4.1 Plane Wave Representation

This section describes the plane wave representations used in the analysis of small-scale wireless channels.

4.1.1 Electromagnetic Fields and Received Signals

The propagation of electromagnetic waves in space must be described using the vector quantities of electric or magnetic fields. However, a radio receiver does not receive a true vector signal. Rather, the input of a radio receiver is a scalar voltage or current emanating from the terminals of the receiver antenna. In essence, an antenna is a device that maps a space-varying, vector field quantity (with units of *Volts* m^{-1} or *Amps* m^{-1}) to a scalar voltage quantity (with units of *Volts* or *Amps*). This mapping is illustrated in Figure 4.1 for an electric field.

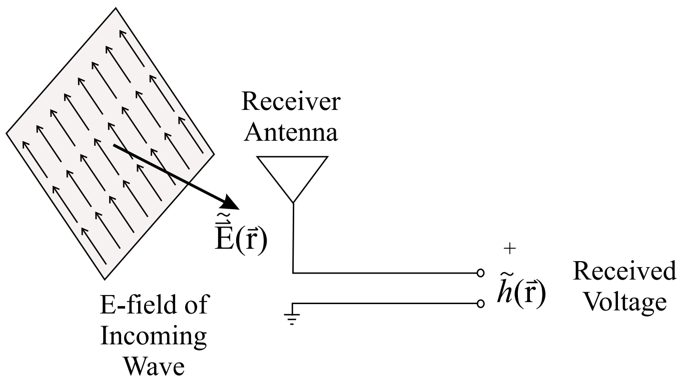

Figure 4.1 An antenna maps the complex electric field vector, $\tilde{\vec{\mathrm{E}}}(\vec{\mathrm{r}})$, to a scalar baseband channel voltage, $\tilde{h}(\vec{\mathrm{r}})$.

To avoid the complications of vector notation, we often use scalar voltage or current of the antenna instead of the vector fields of free-space. This implies that the effects of the antenna - gain, phase change, and polarization mismatch - have already been accounted for in the voltage representation. All of these antenna effects for a propagating wave may be accounted for by a single *polarization vector*, $\tilde{\vec{\mathrm{a}}}$. If the propagation of a single time-harmonic electromagnetic wave is described by an electric field vector, $\tilde{\vec{\mathrm{E}}}(\vec{\mathrm{r}})$, then the baseband voltage, $\tilde{h}(\vec{\mathrm{r}})$, results from the following operation:

$$\tilde{h}(\vec{\mathrm{r}}) = \tilde{\vec{\mathrm{E}}}(\vec{\mathrm{r}}) \cdot \tilde{\vec{\mathrm{a}}} \tag{4.1.1}$$

where $\vec{\mathrm{r}}$ is spatial translation of the receiver, or at least of its antenna. The dot product of the electric field and the polarization vector of the antenna produce the baseband voltage at the terminals of the receiver antenna.

The polarization vector, $\tilde{\vec{\mathrm{a}}}$, of Equation (4.1.1) must have units of distance (meters) to be dimensionally correct. The magnitude of the polarization vector is proportional to the gain of the antenna. Since the polarization vector is complex, it is capable of modeling phase change of the incoming signal. The orientation of the vector is capable of modeling polarization mismatch. The only limitation regarding the polarization vector notation is that $\tilde{\vec{\mathrm{a}}}$ changes as the incident angle of the impinging wave changes. For example, if multiple waves from different directions impinge upon the antenna, then a polarization vector must be calculated for each wave:

$$\tilde{h}(\vec{\mathrm{r}}) = \underbrace{\tilde{\vec{\mathrm{E}}}_1(\vec{\mathrm{r}}) \cdot \tilde{\vec{\mathrm{a}}}_1}_{\tilde{V}_1(\vec{\mathrm{r}})} + \underbrace{\tilde{\vec{\mathrm{E}}}_2(\vec{\mathrm{r}}) \cdot \tilde{\vec{\mathrm{a}}}_2}_{\tilde{V}_2(\vec{\mathrm{r}})} + \underbrace{\tilde{\vec{\mathrm{E}}}_3(\vec{\mathrm{r}}) \cdot \tilde{\vec{\mathrm{a}}}_3}_{\tilde{V}_3(\vec{\mathrm{r}})} + \ldots \tag{4.1.2}$$

Since much of our analysis requires breaking complicated propagating wave solutions into individual wave components, this aspect of polarization vectors is not restrictive.

