ECE 6390 Homework 5: Noise and Digital Coding

Solutions

1.

1. To solve the set of problems in this part, we define the parameters below: $T_{atm,uplink} = 190\ K$, $BW = 6\ MHz$, $f_{uplink} = 6\ GHz$ (typo: if you used 6.2GHz, it's okay), $P_{ES,T} = 55\ dBW$ – these are transmitting earth station parameters.

On satellite, $G_{A1} = 8\ dBi$, $G_{S,LNA} = 20\ dB$, $T_{S,LNA} = 25\ K$, $G_{S,M} = 0\ dB$, $T_{S,M} = 80\ K$, $G_{A2} = 5\ dBi$ where the subscripts M, LNA, $A1$, and $A2$ refer to the mixer, low-noise amplifier, receiving antenna and transmitting antenna respectively on the satellite. Moreover, $P_{PA} = 5\ dBW$ (this is NOT the gain but output power of the *gain-controlled* power amplifier).

On the receiving earth station system, we have $T_{atm,downlink} = 13\ K$, $BW = 6\ MHz$, $f_{downlink} = 3.7\ GHz$, $G_{ES,A} = 35\ dBi$, $G_{ES,LNA} = 30\ dB$, $T_{ES,LNA} = 55\ K$

(a) To find $(C/N)_{dB}$ at the output of the satellite, we note that (C/N) after the gain-controlled power amplifier (PA) is essentially the same before the power amplifier (PA). In other words, the PA does not really affect the (C/N) ratio. We break the carrier power in stages: P_1 is before the LNA on the satellite, P_2 is the carrier power before the mixer, and P_3 is the carrier power after the mixer (before the PA). Thus:

$$\begin{aligned} P_1 &= P_{ES,T} + G_{A1} - 20log_{10}\left(\frac{4\pi}{\lambda}\right) - 20log_{10}(r) \\ &= 55 + 8 - 20log_{10}\left(\frac{4\pi}{\left(\frac{3\times10^8}{6\times10^9}\right)}\right) - 20log_{10}(35786\times10^3) \\ &= -136.08dBW \end{aligned} \tag{1}$$

$$P_2 = P_1 + G_{S,LNA} = -116.08\ dBW$$

$$P_3 = P_2 + G_{S,M} = -116.08\ dBW = C_{dB}$$

Next, we find the total equivalent noise temperature on the satellite system:

$$\begin{aligned} T_s &= T_{ATM,uplink} + T_{S,LNA} + \frac{T_{S,M}}{G_{LNA}} \\ &= 190 + 25 + \frac{80}{10^{\left(\frac{20dB}{10}\right)}} \\ &= 215.80\ K \end{aligned} \tag{2}$$

Noise power becomes:

$$\begin{aligned} N_{dB} &= G_{S,LNA(dB)} + G_{S,M(dB)} + k_{dB} + BW_{dB} + T_{dB} \\ &= 20 + 0 + 10log_{10}(1.38 \times 10^{-23} J/K) + 10log_{10}(6 \times 10^6) + 10log_{10}(215.8) \\ N_{dB} &\approx -117.48dBW \end{aligned} \tag{3}$$

Therefore, $(C/N)_{dB}$ at the output of the satellite $= C_{dB} - N_{dB} = -116.08 - (-117.48) = 1.40dB$

(b) At the receiving ES, we do a similar calculation to part (a) above:

$$\begin{aligned} C_{dB} &= P_{PA} - 20log_{10}\left(\frac{4\pi}{\lambda'}\right) - 20log_{10}(r) + G_{A2} + G_{ES,A} + G_{ES,LNA} \\ &= 5\ dBW - 20log_{10}\left(\frac{4\pi}{\left(\frac{3\times10^8}{3.7\times10^9}\right)}\right) - 20log_{10}(35786 \times 10^3) + 5\ dBi + 35\ dBi + 30\ dB \\ C_{dB} &\approx -119.88\ dBW \end{aligned} \tag{4}$$

