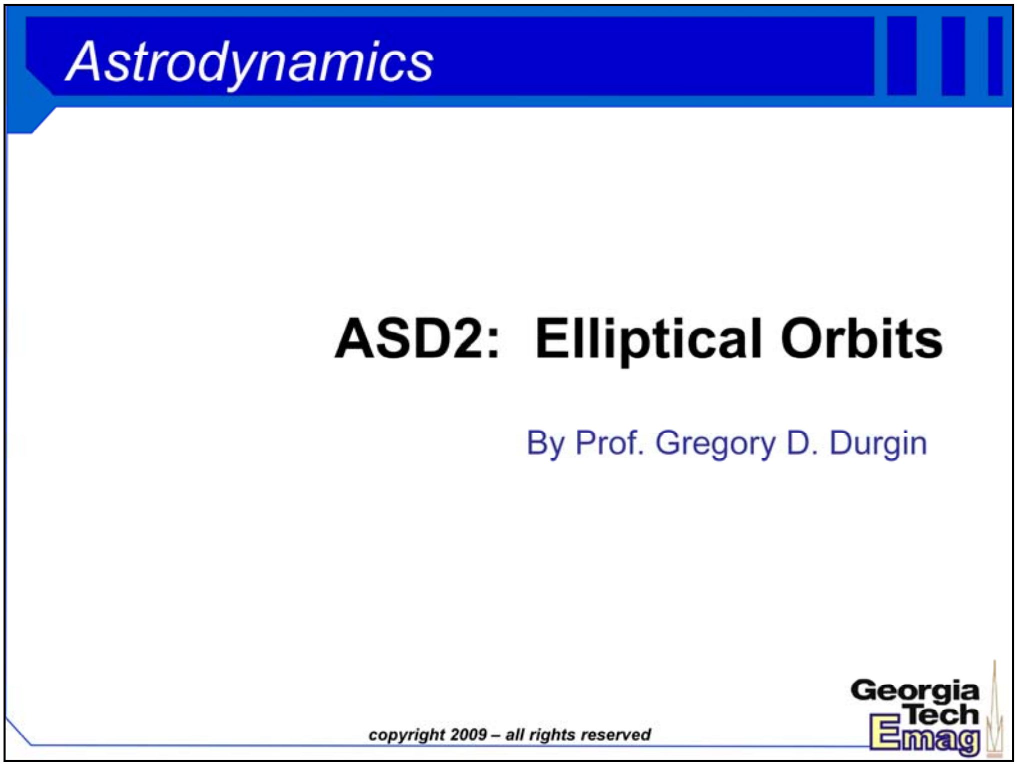
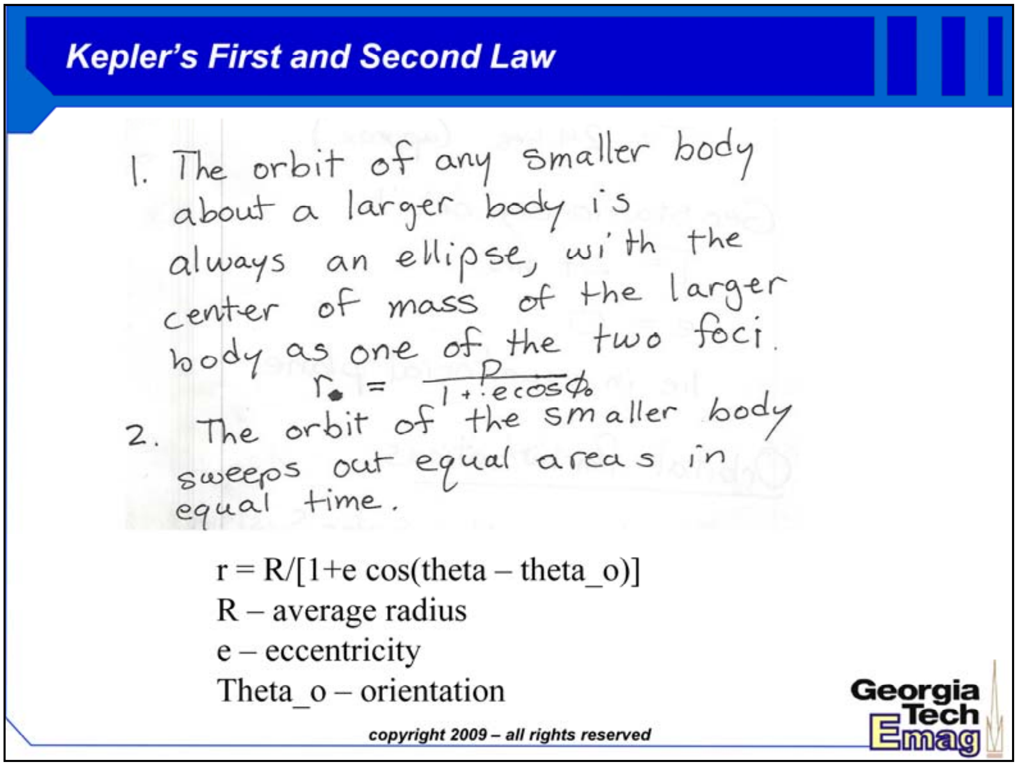