The Physical Channel Convention

The channel in Equation (4.1.2) has units of *Volts* and, as defined, represents the physical voltage excited by waves at the terminals of the antenna. With this definition, the terms *channel* and *received voltage* may be used interchangeably. This convention, which we call the *physical channel convention* (PCC), is convenient. If a unitless information signal, $\tilde{x}(t)$, is sent through the channel according to Figure 2.4, then the received baseband signal, $\tilde{y}(t)$, has the units of *Volts* and represents an actual signal processed by receiver hardware.

The Normalized Channel Convention

Another useful way of defining the radio channel is the *normalized channel convention* (NCC). Using this convention, we define the radio channel model as a normalized version of Equation (4.1.2), dividing out the RMS channel power:

$$\tilde{h}(\vec{\mathrm{r}}) = \frac{\sum_i \tilde{V}_i(\vec{\mathrm{r}})}{\sqrt{\left\langle \left| \sum_i \tilde{V}_i(\vec{\mathrm{r}}) \right|^2 \right\rangle}} \tag{4.1.3}$$

where $< \cdot >$ is an averaging operator (ensemble or spatial). Equation (4.1.3) is useful for application analysis involving fading power levels. In the NCC case, the average power is treated as a multiplying constant that subsumes hardware-dependent constants such as impedance, digital signal filtering, or the $\frac{1}{2}$ multiplier discussed in Section 2.1.4. Most of our discussion is general to both the PCC and the NCC; if not, the text will specify which channel convention is appropriate.

4.1.2 The Maxwellian Basis

In order to describe complicated propagation in a bounded region of linear free-space, it is useful to break down the received voltage levels as a function of space into a *solution basis.* A solution basis is a set of elementary functions whose linear combinations span every possible solution for a set of differential equations - in our study, Maxwell's equations of free-space radio wave propagation. For example, we may postulate that a solution basis for any baseband received voltage is the set of all complex sinusoidal waves:

$$\tilde{h}(\vec{\mathrm{r}}) = \sum_i V_i \exp(j[\phi_i - \vec{\mathrm{k}}_i \cdot \vec{\mathrm{r}}]) \tag{4.1.4}$$

where $\{V_i\}$ are real amplitudes, $\{\phi_i\}$ are real phases, and $\{\vec{\mathrm{k}}_i\}$ are real vectors. According to basic Fourier analysis, *any* 3D function such as complex voltage, $\tilde{h}(\vec{\mathrm{r}})$, may be written in the form of Equation (4.1.4).

The basis of Equation (4.1.4) is purely mathematical. In radio wave propagation, the solution basis for a channel must satisfy Maxwell's equations in addition to spanning all possible functions of received voltage. In fact, only a select few terms of the summation in Equation (4.1.4) will obey the fundamental laws of propagation. A complete solution basis in which every term solves Maxwell's equations is called a *Maxwellian basis* [Bro98].

One Maxwellian basis that may be used to construct any realizeable set of received voltages in bounded free-space is the set of all plane waves. This basis is very similar to Equation (4.1.4):

$$\tilde{h}(\vec{\mathrm{r}}) = \sum_i V_i \exp(j[\phi_i - \vec{\mathrm{k}}_i \cdot \vec{\mathrm{r}}]) \qquad \text{where } \vec{\mathrm{k}}_i \cdot \vec{\mathrm{k}}_i = k_0^2 \tag{4.1.5}$$