The equivalent noise temperature here becomes:

$$\begin{aligned} T_E &= T_{atm,downlink} + T_{ES,LNA} \\ &= 13 + 55 = 68\ K \end{aligned} \tag{5}$$

Therefore noise power N_1 *due to temperature* is:

$$\begin{aligned} N_{1(dB)} &= G_{ES,LNA(dB)} + k_{dB} + T_{E(dB)} + BW_{dB} \\ &= 30 + 10log_{10}(1.38 \times 10^{-23}) + 10log_{10}(68) + 10log_{10}(6 \times 10^6) \\ &= -112.49\ dBW \end{aligned} \tag{6}$$

However, we also have noise, N_2 propagated due to the satellite system (after the PA). From class, we know that the gain-controlled PA basically does not affect the (C/N) at the satellite's output. To get the noise after the PA, we have:

$$(C/N)_{dB} = P_{PA} - N_{afterPA}$$

Therefore, $N_{afterPA} = P_{PA} - (C/N)_{dB(from.part.(a))} = 5\ dBW - 1.40\ dBW = 3.6\ dBW$. Noise travelling to earth station results in:

$$\begin{aligned} N_{2,dB} &= N_{after_PA} + G_{A2} + G_{ES,A} + G_{ES,LNA} - 20log_10(4\pi\lambda') - 20log_{10}(r) \\ &= -121.48\ dB \end{aligned} \tag{7}$$

Therefore, total noise, $N_{dB} = 10log_{10}\left(10^{N_{1(dB)}/10} + 10^{N_{2(dB)}/10}\right) = 10log_{10}(6.348\times 10^{-12}) \approx -111.97\ dBW$

Thus, $(C/N)_{dB} = C_{dB} - N_{dB} = -119.88 - (-111.97) \approx -7.9\ dB$

(c) The weakest link is the downlink. This is seen from total noise power as well as (C/N) at ES.

(d) Here are some of the ways to improve the (C/N): (i) reduce bandwidth for both uplink and downlink; (ii) get LNA and MIXERs with lower noise temperature; (iii) reduce distance to orbit (non-geosynchronous), r; and (iv) reduce carrier signal frequency for uplink and/or downlink.

(e) Prior calculations remain same for the satellite. *carrier signal = same as in part (b)* Noise temperature, $T_N = 10000 + 55 = 10055\ K$ and N_2 remains same as in (b). The change is in N_1 as shown below:

$$\begin{aligned} N_{1(dB)} &= G_{ES,LNA} + k_{dB} + T_{N(dB)} + BW_{dB} \\ &= -112.49 - 10log_{10}(68) + 10log_{10}(10055) \\ &= -90.79\ dBW \end{aligned} \tag{8}$$

Therefore, total noise $N = 10log_{10}\left(10^{N_{1(dB)}/10} + 10^{N_{2(dB)}/10}\right) \approx -90.786\ dBW$. Thus, the new carrier to noise ratio is $(C/N)_{dB} = -119.88 + 90.79 = -29.09\ dB$.

2. Answer:

1. Online eferences used were `http://web.usna.navy.mil/~wdj/reed-sol.htm` and `http://www.united-trackers.org/resources/DAB/sampling.htm` From these sources, the following information about a CD could be made: Sampling rate $= 44.1kHz$ at 16 bits/channel with 2 channels. This means that the bit rate is:

$$\begin{aligned} bit\_rate &= 44.1 \times 1000 \frac{samples}{sec} \times 16 \frac{bits}{channel} \times 2 \frac{channel}{sample} \\ &= 1,411,200 bits/sec \end{aligned}$$

Now, in CD data encoding, there are several redundancies which include a couple of Reed-Solomon (RS) coding and parity checks. Taking these redundancies into account, the actual data rate (information) is:

$$actual\_data\_rate = 150kB/sec = 150 \times 1024 \times 8 = 1,228,800 bits/sec$$

