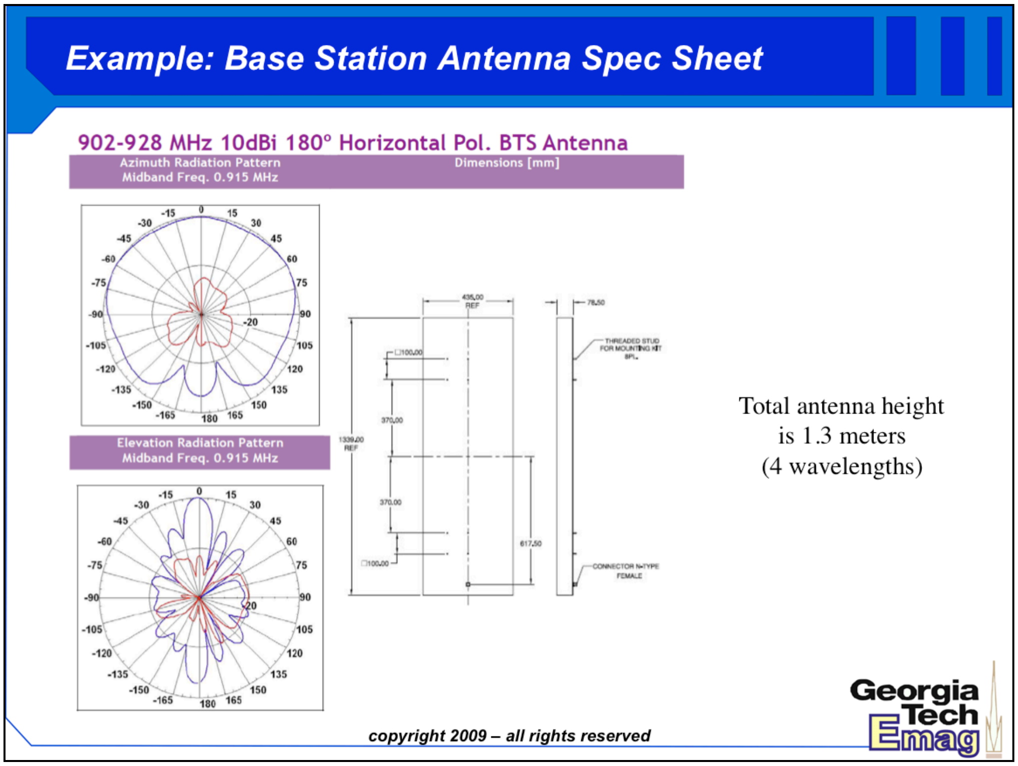
Example: Base Station Antenna Spec Sheet
902-928 MHz 10dBi 180° Horizontal Pol. BTS Antenna
Azimuth Radiation Pattern
Midband Freq. 0.915 MHz
Dimensions [mm]
Elevation Radiation Pattern
Midband Freq. 0.915 MHz
Total antenna height
is 1.3 meters
(4 wavelengths)
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