VID3: Sampling and Quantization

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Claude E. Shannon (1916-2001)

- Mathematician and Electrical Engineer
- Worked for Bell Labs
  - Alumnus of U of Michigan and MIT
  - Eccentric researcher
- "Father of Information Theory"
- Key Paper
  - "A Mathematical Theory of Communication"
  - Published in Bell System Technical Journal, 1948.

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Key Contributions of Shannon's Paper

- Rate Distortion Theorem
  - Given a distortion criterion, what is the minimum number of bits for representing a signal?
- Lossless Data Compression
  - What is the minimum number of irreducible bits that can reproduce a data set?
- Channel Capacity Theorem
  - What is the minimum rate (in bits/sec) that data can be sent across a noisy channel?
- **Bonus:** Nyquist Sampling Theorem

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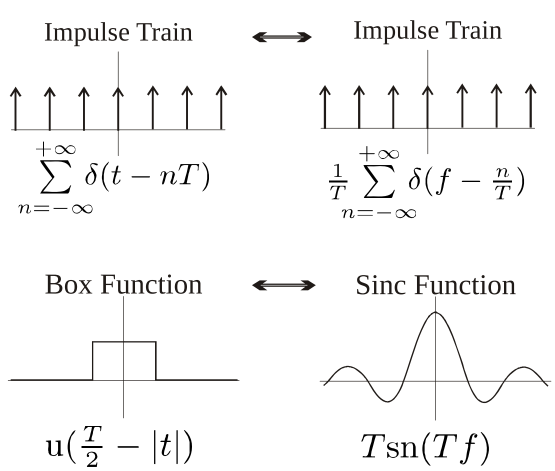
Quick Review of Fourier Transforms…



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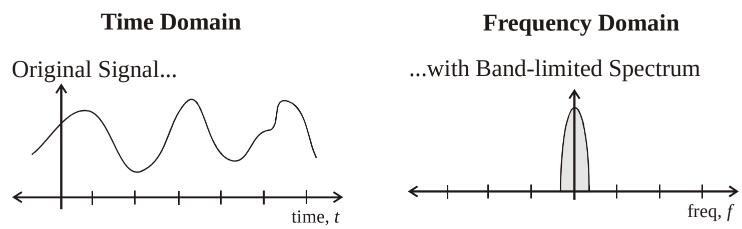
Sampling Theorem



- Start with time-domain signal with band-limited spectrum
- Continuous, real-valued signal

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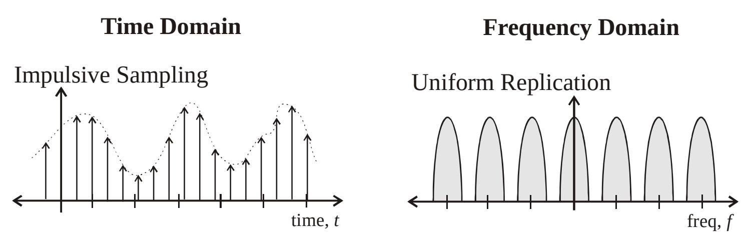
Impulsive Sampling



- Impulsive Sampling: keep only finite number of samples
  - Multiply by impulse train in the time domain
  - Convolve with impulse train in the frequency domain
- Mixing and/or filtering will recover *exact* original signal
- Requires sampling is faster than $2f_{max}$ (Nyquist rate)

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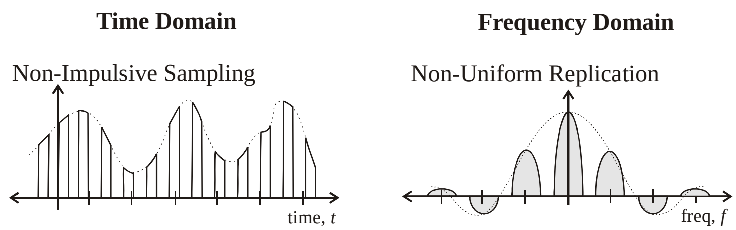
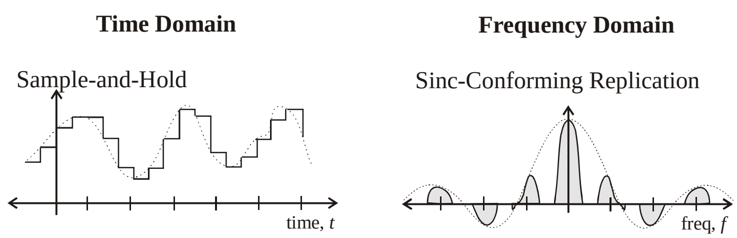
Non-Impulsive Sampling