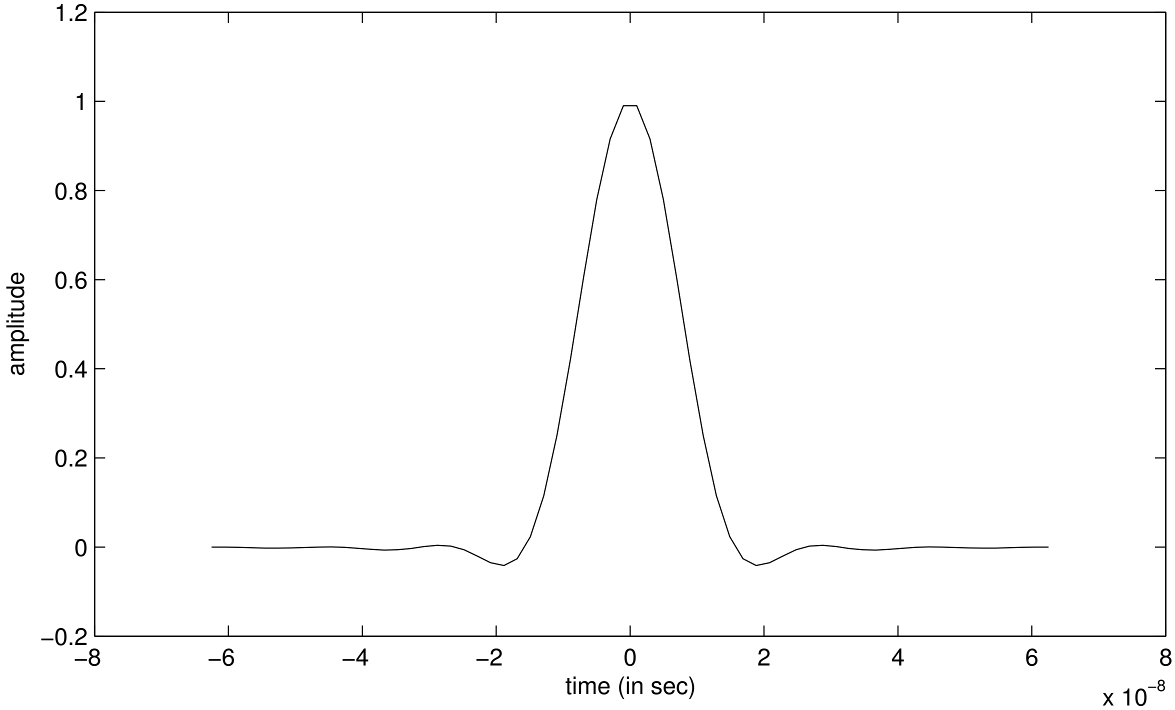
SNR due to quantization noise = 96 dB
The maximum minutes of play that can be obtained using the given information above and the fact that a CD capacity is 700 MB is:

$$\begin{aligned} min\_of\_play &= \frac{700MB}{150kB/sec} \times \frac{1min}{60sec} \\ &= \frac{700 \times 2^{20}}{150 \times 2^{10}} \times \frac{1min}{60sec} \\ &= \frac{70}{15} \times \frac{2^{10}}{60} min = 79.64mins \\ &\approx 80mins \end{aligned}$$