In the previous lecture, we discussed the basics of circular orbits. Mastering even circular orbits provides quite a bit of intuitive behavior about the motion of spacecraft about planets. We learned that orbiting spacecraft speed up as they get closer to their planets, how to classify earth orbits based on their altitudes, and touched upon basic trade-offs of these different orbits.

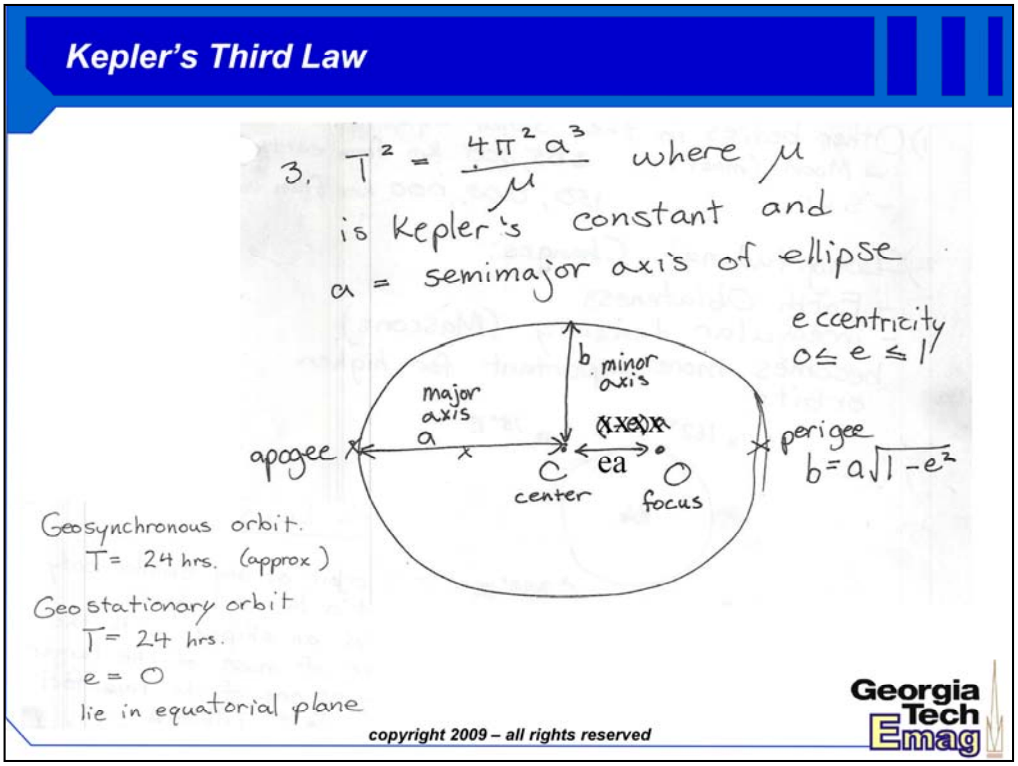
In this lecture, our analysis becomes more general and elegant. We'll deal with the general case of elliptical orbits using Kepler's 3 laws.



Kepler's laws consist of 3 rules that two-body orbits must satisfy.

The first law states that the orbit of a smaller body (spacecraft) about a larger body (planet or sun) is always an ellipse, with the center of mass of the larger body as one of the two foci. Above is the mathematical description of an elliptical path (use lower formula). Recall from geometry that the e-parameter is eccentricity, a measure of how oblong the ellipse is. Always between the values of 0 and 1, inclusive, eccentricity of 0 implies a perfect circle (like the number "0") and an eccentricity of 1 implies an ellipse that has been stretched to zero-thickness along one direction (like the number "1").