- Time-Domain: multiply by a square wave
- Freq-Domain: convolve with non-uniform impulse train
- Nyquist rate still applies or else *aliasing* results

Sample-and-Hold

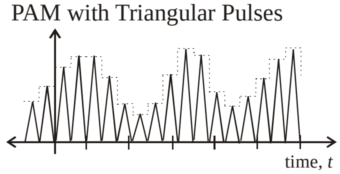


- Time Domain sequence: impulsive sample, convolve with square pulse
- Frequency Domain sequence: convolve signal spectrum with impulse train, multiply by sinc envelope
- Precise signal recovery requires filtering & equalization

Pulse Amplitude Modulation (PAM)

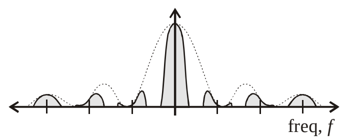
Time Domain

PAM with Triangular Pulses



Frequency Domain

Non-Impulsive Sampling



- Time Domain: impulsive sample, convolve with arbitrary pulse shape
- Frequency Domain: convolve signal spectrum with impulse train, multiply by single-pulse spectral envelope
- Precise signal recovery requires filtering & equalization

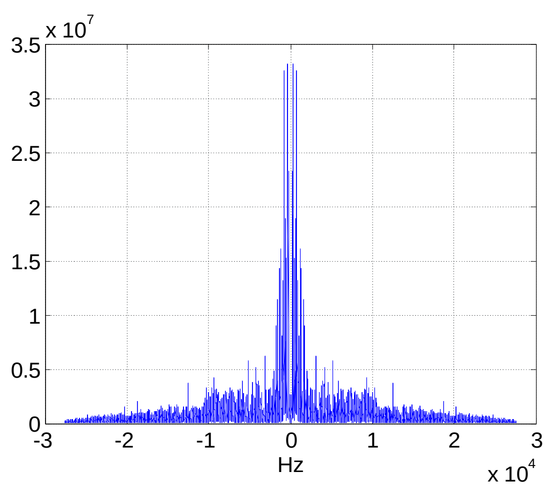
Example: Audio Spectral Content

Filter Levels

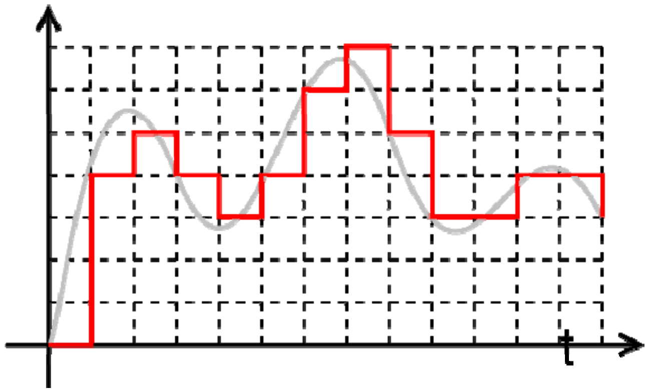
- Full Spectrum
- 20kHz Spectrum
- 10kHz Spectrum
- 5kHz Spectrum

Aliasing

- 10% aliasing
- 100% aliasing



Quantization



Example of Uniform Signal Quantization

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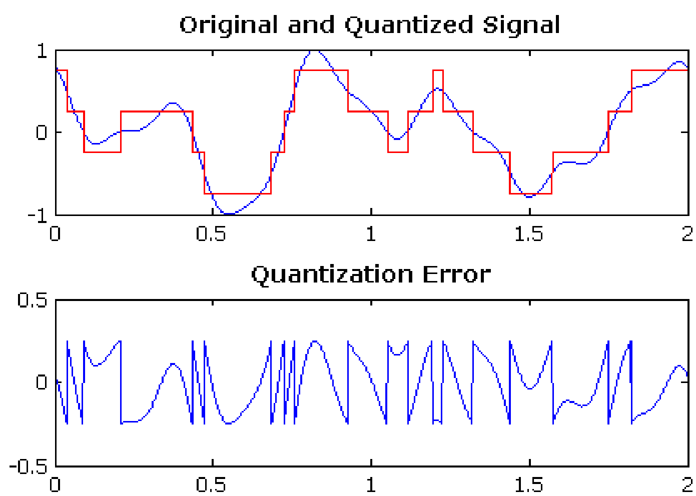
Quantization Noise



