2.1 Antennas in Circuits

Antennas, like many other electrical components, may be modeled in a circuit as linear, lump-parameter equivalent elements consisting of sources, resistances, and reactances. This basic idea of circuit modeling tracks power flow and eventually enhances the physical understanding of the radio link budget, which tracks how much power travels through space from transmitter to receiver.

2.1.1 Antennas as Transmitters

An overall sketch of two equivalent time-harmonic circuits used for antennas – one for transmit antennas and the other for receiver antennas – is shown in Figure 2.1. For a transmit-mode, the antenna is modeled as a complex impedance $\tilde{Z}_A$ that is connected to a sinusoidal voltage source of amplitude V_S and source impedance $\tilde{Z}_S$. The phasors $\tilde{V}_A$ and $\tilde{I}_A$ are the amplitudes and phases of the voltage and current, respectively, at the terminals of the antenna.

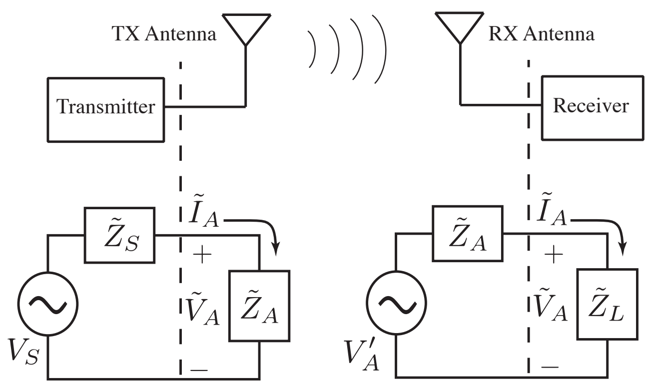

Figure 2.1. Equivalent circuits used to model current, voltage, and power flow for an antenna used as a transmitter (left) or as a receiver (right).

Based on these values, we can calculate the average power delivered into the antenna, P_T, by the source:

$$P_T = \frac{1}{2}\Re\left\{\tilde{V}_A\tilde{I}_A\right\} = \frac{V_S^2 R_A}{2\left|\tilde{Z}_S+\tilde{Z}_A\right|^2} \qquad \begin{array}{l}\tilde{Z}_S = R_S + jX_S \\ \tilde{Z}_A = R_A + jX_A\end{array} \tag{2.1.1}$$

The power P_T is the *transmit* power of the system and includes mismatch losses between the antenna and the source. Not all of this power will necessarily be radiated by the transmit antenna; some of the power may be absorbed, particularly if the antenna has a low *radiation efficiency*. However, a well-designed antenna will radiate most of P_T.

The maximum possible power that the source can deliver occurs when the antenna impedance $\tilde{Z}_A$ is conjugate matched with the source impedance $\tilde{Z}_S$. Under these conditions, the maximum possible power delivered by the source, P_S, is given by

$$\text{Conjugate Match } \tilde{Z}_A = \tilde{Z}_S^*: \quad P_S = \frac{V_S^2}{8R_S} \tag{2.1.2}$$

In terms of the total available source power, P_S, we can express the transmitted power as

$$P_T = \underbrace{\frac{4R_S R_A}{\left|\tilde{Z}_S + \tilde{Z}_A\right|^2}}_{\text{mismatch losses}} P_S \tag{2.1.3}$$

A word of caution: Equation (2.1.3) accounts for the mismatch losses in the electrical connection between transmitter hardware and the transmit antenna. Antenna specification sheets often report *realized gain* values in which the antenna *gain* – used later in RF link budgets – includes mismatch effects. If this is the case, then the maximum available source power, P_S, in Equation (2.1.2) should be used as the transmit power. Otherwise, the electrical mismatch losses will be double-counted in power calculations.

2.1.2 Antennas as Receivers

Power can also be tracked at a receiver antenna using the equivalent circuit on the right-hand side of Figure 2.1. The real average power, P_L, delivered to the load impedance representing the receiver hardware in Figure 2.1 is given by

$$P_L = \tfrac{1}{2}\Re\left\{\tilde{V}_A \tilde{I}_A\right\} = \tfrac{V_A'^2 R_L}{2\left|\tilde{Z}_A + \tilde{Z}_L\right|^2} \qquad \begin{array}{l} \tilde{Z}_L = R_L + jX_L \\ \tilde{Z}_A = R_A + jX_A \end{array} \tag{2.1.4}$$

If an transmit antenna with impedance $\tilde{Z}_A$ is used for receiving purposes, it will have the same impedance value $\tilde{Z}_A$ when its role is reversed. This convenient property is a direct result of *reciprocity* in electromagnetism.

The maximum available received power, P_R, from the antenna can only be delivered to the load under conjugate-matched conditions:

$$\text{Conjugate Match } \tilde{Z}_L = \tilde{Z}_A^*: \quad P_R = \frac{V_A'^2}{8R_A} \tag{2.1.5}$$

This maximum received power, P_R, is commonly available as a specification of a communications link or as the product of a *link budget* calculation. Thus, Equation (2.1.5) gives us a more convenient relationship for V_A' in terms of received power:

$$V_A' = 2\sqrt{2R_A P_R} \tag{2.1.6}$$

Using this result, we can now calculate average power delivered to the receiver load, P_L, in terms of received power, P_R:

$$P_L = \underbrace{\frac{4R_L R_A}{\left|\tilde{Z}_A + \tilde{Z}_L\right|^2}}_{\text{mismatch losses}} P_R \tag{2.1.7}$$

The term *mismatch losses* in Equation (2.1.7) are a unitless value between 0 and 1, inclusive, that represent how efficiently power is being coupled into the receiver from the antenna.

An additional quantity that will be important to future calculations involving RF energy-harvesting circuitry and low-energy communications will be the peak amplitude of sinusoidal voltage at the terminals of the receiver antennas. In terms of received power, P_R, the amplitude of the output antenna voltage, V_A, is given by

$$V_A = \left|\tilde{V}_A\right| = \frac{2\left|\tilde{Z}_L\right|\sqrt{2R_A P_R}}{\left|\tilde{Z}_A + \tilde{Z}_L\right|} \tag{2.1.8}$$

For a given received power, P_R, voltage can be increased by increasing the overal impedance values of both antenna and load. Large voltage amplitudes are particularly desirable for energy-harvesting circuitry.

Example 2.1: Voltage on a 50Ω Coaxial Cable

Problem: A 50Ω coaxial cable connected to an antenna is receiving 20 dBm when connected to a matched load. What is the peak amplitude of its output voltage?

Solution: We first convert 20 dBm into linear Watts by using the following formula:

$$\text{Watts} = 10^{(\text{dBm}-30)/10} \rightarrow P_R = 0.10\,\text{Watts}$$

The amplitude follows from Equation (2.1.8) under the condition that $\tilde{Z}_A = \tilde{Z}_L = 50\Omega$. The peak voltage amplitude is 3.16 V.