In Equation (4.1.5), $\{V_i\}$ are real amplitudes, $\{\phi_i\}$ are real phases, and $\{\vec{\mathrm{k}}_i\}$ are constant wavevectors [Bor80, p. 562]. The terms of Equation (4.1.5) are called *plane waves* due to the geometry of the equiphase surfaces. An equiphase surface of a propagating wave is defined as the set of points, $\vec{\mathrm{r}}$, in three-dimensional space that satisfy the following equation:

$$\arg\{\tilde{h}(\vec{\mathrm{r}})\} = \phi_0 \tag{4.1.6}$$

where ϕ_0 is some arbitrary phase constant. The equiphase surfaces of each term in Equation (4.1.5) form planes in three-dimensional space.

At first glance, Equation (4.1.5) appears to be a small subset of the Fourier basis in Equation (4.1.4). The condition $\vec{\mathrm{k}}_i \cdot \vec{\mathrm{k}}_i = k_0^2$ restricts the solution set to plane waves with wavevectors of a certain magnitude (see Theorem 4.1 for a derivation of this condition). But unlike the Fourier basis, *the wavevectors of Equation (4.1.5) may be complex-valued.* A complex $\vec{\mathrm{k}}_i$ still solves Maxwell's time-harmonic equations and is needed to describe every possible instance of received voltage in free-space.

Theorem 4.1: Wavevector Criterion for free-space

Statement: The free-space wavevectors in a plane wave basis obey the condition in Equation (4.1.5).

Proof: The electric field of a plane wave propagating in free-space takes the following form:

$$\tilde{\vec{\mathrm{E}}}(\vec{\mathrm{r}}) = \tilde{\vec{\mathrm{E}}}_0 \exp(-j\vec{\mathrm{k}}_0 \cdot \vec{\mathrm{r}})$$

where $\tilde{\vec{\mathrm{E}}}_0$ is a complex, constant electric field vector and $\vec{\mathrm{k}}_0$ is a constant-valued wavevector. Any solution of this equation must solve the scalar wave equation for free-space:

$$(\nabla^2 + k_0^2)\tilde{\vec{\mathrm{E}}}(\vec{\mathrm{r}}) = 0$$

where k_0 is the free-space wavenumber of time-harmonic propagation and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. We may then write

$$\begin{aligned} 0 &= (\nabla^2 + k_0^2)\ \tilde{\vec{\mathrm{E}}}_0 \exp(-j\vec{\mathrm{k}}_0 \cdot \vec{\mathrm{r}}) \\ &= \tilde{\vec{\mathrm{E}}}_0\ \left[k_0^2 - \vec{\mathrm{k}}_0 \cdot \vec{\mathrm{k}}_0\right]\ \exp(-j\vec{\mathrm{k}}_0 \cdot \vec{\mathrm{r}}) \end{aligned}$$

which took advantage of the following vector relationship:

$$\nabla^2\ \exp(-j\vec{\mathrm{k}}_0 \cdot \vec{\mathrm{r}})\ =\ -\vec{\mathrm{k}}_0 \cdot \vec{\mathrm{k}}_0\ \exp(-j\vec{\mathrm{k}}_0 \cdot \vec{\mathrm{r}})$$

Thus, the wavevector, $\vec{\mathrm{k}}_0$, must satisfy the following relationship:

$$\vec{\mathrm{k}}_0 \cdot \vec{\mathrm{k}}_0 = k_0^2$$

All plane waves must have wavevectors of this form in order to be a valid solution of Maxwell's equations in free space.