Kepler's second law states that the orbit of the smaller body sweeps out equal areas in equal time. What does this mean? A space craft in orbit around a planet, regardless of that orbit's shape, will speed up when it is closer to the center of the planet and will slow down when it is further from the center. The swept area mathematical definition is an elegant, precise way to formulate this principle; we see this at work trivially in the case of circular orbits where the "pie-slice area" swept out was constant at all points of the orbit.



Kepler's third law allows us to compute the period of a satellite in elliptical orbit given its geometrical properties.

Consider the geometry of an ellipse as described by its semi-major axis (a) and its semi-minor axis (b). The greater the eccentricity, the large (a) becomes relative to (b). In fact, there is a simple relationship (given above) between all 3 of these quantities.

The planet must rest on one of the foci of this ellipse. The perigee (on earth; periapsis in the general case) is the distance between the origin and the spacecraft at the closest distance to the planet. This distance occurs along the major axis and is basically (1-e)a. The apogee (on earth; apoapsis in the general case) is the distance between the origin and the spacecraft at the furthest distance to the planet. This distance also occurs along the major axis on the opposite end of the orbit, for a total distance (1+e)a. Note that the average of apogee and perigee will always produce the semi-major axis a.

At apogee and perigee, the orbiting spacecraft has no radial velocity component – all velocity is tangential. Also note that, although the speed of the spacecraft changes during its orbit, there is a temporal symmetry about the major axis to its travels. A journey from perigee to apogee, and vice versa, takes exactly one half of a period T.

Apsis and Periapsis

Body	Closest approach	Farthest approach
Galaxy	Perigalacticon	Apogalacticon
Star	Periastron	Apastron
Black hole	Perimelasma/Peribothra/Perinigricon	Apomelasma/Apobothra/Aponigricon
Sun	Perihelion	Aphelion[1]
Mercury	Perihermion	Apohermion
Venus	Pericytherion/Pericytherean/Perikrition	Apocytherion/Apocytherean/Apokrition
Earth	Perigee	Apogee
Moon	Periselene/Pericynthion/Perilune	Aposelene/Apocynthion/Apolune
Mars	Periareion	Apoareion
Jupiter	Perizene/Perijove	Apozene/Apojove
Saturn	Perikrone/Perisaturnium	Apokrone/Aposaturnium
Uranus	Periuranion	Apouranion
Neptune	Periposeidion	Apoposeidion
Pluto	Perihadion	Apohadion

from Wikipedia (August 2009)

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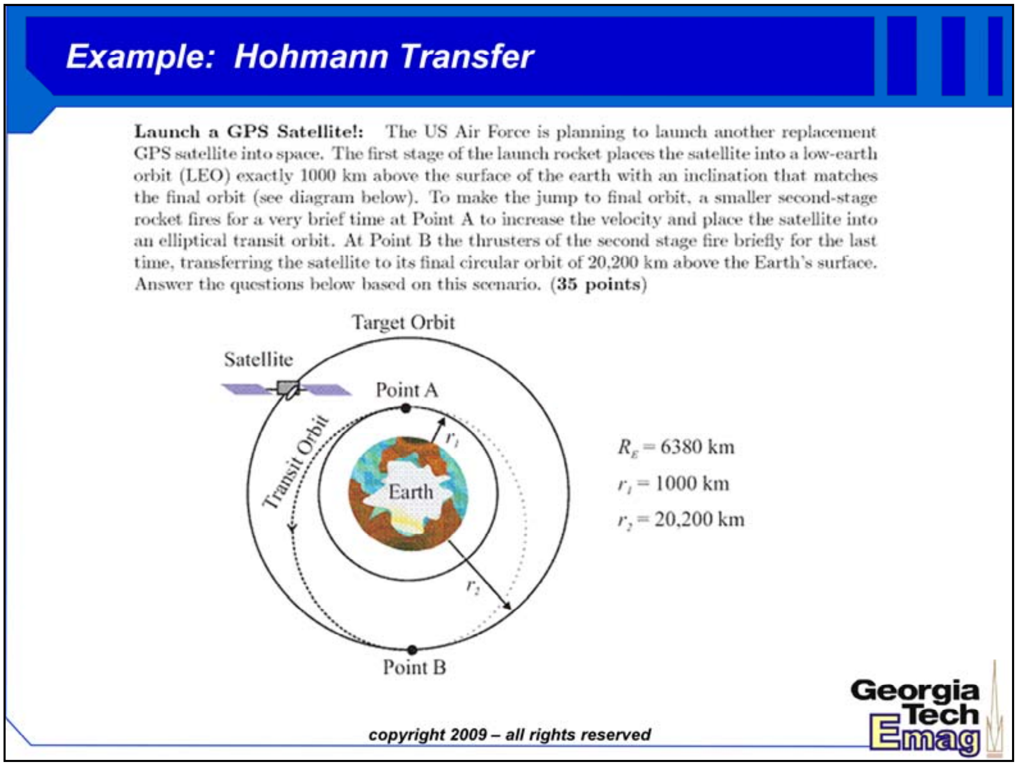
It is an unavoidable fact of life in a fallen world that much of learning in a profession is couched in specific jargon, the language of which often obscures what is actually very simple concepts. In some fields, practitioners take advantage of this jargon and use it as a sort of blockade to any novice from entering a profession that is otherwise straightforward and even a little lite on concepts (fill in your own joke here).