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Quantization Noise

Quantization Levels

- 8-bit quantization (SNR of 48 dB)
- 6-bit quantization (SNR of 36 dB)
- 4-bit quantization (SNR of 24 dB)
- 3-bit quantization (SNR of 18 dB)
- 2-bit quantization (SNR of 12 dB)
- 1-bit quantization (SNR of 6 dB)

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Properties of Uniform Quantization

- Provides uniformly-spaced quantization levels
  - Can span [0,+Vmax] or [-Vmax,+Vmax]
  - Works best when signal levels uniformly distributed
- Typical # of levels is related to # of bits/sample
  - 2^*M*, where *M* is number of bits/sample
  - Spacing of [Vmax – Vmin]/(2^M - 1)
- Quantization Signal-to-Noise Ratio is 6*M* (ideal)
- Non-uniform signals must be *companded*

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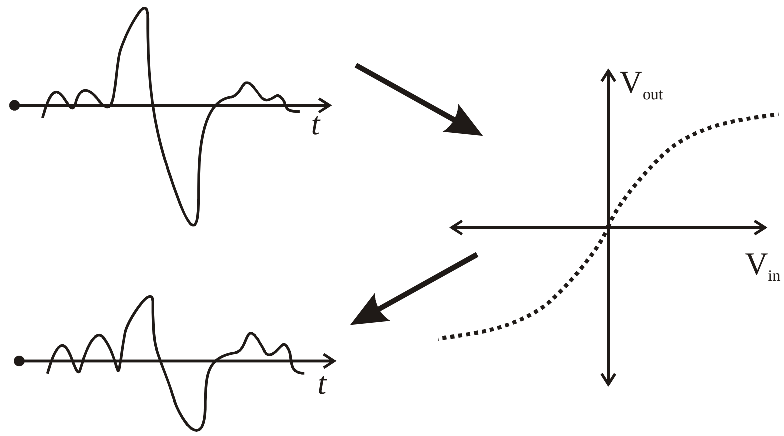
Companding a Signal



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Non-Uniform Quantization

- Alternative to companding
- Requires Lloyd-Max algorithm
  - Iterative procedure for choosing optimal levels
  - Could use results to design optimal compander as well

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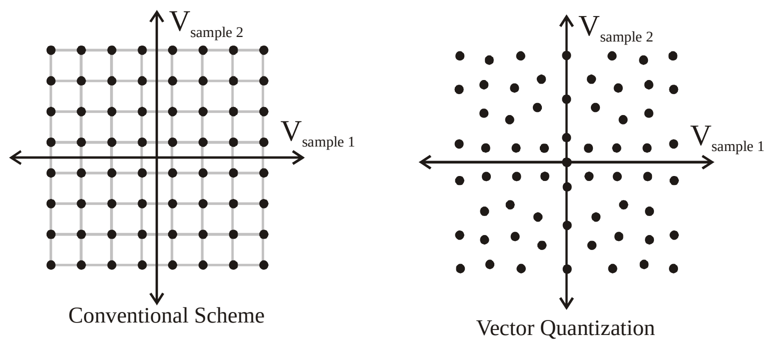
Vector Quantization

- Quantize 2 or more samples simultaneously
- Only way to approach Shannon Rate Distortion limit



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Example: Digitizing Analog Video

- Baseband signal has 5 MHz maximum frequency
  - Remember: starting point is a lousy analog signal
  - Nyquist sampling rate is 10 Msamples/sec
- Let's assume 8-level quantization
  - Requires 8 bits/sample
  - Visible SNR of 48 dB – pretty good picture
- Requires uncompressed bit rate of 80 Mbits/sec
  - Way too fast for many wired connections
  - Signal is still poor analog video ***plus*** quantization noise

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