Note: Frequency, Wavelength, and Wavenumber

Recall the basic result from physics that all waves traveling in a linear medium satisfy $f_c\lambda = c$, where f_c is carrier frequency, λ is wavelength, and c is the speed of propagation (for electromagnetic propagation, this is the speed of light, 3.0×10^8 m/s). Since k_0 is the free-space wavenumber, we may write this simple relationship in an alternative form:

$$\frac{2\pi f_c}{k_0} = c \qquad \left(k_0 = \frac{2\pi}{\lambda}\right)$$

The terms of the Maxwellian plane wave basis neatly divide into two different types of plane waves. The first group consists of *homogeneous* plane waves with strictly real-valued wavevectors. The second group consists of *inhomogeneous* plane waves with complex-valued wavevectors. Thus, Equation (4.1.5) may be rewritten as

$$\tilde{h}(\vec{\mathrm{r}}) = \underbrace{\sum_l V_l \exp(j[\phi_l - \vec{\mathrm{k}}_l \cdot \vec{\mathrm{r}}])}_{\substack{\text{Homogeneous,} \\ \text{Real } \vec{\mathrm{k}}_l}} + \underbrace{\sum_m V_m \exp(j[\phi_m - \vec{\mathrm{k}}_m \cdot \vec{\mathrm{r}}])}_{\substack{\text{Inhomogeneous,} \\ \text{Complex } \vec{\mathrm{k}}_m}} \qquad (4.1.7)$$

Each classification of plane wave is discussed below.

> *Note: Definition of Free-Space*
>
> The term *free-space* in propagation implies the following three characteristics:
>
> 1. There are no current sources or charges present in the medium.
> 2. The propagation medium is *linear*, *isotropic*, and *lossless*.
> 3. The material parameters of permittivity and permeability are constants and are equal to that of vacuous space ($\epsilon = \epsilon_0$ and $\mu = \mu_0$).
>
> It is generally accepted that the atmosphere - despite the presence of gases - behaves as a free-space medium for short-range propagation.

4.1.3 Homogeneous Plane Waves

Homogeneous plane waves are also called *uniform* plane waves because their envelope is a constant value that does not depend on position in space. Since the wavevector of a homogeneous plane wave, $\vec{\mathrm{k}}$, is real-valued and satisfies the restriction noted in Equation (4.1.5), the plane wave may be written as

$$V(\vec{\mathrm{r}}) = V_0 \exp(j[\phi_0 - k_0 \hat{\mathrm{k}} \cdot \vec{\mathrm{r}}]) \tag{4.1.8}$$

where $\hat{\mathrm{k}}$ is a unit vector that points in the direction of propagation and k_0 is the free-space wavenumber. In a time-harmonic free-space analysis, k_0 is related to the wavelength of radiation, λ:

$$k_0 = \frac{2\pi}{\lambda} \tag{4.1.9}$$

Regardless of the direction of travel, all homogeneous plane waves propagate at the same speed with the same wavenumber. This is not true of inhomogeneous plane waves.

4.1.4 Inhomogeneous Plane Waves

Inhomogeneous plane waves are only slightly more difficult to understand than homogeneous plane waves. First, it helps to break the complex wavevector into a real and imaginary part:

$$\vec{\mathrm{k}} = \beta\hat{\beta} - j\alpha\hat{\alpha} \tag{4.1.10}$$

where $\hat{\beta}$ is a unit vector pointing in the direction of the real part of $\vec{\mathrm{k}}$ and $\hat{\alpha}$ is a unit vector pointing in the direction of the imaginary part of $\vec{\mathrm{k}}$. This forces α and β to be real quantities.

The form of Equation (4.1.10) illuminates some basic properties of inhomogeneous plane waves. Imposing the wavevector restriction of the Maxwellian basis from Equation (4.1.5) leads to two descriptive conditions for a vector of this form:

$$\vec{\mathrm{k}} \cdot \vec{\mathrm{k}} = k_0^2 \;\longrightarrow\; \begin{array}{ll} \text{Condition 1:} & \hat{\alpha} \cdot \hat{\beta} = 0 \\ \text{Condition 2:} & k_0^2 = \beta^2 - \alpha^2 \end{array}$$

First, we see that the unit vectors $\hat{\alpha}$ and $\hat{\beta}$ must be orthogonal to one another (their dot products are zero). Second, while α and β may take on any positive value, they are dependent on one another. Specifically, the value $\sqrt{\beta^2 - \alpha^2}$ must be equal to the free-space wavenumber, k_0.