Normally we engineers and the more practical scientists try to avoid this, but in the case of apsis and periapsis, it would appear that we have tried to make this as complicated as possible. Vastly different terms are applied to this very simple concept depending on what the spacecraft is orbiting. In fact, it gets so ridiculous, that one usually must delve into the legal profession or the field of biology to find this level of nomenclature chicanery.

The terms apsis and periapsis are general; apogee and perigee are applied when the orbit is around earth; aphelion and perihelion are applied when the orbit is around the sun; and so forth. Observe the enormous variety in the table above. As soon as scientists discover something new to orbit around, they feel compelled to invent new terminology to re-describe an ellipse.

Particularly humorous is the apparent multiplicity of orbital terms for black holes. We've never sent a satellite into orbit about a black hole and likely have no intentions of doing so for the next few millennia, and yet there appears to already exist three competing terminologies for such an orbit.



An initial rocket stage launches satellites into LEO. From there, depending on how high the final orbit is, a transfer orbit or trajectory is required to move the satellite into its final orbit. A Hohmann transfer orbit is one such example, commonly used because of its low-energy requirements and short transit time.

The Hohmann transfer orbit requires a short, energetic burst of thrust from the spacecraft in LEO. By adding kinetic energy at what is nearly a single point in the orbit (Point A), the spacecraft has been transferred to an elliptical orbit, swinging round the planet to a much higher altitude at apogee (Point B). Another burst of thrust at the apogee of the elliptical orbit transfers the spacecraft into a higher-energy orbit. If the amount of thrust is carefully calculated, this orbit can be "circularized" and the final MEO, GEO, or HEO achieved.

The Hohmann transfer requires minimal energy because both thrusts occur parallel to the direction of spacecraft travel (recall that at perigee and apogee of the elliptical orbit, the spacecraft only has a radial velocity component). The transfer does not waste any energy changing directions of the spacecraft – only in bringing the kinetic energy of the craft up to the level to maintain first the elliptical orbit and then the circular orbit.

The Hohmann transfer is also relatively expedient. Although the spacecraft could be placed indefinitely into the elliptical transfer orbit for any number of cycles before performing the transfer to the target orbit, it only takes half a period T to complete the journey to the final trajectory in space.

Example: Hohmann Transfer

(c) We use our geometrical formulas relating apogee and perigee to eccentricity:

$$\frac{\text{Perigee, } r_p}{\text{Apogee, } r_a} = \frac{a(1-e)}{a(1+e)} \longrightarrow$$

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{(6380\text{ km} + 20{,}200\text{ km}) - (6380\text{ km} + 1000\text{ km})}{(6380\text{ km} + 20{,}200\text{ km}) + (6380\text{ km} + 1000\text{ km})} = 0.56$$

(d) We use the Kepler's law equation for semi-major axis:

$$T^2 = \frac{4\pi^2 a^3}{\mu} \quad \text{where } a = \frac{r_a + r_p}{2}$$

Since we are using exactly half the orbit, we only need to use half the period. The result is a transit time of approximately 11,000 seconds (3 hours and 3 minutes) long.

(e) The final orbit speed is approximately 3.8 km/s.

