

## ECE 6390 Homework 1: Orbital Mechanics

**Numerical Analysis of Orbits:** In class we derived the basic system of differential equations that describe orbits around a single planet in polar coordinates. Recall this pair of equations:

$$\ddot{r} = r\dot{\theta}^2 - \frac{GM_P}{r^2} \quad \ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r}$$

We know from class that the trajectories are an ellipse. If we want to simulate the time-evolution of the orbit, we can discretize this system of differential equations. Below is a method for doing this that is easily extendible to more complicated systems.

First, we choose a uniform time increment  $\Delta t$  to sample our orbit. Since we have two variables,  $(r, \theta)$ , that are both second-order in the differential equation, we can uniquely determine the state of the system with four pieces of information:  $r$ ,  $\dot{r}$ ,  $\theta$ , and  $\dot{\theta}$ . For our discrete simulation, we will characterize the  $n$ th sample with the following pieces of information:  $r_n$ ,  $\Delta r_n$ ,  $\theta_n$ , and  $\Delta\theta_n$ , where we use the following approximations to the time derivatives of  $r$  and  $\theta$ :

$$\frac{\Delta r}{\Delta t} = \dot{r} \quad \frac{\Delta\theta}{\Delta t} = \dot{\theta}$$

With these definitions, we can use the following set of recursive relationships to calculate the next state in time ( $\Delta t$  seconds later) from a previous set of samples:

$$r_{n+1} = r_n + \Delta r_n$$

$$\theta_{n+1} = \theta_n + \Delta\theta_n$$

$$\Delta r_{n+1} = \Delta r_n + \left( \left[ r_n + \frac{1}{2}\Delta r_n \right] \Delta\theta_n^2 - \frac{GM_p}{r_n^2} \Delta t^2 \right)$$

$$\Delta\theta_{n+1} = \Delta\theta_n - \frac{2\Delta r_n \Delta\theta_n}{r_n + \frac{1}{2}\Delta r_n}$$

Now simply define a state 0 that matches your initial conditions and begin calculating successive states in time. If you find that the orbit does not “close up” into a nice ellipse after running your program, it is possible that you have chosen too coarse of a  $\Delta t$ . A value of 5 seconds should work pretty well for most of our orbital problems.

For this homework assignment, write some computer code that plots the orbits that result from the initial conditions listed below. Be sure to record period and eccentricity for each orbit. All computer code must be original. Please print copies of any computer code and attach it to your assignment. (20 points)

- a. Earth Orbit:  $r = 20,000\text{km}$ ,  $\theta = 0^\circ$ ,  $V_r = 0.000\text{km/s}$ ,  $V_\theta = 5\text{km/s}$ .
- b. Earth Orbit:  $r = 20,000\text{km}$ ,  $\theta = 0^\circ$ ,  $V_r = -3.000\text{km/s}$ ,  $V_\theta = 5\text{km/s}$ .
- c. Earth Orbit:  $r = 20,000\text{km}$ ,  $\theta = 0^\circ$ ,  $V_r = -6.000\text{km/s}$ ,  $V_\theta = 5\text{km/s}$ .
- d. Do some internet or library research to track down the orbital specifications of Halley's Comet around the sun. Try to reconstruct this orbit with your simulation program.