Armed with this information, it is now useful to introduce the split wavevector of Equation (4.1.10) into the expression for a propagating plane wave:

$$\tilde{h}(\vec{r}) = V_0 \overbrace{\exp(-\alpha[\hat{\alpha} \cdot \vec{r}])}^{\text{Attenuation}} \underbrace{\exp\left(j[\phi_0 - \beta(\hat{\beta} \cdot \vec{r})]\right)}_{\text{Phase Progression}} \tag{4.1.11}$$

Now the physical meanings of α and β are apparent. The value of β is the actual wavenumber of the inhomogeneous plane wave, determining the rate of phase progression through space in the direction $\hat{\beta}$. The value of α represents the rate of amplitude decay. As Equation (4.1.11) shows, the amplitude of the plane wave attenuates in the direction of $\hat{\alpha}$. The direction of amplitude attenuation must always be orthogonal to the direction of propagation for an inhomogeneous plane wave.

Figure 4.2 illustrates a comparison of homogeneous and inhomogeneous plane waves. Notice the two key differences between the types of plane waves. The wavenumber, β, of the inhomogeneous plane wave is always greater than the free-space wavenumber, k_0. It is useful to refer to an "effective wavelength," λ_{eff}, for inhomogeneous plane waves, defined as

$$\lambda_{\text{eff}} = \frac{2\pi}{\beta} \tag{4.1.12}$$

The effective wavelength of an inhomogeneous plane wave is always smaller than the free-space wavelength of radiation. As a result, the equiphase fronts of the inhomogeneous plane wave are more densely packed than the homogeneous plane wave.

The second key difference between homogeneous and inhomogeneous plane waves is the exponential decay of amplitudes perpendicular to the direction of propagation. If the decay rate, α, is small, then the wavenumber, β, approaches the homogeneous free-space wavenumber, k_0. If the decay rate, α, is large, then the wavenumber, β, is much larger than k_0. This principle is crucial for later sections in this chapter that develop physically-based small-scale channel models.

4.1.5 Homogeneous Versus Inhomogeneous Plane Waves

If free-space were unbounded, having infinite extent in every direction, then inhomogeneous plane waves would not exist. A quick study of Equation (4.1.11) tells us why. A wave that decays exponentially in one direction actually grows exponentially in the opposite direction. Thus, in unbounded free-space, the power density of an inhomogeneous plane wave becomes infinite towards one side of space - clearly a

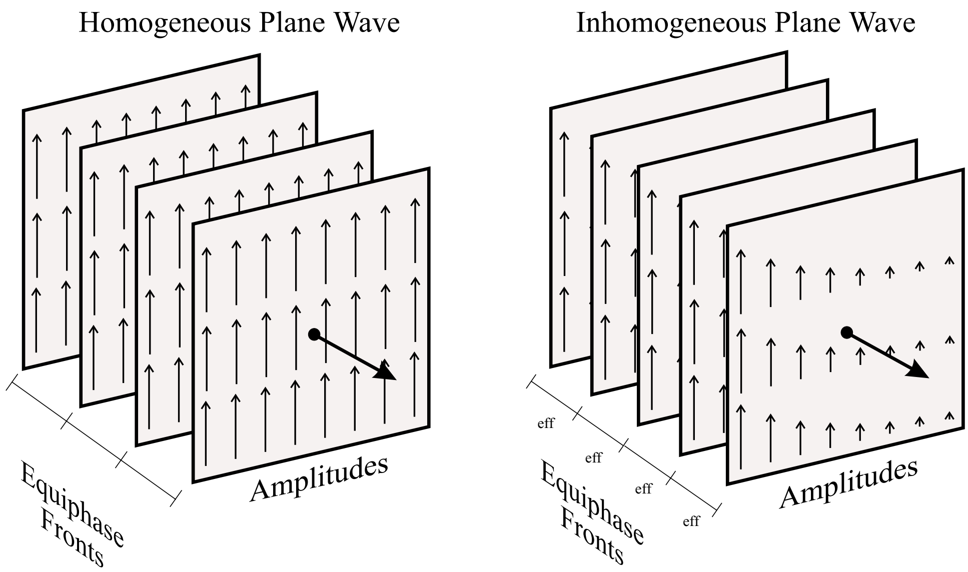

Figure 4.2 Homogeneous and inhomogeneous plane waves.

nonphysical result. Homogeneous waves have no such imbalance, having a constant, finite power density in all of space.