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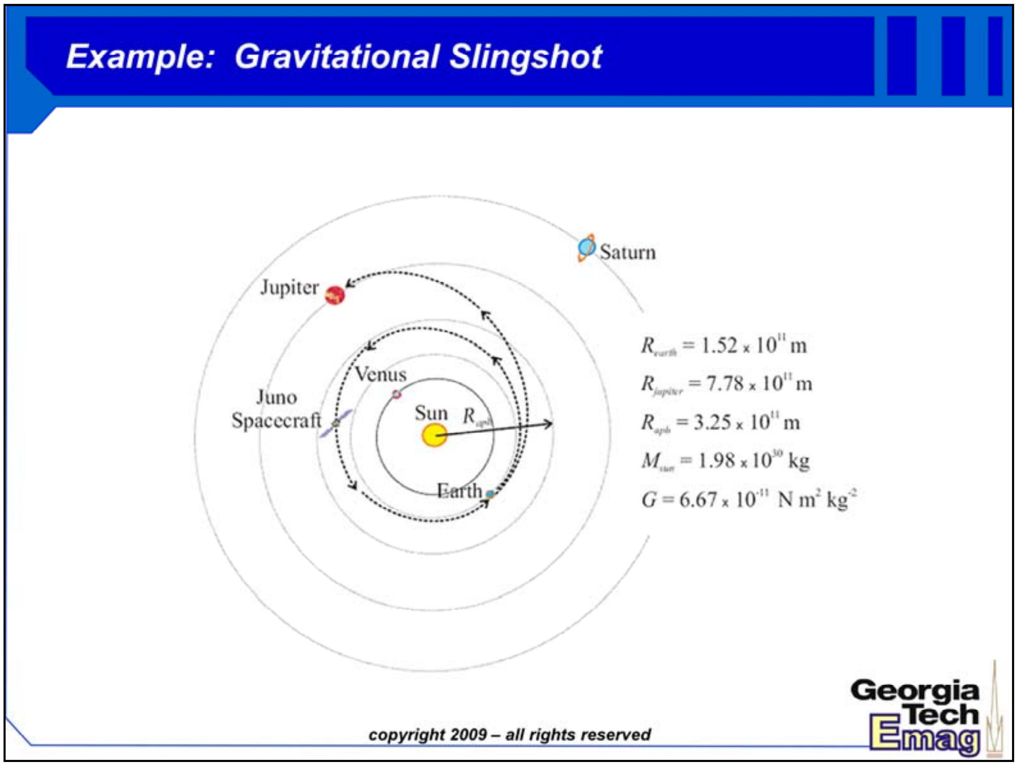
Let us calculate the eccentricity of this required transfer orbit. We know that perigee is at 1000 km + 6380 km and that apogee is 20,200 km + 6380 km. See above for how to solve for eccentricity in terms of perigee and apogee.

Now we can use Kepler's third law to calculate the total transit time. The formula predicts a total period of 22,000 seconds, but we only need half the period (3 hours and 3 minutes) to achieve the final orbit. Once at its final circular orbit at 26,580 km from the center of the earth, the satellite will be full of potential energy, but require "only" 3.8 km/s.

The Hohmann transfer is one of the most common transfer orbits used in spacecraft deployment, but other types are possible. A High Energy transfer could be used to speed things up; in this case, more thrust is generated at point A, allowing the spacecraft a shorter, faster trajectory to the target orbit. Once it achieves final orbit however, the spacecraft would have to spend quite a bit of fuel braking and changing its velocity vector to achieve the circular trajectory about the planet.