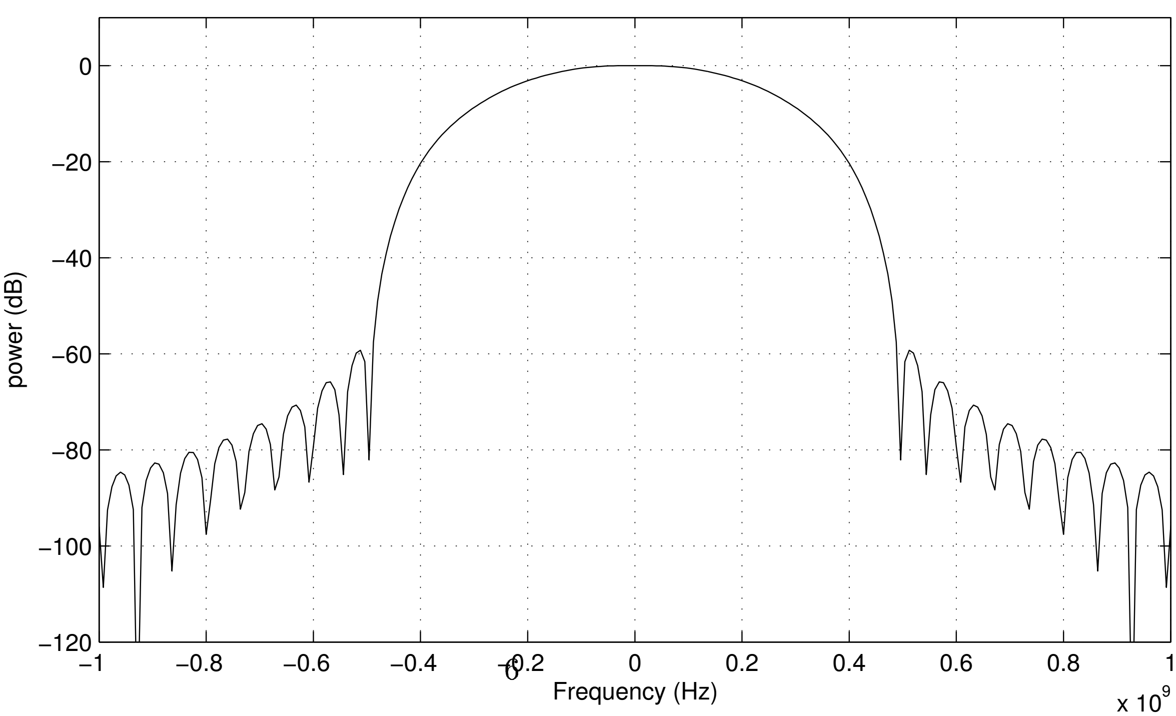
Any values from around 60 to 90 mins is correct as long as your calculation is consistent.

3. There are a number of good pulses that satisfy this design criteria. One example is a raised cosine pulse with 32 Msamples/sec and a roll-off factor of 0.9. Sample plots are included on the following pages.

```
% Solution to part 2 of HW #5
% Using a raised cosine pulse or root raised cosine pulse solves this
% problem
close all, clear all
f0 = 32e6;   % pulse signal frequency
alpha = 0.9; % roll-off factor
% bear in mind that we only use 4 clock cycles
t1 = -2/f0; % first sample
t2 =  -t1; % second sample

N = 512;    % number of time-domain samples

t = linspace(t1,t2,64);       % creates a time-domain axis
f = (-N/2:N/2-1)/(t2-t1);    % creates a frequency-domain axis

x = (sin(2*pi*f0*t)./(2*pi*f0*t)).*(cos(2*pi*alpha*t*f0)./(1-(4*alpha*f0*t).^2)); % raised co

subplot(2,1,1)
plot(t,x)
xlabel('time (in sec)'), ylabel('amplitude')

X = fft(x,N);
XX = abs(fftshift(X))*(t2-t1)/N;  % shifts/scales FFT output for plotting
XX_new = 20*log10(XX/max(XX));
subplot(2,1,2)
plot(f,XX_new), axis([-1e9 1e9 -120 10]), grid on
xlabel('Frequency (Hz)'), ylabel('power (dB)')
```



1.2
1
0.8
0.6
0.4
0.2
0
−0.2
amplitude
−8
−6
−4
−2
0
2
4
6
8
time (in sec)
x 10^−8



0
−20
−40
−60
−80
−100
−120
power (dB)
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
Frequency (Hz)
x 10^9