Although inhomogeneous plane waves solve Maxwell's equations in bounded free-space, these waves are caused by scatterers or sources in the propagation medium *outside* the bounded free-space region. The larger wavenumbers of inhomogeneous plane waves help meet the boundary conditions of electromagnetic fields close to material surfaces with fine spatial structure [Bor80, p. 563]. This physical characteristic leads to a rule of thumb for inhomogeneous plane wave propagation:

> *Boundary conditions near large scatterers typically introduce inhomogeneous plane waves into a region of bounded free-space. The direction of decay, $\hat{\alpha}$, for these waves tends to point away from the scatterers.*

The homogeneous waves follow a different rule of thumb:

> *The direction of propagation, $\hat{k}$, for homogeneous plane waves is typically away from the scatterers.*

These two rules of thumb are illustrated in Figure 4.3.

An Analogy From Circuit Theory

A useful analogy exists between free-space propagation and linear circuit theory that aids in understanding the nature of homogeneous and inhomogeneous plane

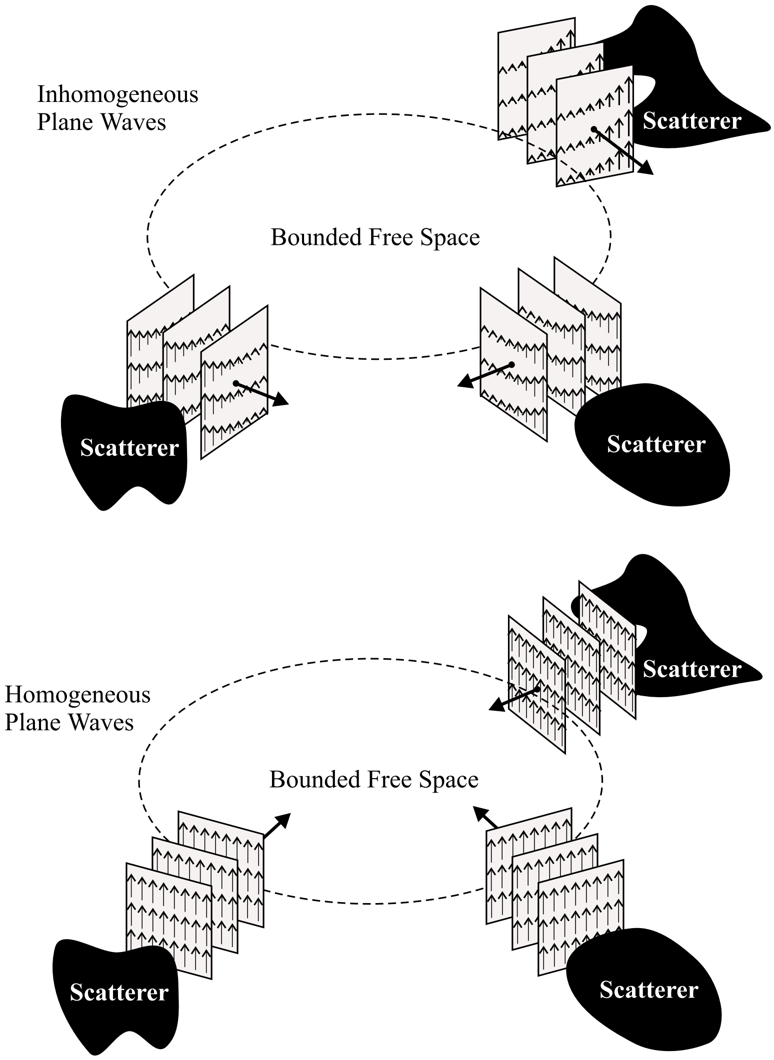

Figure 4.3 Rules of thumb for homogeneous and inhomogeneous plane wave propagation.