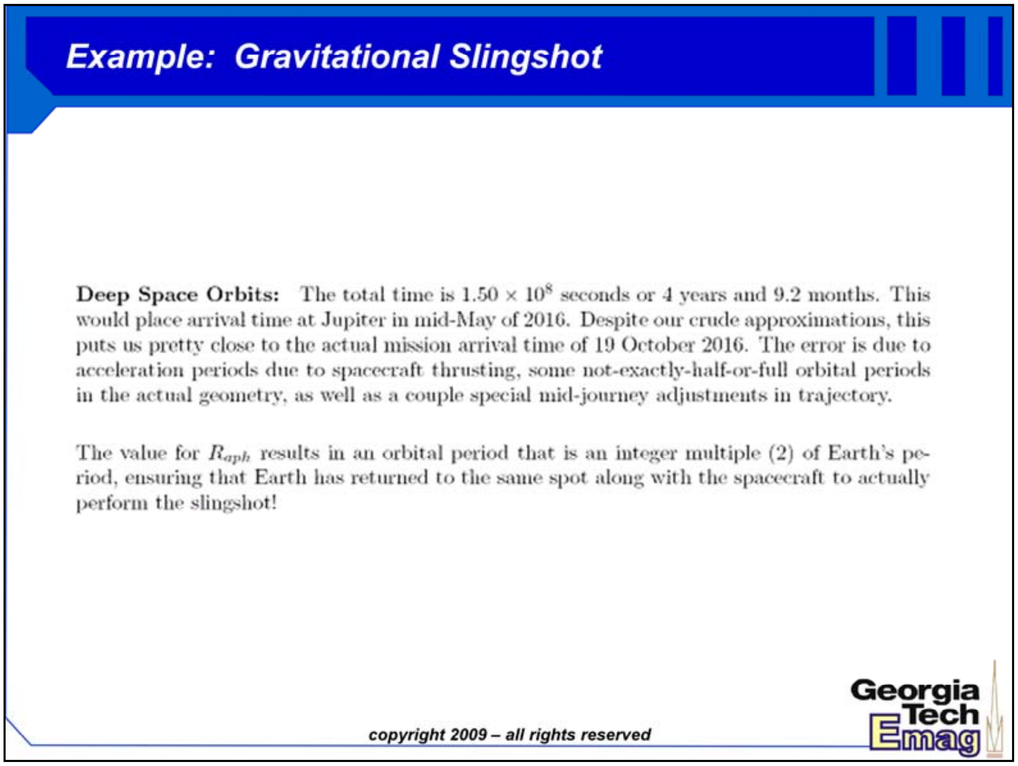
Other low-energy transfers are possible, where the spacecraft fires engines continuously at point A and spirals slowly out to the final orbit. These orbits are typically reserved for spacecraft with low-power drives (such as ion enginees).



Let’s cut our new-found powers of orbital analysis on a deep space mission. Above is a diagram charting the trajectory of the proposed Juno mission to Jupiter. A *gravitational slingshot* is a method for propelling a spacecraft toouter planets without using extraordinary amounts of fuel, cost, and propulsion complexity. Under most circumstances, the orbit of a satellite around the solar system is an ellipse with the massive sun at one of the focii. The sun provides the principle gravitational forces to maintain the orbit, unless the spacecraft approaches very close to a planet. For a brief time period, the spacecraft can get a “free" boost in its relative velocity with respect to the sun by getting “slung forward" by the nearby gravity well of a planet in motion. This will transfer the satellite to a higher orbit without firing thrusters. Conservation of energy still holds – the spacecraft is simply borrowing some of the momentum of the massive, moving planet.

Above is a series of approximate proposed slingshot and orbits for use in NASA's Juno mission to Jupiter, with a proposed launch in August 2011. The spacecraft will first travel a full elliptical orbit with aphelion of *Raph* back to Earth to receive its slingshot. After this boost, the spacecraft completes a half-orbit that will rendezvous with Jupiter. Clearly, this is a very effective albeit time-consuming method for traveling to distant planets. Below is a diagram of Juno's approximate path through the solar system, as well as all the pertinent planetary data. Estimate the year and month that the spacecraft first arrived at Jupiter.

Also, why did the designers pick this precise value of *Raph*?



If we approximate these as Hohmann-like transfers (where the thrust is borrowed from the planets instead of propulsion drives) then we can make a series of estimates for each leg of the trip.

Leg 1: A single elliptical period from earth to earth (2 years)

Leg 2: Another half-elliptical period from earth to Jupiter (2 years and 9 months)

Note that with this crude model, we can estimate that the trip would take 4 years and 9 months. This would place arrival time at Jupiter in mid-May of 2016. The actual NASA predictions state an arrival date of 19 October 2016. The extra 5 months is a function of less-than-ideal assumptions made by our analysis (there are not perfectly aligned half-ellipses in the transfer orbit) along with some mid-journey planned thrusting adjustments.

Note that the value of Raph is non-negotiable. If we ran the calculations, we would find that the period for that orbit is exactly 2 years. Why? Because when the spacecraft returns to perihelion, it needs to encounter the earth again if it is to slingshot to a higher level orbit. Thus, an integer number of *n* years needs to elapse before the spacecraft returns to this point. Since *n*=1 is not possible, and an overly large *n* would result in an elliptical orbit with aphelion beyond Jupiter (and likely too much energy for a realistic single slingshot), *n*=2 seems like a nice compromise.