waves. Consider as an example the linear circuit of Figure 4.4. This circuit contains an AC voltage source, which switches on at $t = 0$. To solve for an output voltage, $V(t)$, as a function of time, we must solve the governing differential equation of the system. The order of a circuit differential equation is equal to the total number of inductors and capacitors. Thus, the example in Figure 4.4 has a seventh-order differential equation governing the input–output relationships.

While the source of Figure 4.4 is a time-harmonic AC source, the closing switch at time $t = 0$ introduces transient behavior into the circuit. The solution of this

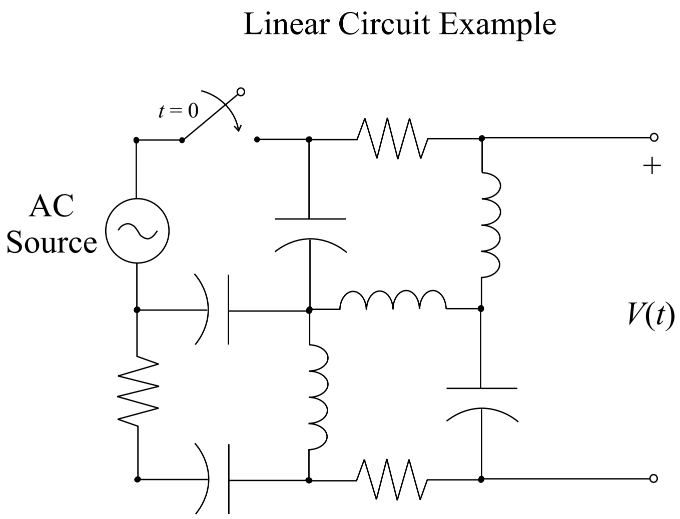

Figure 4.4 A linear circuit contains capacitors, inductors, resistors, and an AC source.

circuit for $t \geq 0$ may be written in the following form:

$$V(t) = [\text{Steady-State Solution}] + [\text{Transient Solution}] \quad \text{for } t \geq 0$$

A *steady-state solution* is a time-harmonic, sinusoidal voltage that eventually becomes the dominant solution for large values of t. The transient solution is an exponentially decaying set of voltages that transfers the circuit from its zero-excited state ($t < 0$) to its steady state ($t \gg 0$). The concept of steady-state and transient solutions are illustrated in Figure 4.5. If we make several modifications in terminology, then the linear circuit becomes a powerful analogy for understanding free-space propagation.

Analogy to Free-Space Plane Waves

The key difference between free-space propagation and the linear circuit example is that the plane wave propagation is a function of space rather than time. Homogeneous plane waves are like the steady-state solutions in the linear circuit. These plane waves are uniformly periodic (space-harmonic) with respect to space just as the steady-state solution is uniformly periodic (time-harmonic) with respect to time. Inhomogeneous plane waves are like the transient solutions in the linear circuit. The amplitudes of these plane waves decay exponentially with respect to space just as the transient solution decays in time.

In the propagation problem, the electromagnetic scatterer is like a "switch in space." The position of the scatterer in space is like the time that the circuit switch is thrown. Solutions for fields that are very close to the scatterer must pay attention

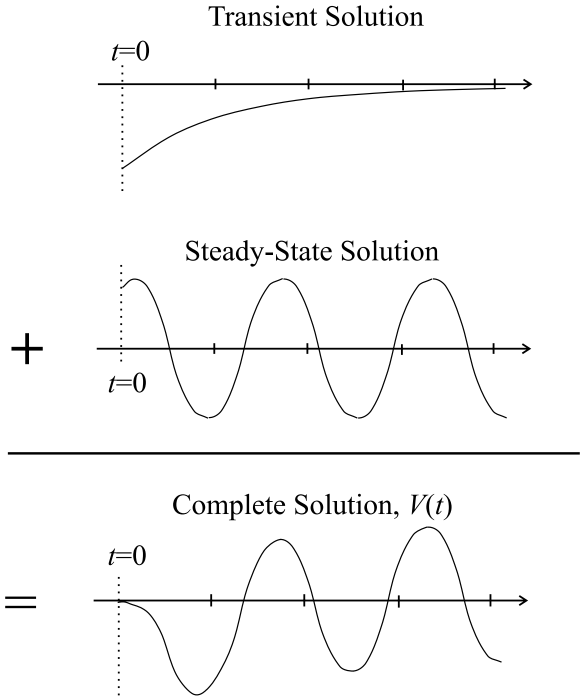

Figure 4.5 A linear circuit solution may be broken into a steady-state and transient solution.

to the behavior of inhomogeneous plane waves, just as the transient circuit solutions dominate for times close to the switching time of $t = 0$. Solutions for fields that are further from the scatterer may ignore the inhomogeneous plane waves, since they decay exponentially like the transient circuit solution.

There are numerous analogies to be made between the linear circuit of Figure 4.4 and the free-space propagation of plane wave solutions. Below is a summary of many of these parallel concepts:

LINEAR CIRCUIT CONCEPT	FREE-SPACE PROPAGATION CONCEPT
time	position
switch	scatterer
$t > 0$	bounded free-space
time elapsed since switch	distance from scatterer
steady-state solution	homogeneous plane waves
transient solution	inhomogeneous plane waves

The goal of this circuit analogy is, by using familiar engineering constructs, to make an esoteric concept (wave propagation in three dimensional space) easier to understand. Another motive is to help justify the key assumption in the definition of a local area, namely, the removal of inhomogeneous plane waves for receivers that operate away from significant scatterers.

> *Note: Analogy from Waveguide Theory*
>
> A reader familiar with dielectric waveguide theory may recognize inhomogeneous plane waves as the *evanescent* waves that propagate along a material interface of the waveguide and an open medium. Much like Figure 4.3, these waves are bounded to the material surface and decay into free-space exponentially away from the waveguide.

4.2 The Local Area

Used often in the wireless literature without definition, the term *local area* is one of the most important concepts in small-scale channel modeling. This section presents a definition of the local area and analyzes its usefulness and validity.

4.2.1 Definition of a Local Area

Small-scale fading due to spatial selectivity is the focus of a *local area* propagation analysis. Characterization of a channel autocorrelation function is just one example of a local area propagation analysis. This is in contrast to a *macro area* analysis, which focuses on the differences in propagation from one radio environment or location to the next. A comparison of average power received in the front of a shopping mall to average power received on the top story of a parking garage is an example of a macro area analysis.

Before proceeding, it is crucial to define exactly what is meant by a local area, since the definition can be quite arbitrary. There is no magical distance quantity that can be applied to any region of space and called a local area. Rather, the following definition will be applied throughout this work:

> A *local area* is the largest volume of free-space about a specified point ($\vec{\mathrm{r}} = \vec{0}$) in which the wireless channel can be modeled accurately as the sum of homogeneous plane waves of the form
>
> $$\tilde{h}(f, \vec{\mathrm{r}}) = \sum_i V_i \exp\left(j\left[\phi_i - \vec{\mathrm{k}}_i \cdot \vec{\mathrm{r}} - 2\pi f \tau_i\right]\right)$$

The justification for this definition of the local area channel is discussed next.

> *Note: A Multipath Wave*
>
> The term *multipath* is used by wireless engineers to describe the multiple wave components, each traveling different paths, that impinge upon a receiver antenna. Hence, *multi*ple + *path*s = *multipath*. When the term *multipath wave* is used in this work, we are speaking of a single wave term in the local area representation. With this definition, it is possible that a single scatterer may produce more than one *multipath wave*. Therefore, be aware that the *multi* in multipath refers to the number of waves and not the number of scatterers.